SEISMIC ARRAY THEOREM AND 
OPTIMAL SURVEY DESIGN

by

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ABSTRACT

The acquisition footprint noise in migrated sections consists of migration artifacts associated with a discrete recording geometry. Such noise can corrupt the interpretation of seismic sections for Amplitude Variation with Offset (AVO) studies and enhanced oil recovery operations. I show that the point scatterer response of the farfield Kirchhoff migration operator, which reveals the acquisition footprint noise, is proportional to the stretched Fourier transform of the source and geophone sampling function. Using the Array theorem developed by Optical/Electrical Engineers, the Fourier transform for an orthogonal recording geometry can be quickly calculated by a concatenation of 1-D analytical functions. I use the so-called Seismic Array theorem to rapidly calculate the acquisition footprint noise for different seismic surveys. The results of numerical tests for monochronic and transient sources show that the Seismic Array theorem image is a good approximation to the Kirchhoff migration image. By a rapid trial and error procedure, I use this theorem to determine the best shooting geometries from a variety of seismic survey geometries. Because of the high computing efficiency of the Seismic Array theorem, I also present a constrained optimization method that seeks to identify the optimal survey design. In this procedure, the aperture size and number of traces are constrained and the optimization algorithm searches for the receiver and source spacings that minimize the alias energy of the acquisition footprint. Examples with actual field data geometries show a rapid convergence to the optimal survey geometry, and this geometry results in a significantly reduced acquisition footprint relative to that for the starting geometry.
For my family.
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CHAPTER 1

INTRODUCTION

The "acquisition footprint" is the numerical noise introduced into the migration section by the discrete sampling of wavefields measured over a finite-width aperture. Such noise can seriously degrade the interpretation of three dimensional (3-D) seismic images, especially for AVO studies that interpret horizontal slices of 3-D migration sections. The noise can be quite troublesome if traces are severely aliased in at least two of the four sampling coordinates, which is typical of 3-D land data.

The 1996 SEG workshop on 3-D Seismic Sampling addressed the acquisition footprint problem, where a variety of speakers presented different sampling strategies and their effects on the migrated section, theoretical formulae and their implications for resolution, and field study results. In the final analysis, everyone agreed that the ultimate remedy to the acquisition footprint problem is to sample at large apertures and unaliased station intervals. In practice, however, cost considerations force many surveys to undersample the wavefields in at least two of the four sampling coordinates for geophones \((x_g, y_g)\) and sources \((x_s, y_s)\). So, how do we design a cost-effective survey that minimizes the acquisition footprint problem?

In this thesis I present simple formulae that can be used to compute the acquisition footprint, i.e., the point scatterer response of the farfield migration operator for a given sampling strategy. The migration energy away from the point scatterer location is considered to be the characteristic signature of the acquisition footprint. The formulae show that the farfield acquisition footprint in migration sections is proportional to the stretched Fourier transform of the source and geophone sampling function. Using the Seismic Array theorem (SAT), this Fourier transform for orthogonal shooting and recording grids can be quickly calculated by a concatenation of 1-D analytical functions. I also use the SAT to rapidly compute not only the point scatterer responses for a monochromatic sources, but also those for transient sources and the responses of irregular arrangements scatterers.

In Chapters 3 and 4, I demonstrate the use of the SAT in designing survey geometries
with the weakest acquisition footprint. Two design strategies are used: trial and error analysis and a constrained optimization method. Examples of survey geometries used by ARCO and Phillips Oil Co. are used to illustrate the first method, i.e., rapid trial and error calculations of the acquisition footprints allows us to identify the best seismic survey from a given set of surveys. A more efficient approach is to automatically seek, by nonlinear optimization, the best survey parameters by minimizing the alias energy of the acquisition footprint. Examples with both point scatterer and meandering river channel models show a rapid convergence to the optimal survey geometry. Thus, the exploration geophysicist now has, for perhaps the first time, a set of tools to rapidly design 3-D seismic surveys with minimal acquisition footprint noise.
CHAPTER 2

SEISMIC ARRAY THEOREM

In this chapter, I use the formula of the diffraction stack migration to develop the SAT which can be used to rapidly calculate the 3-D migration image of a point scatterer. The SAT says that the migrated image of a point scatterer is a product of two 2-D Fourier transforms, and in the case of an evenly sampled orthogonal array can be represented as an analytical formula. After that, I will use two numerical tests to verify the correctness of the SAT. At last, this theorem is used to calculate the point scatterer response for a transient source, and calculate the migration image for an irregular set of scatterers that represents a meandering stream.

2.1 Migration Imaging Formula

The migration image $m(x)_{mig}$ can be represented by a weighted linear combination of the reflectivity distribution $m(x)$:

$$m(x)_{mig} = \int G(x \mid x')m(x')dx',$$

where $G(x \mid x')$ is the point scatterer response of the migration operation, or the migration Green's function (Schuster, 1997). In other words, if seismic data are recorded over a point scatterer model then $G(x \mid x')$ is the resulting image at $x$ after migrating these data.

