Daylight Imaging in $V(x, y, z)$ Media

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ABSTRACT

Previous authors have tried to image seismic reflectivity by crosscorrelating passive seismic data, and treating the resultant correlograms as active source seismograms. We provide a mathematical framework for working with passive seismic crosscorrelograms that is both appropriate for $V(x, y, z)$ media, and arbitrary source location. Under this framework, correlograms can be migrated with an imaging condition that is tuned to image particular events. For example, tuning the imaging condition to the kinematics of the Direct-Direct correlation event allows direct imaging of the seismic sources. Similarly, tuning to the Direct-Ghost correlation event allows imaging of subsurface reflectivity. Numerical results with synthetic data partly verify the effectiveness of crosscorrelation migration, but also suggest worse resolution of the image compared to standard Kirchhoff migration.

INTRODUCTION

Passive seismic imaging can be split into two categories: firstly, attempts to image the spatial locations of passive seismic sources themselves, and secondly, attempts to image the subsurface reflectivity that is illuminated by passive seismic energy.

Passive seismic source imaging

Passive seismic source imaging has the unique potential to provide direct measurements of subsurface permeability (e.g. Shapiro et al., 1999). Fluid flow
causes fracturing; you image the fracturing; therefore, you are imaging the fluid flow. This, along with the growth of (both surface and borehole) time-lapse seismic, has led to the drive towards the “electric oilfield” permanently instrumented and continually monitoring itself (Jack and Thomsen, 1999).

To date, however, most of the published case studies of microseismic fracture imaging rely on earthquake-style hypocentral event triangulation. For example, Maxwell et al. (1998) describes the successful application of such technology to the Ekofisk field in the North Sea. These approach require automated event picking algorithms, and may run into problems if microseismic events are not localized in time.

**Reflectivity imaging with passive seismic energy**

Baskir and Weller (1975) describe possibly the first published attempt to use passive seismic energy to image subsurface reflectivity. They briefly describe crosscorrelating long seismic records to produce correlograms that could be processed, stacked and displayed as conventional seismic data. Unfortunately their field tests seem to have been inconclusive.

Dating from about the same time, an exercise in Claerbout’s first book (1976) asks the reader to prove that the temporal autocorrelation of a transmission seismogram for a layered model with a source deep underground is equivalent to a reflection seismogram. This may have inspired his conjecture that by crosscorrelating two passive traces, we can create the seismogram that would be computed at one of the locations if there was a source at the other. Cole (1975) attempted to verify this conjecture with data collected using a 4000 channel 2-D field array on Stanford campus. Unfortunately, again, possibly due to the short (20 minute) records or bad coupling between the geophones and the dry California soil, his results were inconclusive.

Following Cole’s work, Rickett and Claerbout (1996) generated synthetic data with phase-shift method. Their earth reflectivity models consisted of (both flat and dipping) planar layers and point diffractors embedded in a $v(z)$ velocity function, and illuminated by random plane waves from below. They generated both *pseudo shot gathers* (by crosscorrelating one passive trace with many others nearby), and *pseudo zero-offset sections* (by autocorrelating many traces). In these crosscorrelated domains, the kinematics for both point diffractors and planar reflectors, were identical to those predicted for real shot gathers and zero-offset sections. Rickett and Claerbout (1999) then experimented with moving
the passive source location close to the receivers and reflectors, and included modeling with a $v(x, z)$ velocity model. He observed that these changes did indeed affect the kinematics of the correlograms; however, changes were small, and would probably not cause the method to fail in most situations.

The idea that a reflection seismogram could be created by crosscorrelating two passive seismic records was rediscovered independently by the helioseismologists (Duvall et al., 1994), who created time-distance curves by cross-correlating passive solar dopplergrams recorded by the Michelson Doppler Imager (Scherrer et al., 1995). Point-to-point traveltimes derived from these time-distance curves could then be used in a range of helioseismic applications (e.g. Giles et al., 1997 and Kosovichev, 1999). If helioseismic time-distance curves are averaged spatially, the result is equivalent to a multi-dimensional autocorrelation. Rickett and Claerbout (2000) demonstrated that multi-dimensional spectral factorization provides spatially averaged time-distance curves with more resolution than those calculated by autocorrelation. Their demonstration was restricted to layered models with no lateral velocity variation.

Extending daylight imaging

In this report, the daylight imaging method is represented mathematically and extended to image arbitrary reflectivity and source distributions in $v(x, y, z)$ media. We believe this is the first theoretical formulation of daylight imaging in 2-D and 3-D media. Seismic traces are temporally crosscorrelated to form pseudo-shot gathers. These gathers are then migrated according to a crosscorrelogram migration operator (Schuster, 1999). The migration operator can be tuned to image either the source locations, or the subsurface reflectivity distribution. Simple synthetic examples are given to clarify the benefits and limitations of this daylight imaging procedure. A special case of crosscorrelation migration is that of autocorrelation migration (Schuster et al., 2000 and Yu et al., 2000), except that now the source location does not need to be known.

THEORY OF DAYLIGHT IMAGING

We will now describe how to image either the source distribution or the reflectivity distribution from passive seismic data in a $v(x, y, z)$ medium. The sources are assumed to be distributed anywhere in space, and the time histories of each
source are assumed to be uncorrelated. Neither the source location or time history are known. Without loss of generality we conveniently assume one source and one scatterer, but the resulting migration formula are also applicable to multiple sources and scatterers that emulate reflector interfaces.

The high-frequency Green’s function for a single point scatterer at $x_o$ (see Figure 1) in a smoothly varying velocity distribution and bandlimited by the source spectrum $F(\omega)$ is given by

$$
G(\vec{r}_g'|\vec{r}_s)F(\omega) \approx \text{Direct}_{g'} + \text{Primary}_{g'} + \text{Ghost}_{g'},
$$

where $G(\vec{r}_g'|\vec{r}_s)$ is the Green’s function for an impulsive source at $\vec{r}_s$ and a geophone at $\vec{r}_g'$. The first term on the RHS represents the direct arrival (see top figure in Figure 1), the second term represents the scattered field excited by the direct arrival, and the third term represents the scattered arrival that is generated by a specular free-surface reflection (see middle figure in Figure 1). Here, $R$ is the scattering coefficient, $\tau_{gg'}$ is a solution to the eikonal equation for a source at $g$ and a receiver at $g'$, and the geometrical-spreading factors have been harmlessly dropped. The specular-reflection point on the free surface is denoted by $g''$ and the location of the recording geophone is denoted by $g'$. To eliminate the phase of the unknown source spectrum we take the crossproduct of the data $G(\vec{r}_g'|\vec{r}_s)F(\omega)$ recorded at $\vec{r}_g'$ with $G(\vec{r}_{g_o}|\vec{r}_s)^*F(\omega)^*$ measured at $\vec{r}_{g_o}$ to get

$$
G(\vec{r}_g'|\vec{r}_s)G(\vec{r}_{g_o}|\vec{r}_s)^* \approx \text{Direct}_{g'} \text{Direct}_{g_o}^* + \text{Ghost}_{g'} \text{Direct}_{g_o}^* + \text{other terms},
$$

where the first expression on the RHS corresponds to the correlation of the conjugate direct wave recorded at geophone $g_o$ with the direct wave at $g'$; and the second term corresponds to the correlation of the conjugate direct wave at $g_o$ with the ghost reflection recorded at $g'$. We conveniently assume that the source spectrum magnitude is equal to the value one for all frequencies. The other terms correspond to the other correlations which will not be needed for imaging, and are presumed to cancel upon migration. This last assumption about cancellation is similar to the standard wishful assumption in migration,
i.e., all migrated arrivals incoherently superimpose except those that are tuned to the specified migration imaging condition. For the above equation, we will tune the imaging conditions so that either the $Direct_{g_o}^* Direct_{g'}$ terms are migrated to image the source distribution, or the $Direct_{g_o}^* Ghost_{g'}$ terms are migrated to image the reflectivity distribution.

Source location imaging.
To image the unknown source location at $\vec{r}_s$ from the data given in equation 2, we simply identify the migration operator which, when applied to the above equation, cancels the phase of the $Direct_{g_o}^* Direct_{g'}$ term. Such a migration operator is given by

$$e^{-i\omega[\tau_{s'g'} - \tau_{s'g_o}]}$$

(3)

where $s'$ denotes the trial source-point location. Application of this migration operator to the crosscorrelated data in equation 2 will annihilate the phase of the $Direct_{g_o}^* Direct_{g'}$ term when the trial image point $\vec{r}_{s'}$ coincides with the actual source location denoted by $\vec{r}_s$. The migration section is then given by summation over all geophone pairs

$$m(\vec{r}_{s'}) = \sum_{g_o} \sum_{g'} G(\vec{r}_{s'} | \vec{r}_{g'}) G(\vec{r}_s | \vec{r}_{g_o}) e^{-i\omega[\tau_{s'g'} - \tau_{s'g_o}]}$$

(4)

The migration operator in equation 3 is "tuned" to image the source location so that as $\vec{r}_{s'} \rightarrow \vec{r}_s$, the $Direct_{g_o}^* Direct_{g'}$ term will constructively interfere while the all other terms tend to cancel.

Reflectivity distribution imaging.
To image the unknown scatterer location and strength at $\vec{x}_o$, we simply identify the migration operator which, when applied to the crosscorrelated data in
Figure 1: (Top) Direct ray and a (middle) scattered ray excited by a specular free-surface reflection associated with a source at $s$ and a scatterer at $x_0$. Bottom figure denotes the rays associated with the migration operator for free-surface reflections.
equation 2, cancels the phase of the $\text{Direct}_{g_0} \ast \text{Ghost}_{g'}$ term. Such a migration operator is given by

$$e^{-i\omega(\tau_{g_0x}+\tau_{xg'})}$$

(5)

where $x$ and $g_0$ denote, respectively, the trial scatterer and specular-reflector point locations. Application of this migration operator to equation 2 will annihilate the phase term of the $\text{Direct}_{g_0} \ast \text{Ghost}_{g''}$ term in equation 2 when the trial image point $\bar{x}$ coincides with the actual source location denoted by $\bar{x}_o$, i.e.,

$$\bar{x} \rightarrow x_o,$$

(6)

and when the trial specular-reflection point $\bar{r}_{g_0}$ coincides with the actual specular-reflection point at $\bar{r}_{g''}$

$$\bar{r}_{g_0} \rightarrow \bar{r}_{g''}.$$  

(7)

This can be understood more clearly by noting that the migration section is given by summation of the migrated data over all geophone pairs

$$m(\bar{x}) = \sum_{g_0} \sum_{g'} G(\bar{r}_{g'}|\bar{r}_s)G(\bar{r}_{g_0}|\bar{r}_s)^* e^{-i\omega[\tau_{g_0x}+\tau_{xg'}]},$$

$$= -R \sum_{g_0} \sum_{g'} \left[ e^{-i\omega[\tau_{g''}+\tau_{g'x_0}+\tau_{xg''}-\tau_{gox_0}-\tau_{xg'}]} + \text{all other terms} \right].$$

(8)

If one of the summation’s trace indices $g_0$ coincides with the actual specular reflection point at $g''$, then for $\bar{x} \rightarrow x_o$ and $g_0 = g''$ the above exponent goes to zero, leading to constructive interference at the scatterer’s location. The $g_0 = g''$ condition will be satisfied if the specular-reflection point is within the aperture of the recording array and the geophone array is sufficiently dense. The stationary phase justification for migrating free-surface reflections in crosscorrelograms is given in the appendix.

**NUMERICAL RESULTS**

Synthetic data will be generated to test the feasibility of imaging the location of unknown sources and of imaging the reflectivity distribution from free-surface
Figure 2: Specular ghost-reflection rays associated with a source at \( s \), a trial image point at \( x \) (dashed rays), and a scatterer at \( x_o \) (solid rays). The rays intersect points the actual specular-reflection point \( g'' \) or the trial specular point \( g_o \). Note, only the specular rays are allowed by the stationary phase analysis in the appendix.

Figure 3: As \( x \) approaches \( x_o \) then \( g_o \) approaches \( g'' \) in Figure 2; thus, the dashed rays will coincide with the solid rays. Therefore, the *Direct Ghost* correlation term in equation 8 (or equation 12) will have zero phase, leading to maximal constructive interference of the migration section at the actual scatterer point.
multiples. The first example imitates the scenario where a fluid is injected into a medium to open cracks, and the goal is to use passive seismic data to image the location of the opened cracks. The second example approximates the situation where the reflectivity distribution is imaged from seismic data generated by a drill bit with unknown location.

**Imaging the Location of Seismic Sources**

Figure 4a shows synthetic data generated for a point exploder centered 1050 m below a 2100 m wide array. There are 70 geophones in the array with a geophone spacing of 30 m. The traces are computed for 1 second of duration with a 30 Hz Ricker wavelet source. The point scatterer response of the Kirchhoff operator and the crosscorrelation migration operator in equation 3 are shown in Figures 4b and 4c, respectively. Note, the crosscorrelation image of the point scatterer is smeared over a larger depth range than that of the Kirchhoff image. This is because the crosscorrelation of one trace with another smears the source wavelet into a longer wavelet, and also because the crosscorrelation operator has poor resolution in the depth direction. Nevertheless, the crosscorrelation point-scatterer image is acceptable.

In practice, the trace at the master trace and its two nearest neighbors were muted because the direct wave migration operator in equation 3 has zero or nearly zero phase when \( g \approx g' \). This is undesirable because any energy from these traces will be smeared uniformly throughout the model, not just at the exploder points. Also, a second derivative in time was applied to the crosscorrelation traces to partly compensate for the smoothing effects of crosscorrelation and migration.

Figure 5 is the same as Figure 4 except the source wavelet is a long random time series. The crosscorrelation of traces collapses the ringy time series to an impulse-like wavelet so that the associated migration image in Figure 5c has good spatial resolution compared to the Kirchhoff image in Figure 5b.

In the previous examples, the scatterer exploded at time zero. Now, there are ten scatterers and all are assumed to explode at random times with a random time series as a source wavelet. The resulting data for 1 second is shown in Figure 6a. Figure 6b shows these data after crosscorrelation migration of 1 second of data, and roughly locates the location of the 10 point exploders. Repeating this crosscorrelation migration for fifteen data sets, each with 1 second of data generated from ten point scatterers with distinct random time histories,
Figure 4: (Top) Synthetic 30 Hz data generated by an impulsive-like point exploder (*) at a depth of 1050 m. The point exploded at time zero. (Middle) Kirchhoff migration image, and (bottom) crosscorrelation migration image. The Kirchhoff image is better resolved partly because temporal crosscorrelation of traces will broaden the wavelet.
Figure 5: Same as previous figure, except a long random time series is used for the source wavelet that is excited at time zero. Note, that the crosscorrelation of traces collapses the ringy source wavelet into an impulsive-like wavelet, leading to a better resolved migration image in the crosscorrelogram image.
yields the stacked images in Figure 6c and Figure 7. As expected, averaging
the migration images tends to cancel migration noise and reinforce the energy
at the location of the point exploders.

Finally, the fault-like structure denoted by stars in Figure 8 is assumed to
emanate seismic energy randomly in time with random strength. This might
approximate the situation where fluid is injected along a reservoir bed and
seismic instruments are passively monitoring the location of the injection front.
Figure 8 shows the results after crosscorrelation migration of (middle) 1 second
of data and (bottom) 40 stacks of 1 second records. The fault boundaries are
much better delineated in the 40-stack migration image, although the resolution
is much worse than that of an ordinary seismic survey. Figure 9 shows detailed
contour images of these migration images.

Poor resolution of the crosscorrelation images is consistent with the poor
resolution predicted by the crosscorrelation migration impulse response shown
in Figure 10. A possibility for improving resolution is to measure the incidence
angle of energy in the crosscorrelograms and use this angle as a constraint in
smearing data into the model. This strategy is similar to that of ray-map
migration, but it remains to be seen if this is a practical strategy with crosscor-
relograms.

Imaging the Reflectivity from Free-surface Reflections

A single impulsive source is located somewhere in depth, and the synthetic
data generated in a 4-layer channel model is shown at the top of Figure 11.
The interface boundaries are delineated by the dash-dot lines in the bottom
figure, including the semi-circle river channel along the third interface. Using
the migration operator for ghost reflections given by equation 5, the data are
migrated to give the image at the bottom of Figure 11. Note, the source location
was unknown, represented by the star symbol in the left bottom part of the
migration image. It might be surprising that the single source generates enough
data so that the model is almost entirely imaged. Part of the reason for this
is that the ghost reflections illuminate a much greater part of the medium (for
a fixed recording array) than primary reflections alone. Each point on the free
surface acts as a virtual source.

Finally, ten point sources are buried at intermediate depths and their emis-
sions are recorded on the surface. Each source wavelet is governed by a distinct
random time series Applying the migration operator for free-surface reflections
Figure 6: Similar to previous figure, except the source wavelets of ten point exploders (*) are generated by a random number generator. The middle figure shows the crosscorrelogram image computed from 1 second of data, while the bottom images shows the result after 15 stacks of 1-second data. The stacked image is better resolved because stacking tends to cancel noise and reinforce migration energy at the point exploder locations.
Figure 7: Similar to previous figure except the migration images are contoured.

(equation 5) to the data shown at the top of Figure 12 yields the result shown in the bottom figure. Fifty one-second records were migrated and stacked with one another, and show that the sand channel boundary is well imaged.

**DISCUSSION**

A new methodology is presented for using daylight data to image source locations or reflector boundaries for $v(x, y, z)$ media. Traces are crosscorrelated in time to form the correlation kernel in equation 2, this kernel is weighted by the appropriate migration operator, and summation over the $g$ and $g'$ indices is carried out to give the migrated image. Our theoretical formulas for free-surface migration partly validate the conjecture: a crosscorrelogram is a trace that can be migrated in the same fashion as a trace generated by a source at $g$ and a receiver at $g'$. In this case, the direct waves correlated with free-surface reflections play the role of primary reflections in the crosscorrelograms.

Other possibilities can be explored using the mathematical formalism in this paper. It is straightforward to develop a non-linear inverse methodology by using crosscorrelation misfit functions and obtaining its gradient. It is also obvious from this formulation that an automatic way of migrating free-surface reflections is now available. Can we usefully use this to simultaneously migrate
Figure 8: (Top). Synthetic 30 Hz data generated by 55 point exploders located along a fault-like boundary. The points exploded at random times with random weighting amplitudes. (middle) Crosscorrelation migration image obtained from 1 second of data, and (bottom) crosscorrelation migration image after 40 stacks of 1-second data. The stacked image appears to be less noisy and a better approximation to the fault geometry delineated by stars.
Figure 9: Similar to previous figure except the migration images are contoured, and the fault boundary is delineated by a dashed line.
Figure 10: Isotime contours (in seconds) of the impulse response of the (top) crosscorrelation migration and (bottom) prestack Kirchhoff migration operators. The + and * symbols represent the locations of the source and receiver, respectively, where the source location for the crosscorrelograms is the same as the master trace. The crosscorrelation operator is dominated by nearly vertical contours, so its resolution should be poorest in the vertical direction.
Figure 11: Similar to previous figure except now a 4-layer sand channel model is imaged for one buried source. The source time history is a 30-Hz Ricker wavelet.
Figure 12: Similar to previous figure except there are now ten buried point sources scattered about at intermediate depths. Each source is governed by a distinct random time series.
free-surface reflections and primary reflections in CDP data? We hope to find out at the annual UTAM meeting in February, 2001. Along these lines, Yi Luo of Aramco highlighted this important possibility and has obtained some interesting but preliminary results with simple synthetic data.

It is interesting to note the analogy of inverting correlograms with that of constructing holograms. Light intensity images are used to reconstruct holograms, which are images of the object polluted by noisy interference terms. Seismic correlograms can also be used to reconstruct images of the object, but the other terms in equation 2 suggest that the data are polluted by noisy interference terms. However, we can design migration operators to reconstruct the object function (e.g., reflectivity) or information from the interference terms (e.g., source locations). As in holography, the challenge with seismic crosscorrelograms is to reduce the noise from the interference terms and enhance the signal from the events tuned to the migration operator.

References


Rickett, J., 1996, The effects of lateral velocity variations and ambient noise source location on seismic imaging by cross-correlation: SEP 93, 137-150.


**APPENDIX: Stationary Phase Approximation**

To mathematically justify, in a stationary-phase sense, the migration of free-surface reflections, we can apply the free-surface reflection migration operator

\[ e^{-i\omega(\tau_{gox} + \tau_{g'x'})}, \]  

(9)

(see bottom illustration in Figure 1) to the correlated data \( \Phi_{go,g'} = G(\vec{r}_{go} | \vec{r}_s)G(\vec{r}_{g'} | \vec{r}_s)^* \)
and integrate over \( g' \) traces and \( go \) shots to get

\[ m(\vec{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{go,g'} e^{-i\omega(\tau_{gox} + \tau_{g'x'})} dg_{o} dg_{g'}, \]  

(10)

and substituting in \( \Phi_{go,g'} = G(\vec{r}_{go} | \vec{r}_s)G(\vec{r}_{g'} | \vec{r}_s)^* \) in equation 2 we get

\[ = -R \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\omega(\tau_{gox} - \tau_{sxo} + \tau_{g'x'} + \tau_{syo} - \tau_{goz} - \tau_{g'z'})} dg_{o} dg_{g'} + \text{other terms}. \]  

(11)

Here \( m(\vec{x}) \) denotes the migration image, and \( x \) denotes the trial image point.

At high frequencies we can apply the stationary phase approximation to the integrals to get

\[ m(\vec{x}) \approx -Re e^{i\omega(\tau_{gox} - \tau_{syo} + \tau_{goz} - \tau_{sxo} + \tau_{g'x'} - \tau_{syo} - \tau_{goz} - \tau_{g'z'})}, \]

\[ = -Re e^{i\omega(\tau_{gox} - \tau_{syo} - \tau_{goz} - \tau_{sxo} + \tau_{xgo} - \tau_{g'x'} + \tau_{syo} - \tau_{goz} - \tau_{g'z'})}, \]  

(12)

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where other terms is ignored, and the superscript \( \ast \) denotes the stationary phase values of the traveltimes. These stationary phase values correspond to the specular rays shown in Figure 2 where Snell’s law is satisfied for the free-surface reflections at \( g'' \) and \( g_o \).

Note, the specular rays (dashed rays in Figure 2) associated with the stationary phase approximation will coincide with the actual rays (solid lines) when the image point at \( x \) coincides with the actual scatterer location at \( x_o \), as demonstrated in Figure 3. Thus, the time terms in the exponent of equation 12 will cancel one another leading to constructive interference of migrated free-surface reflections at the scatterer location. Conversely, if the image point \( x \) is not coincident with \( x_o \) then the solid rays do not coincide with the dashed rays in Figure 2, leading to mostly destructive superposition of migrated free-surface reflections away from the actual scatterer location.

The above analysis suggests that we can roughly think of the correlation kernel \( \Phi_{gg'} \) as a trace generated by a source at \( g \) and recorded at \( g' \). In this case, the direct waves correlated with free-surface reflections play the role of primary reflections in the crosscorrelograms. The migration image of the point scatterer is obtained by a weighted spatial correlation of the crosscorrelograms, where the weights are the migration operators for a source at \( g \) and a receiver at \( g' \). This is the correlation equivalent of prestack migration, while weighting the correlation kernel \( \bar{\Phi}_{gg} \) and summing over the \( g \) index is the correlation equivalent of poststack migration.

Note that the migration operator does not depend on the source position or the scatterer location or depth so this procedure also applies to data generated by a random distribution of sources and a medium with many scatterers. It is straightforward to append a summation over scatterers to generalize this procedure to arbitrary reflector boundaries. A similar stationary phase analysis can be applied to the migration operator associated with the problem of determining the source location.