For a point scatterer $m(x') = \delta(x_0 - x')$ equation 2.1 becomes

$$m(x)_{mig} = G(x \mid x').$$

In this case, the ideal image is $\delta(x - x_0)$, but in practice it is a smeared delta function surrounded by aliasing artifacts. These artifacts represent the acquisition footprint noise introduced by the discrete acquisition geometry. Therefore, it seems reasonable to define the best survey geometry as the one which has the weakest alias artifacts in $G(x \mid x_0)$.

The acquisition footprint will now be evaluated for a point scatterer centered at a depth of $z_0$ beneath a 3-D recording and shot pattern (see Figure 2.1). The background
Point Scatterer Model

Figure 2.1. Point scatterer at \( r_0 = (x_0, y_0, z_0) \) and interrogation point at \((x, y, z)\).
is a homogeneous medium with a velocity of \( c \). The shot and geophone patterns cover
an area given by \( L_x \times L_y \), and each shot consists of a point source that is harmonically
oscillating with angular frequency \( \omega \).

The diffraction stack migration of 3-D prestack seismic data for this point scatterer is
described by (French, 1974)

\[
m(\mathbf{x}, \omega) = \int_{\Gamma_s} \int_{\Gamma_g} e^{ik(r_g-x-r_{g0}+r_{s0}-r_{so})} h'(\mathbf{x}_s, \mathbf{x}_g)/(r_{gs}r_{go}r_{sx}r_{so})d\mathbf{x}_s d\mathbf{x}_g, \tag{2.3}
\]

where \( r \) and \( s \) represent the areal extent of the geophone and source arrays, respectively;
the function \( h'(\mathbf{x}_s, \mathbf{x}_g) \) represents the 4-D sampling function of the source and receiver
distribution; \( r_{ij} = |\mathbf{x}_i - \mathbf{x}_j| \); \( k = \omega/c \); \( \mathbf{x}_g = (x_g, y_g, 0) \), \( \mathbf{x}_s = (x_s, y_s, 0) \); and the horizontal
wavenumbers are given by \( k_g = (k_{gx}, k_{gy}, 0) \), and \( k_s = (k_{sx}, k_{sy}, 0) \). Because of its integral
form, we will also refer to this type of migration as Kirchhoff migration. Under the farfield
approximation (Chen, 1996) this equation can be written, for large \( r \) and \( s \), as

\[
m(\mathbf{x}, \omega) = A \int_{\Gamma_s} \int_{\Gamma_g} e^{-i(\mathbf{k}_s \cdot \mathbf{x}_s + \mathbf{k}_g \cdot \mathbf{x}_g)} h'(\mathbf{x}_s, \mathbf{x}_g) d\mathbf{x}_g d\mathbf{x}_s, \tag{2.4}
\]

where \( A = e^{ik(2x_0dx+2y_0dy+2z_0dz-x_0dx-y_0dy)}/r_0^4 \), and

\[
k_{gx} = k_{sx} = kdx/r_0 \quad \text{and} \quad k_{gy} = k_{sy} = kdy/r_0. \tag{2.5}
\]

Here \( H(\mathbf{k}_s, \mathbf{k}_g) \) represents the Fourier transform of the sampling function; \( m(\mathbf{x}, \omega) \) repre-
ts the migrated image at \( \mathbf{x} = (x, y, z) \); \( r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} \); \( dx = x - x_0 \); \( dy = y - y_0 \); and
\( dz = z - z_0 \). Equation 2.4 says that the migrated image of the point scatterer is
approximated by the stretched Fourier transform of the 4-D sampling function, where the
coordinate stretch factors are given in equation 2.5.

As the depth of the scatterer increases (i.e., as \( r_0 \) increases), the stretched wavenumbers
given by \( k_y/r_0 \) or \( k_x/r_0 \) go to zero. Consequently, the evaluation of \( H(\mathbf{k}_s, \mathbf{k}_g) \) beneath the
recording array at depth \( z_0 \) samples the very low wavenumber parts of the transformed
sampling comb, which resembles that of the desired delta function. This confirms the
well-known fact that the migrated images of deeper targets are less distorted by the
seismic footprint because of the nearly normal incidence angles of the reflected waves.
Steeply incident waves are less likely to be aliased by measurements from coarsely spaced
phones.
The survey design goal is to devise a source and geophone geometry that spikes the footprint noise so that:

$$\bar{H}(k_s, k_g) \approx \delta(x - x_0)\delta(y - y_0)\delta(z - z_0).$$  \hspace{1cm} (2.6)

A design strategy might be to compute the 4-D FFT of different source and geophone patterns $h'(\mathbf{r}_s, \mathbf{r}_g)$ and choose the one that best satisfies equation 2.6. But a 4-D FFT can still be expensive to compute so that the next section presents a shortcut to this computation, namely the SAT.

2.2 Seismic Array Theorem

The Array theorem (Reynolds et al., 1989) is used by optical engineers to rapidly compute the optical impulse response of an aperture composed of a regular array of openings. I modify this theorem for application to survey design and the rapid computation of the 4-D sampling function. The theorem presented below is modified for seismic migration (Schuster, 1997).

Seismic Array Theorem 1 Assume a 3-D seismic experiment where each shot shoots into a fixed orthogonal array of geophone stations with area $L_x \times L_y$, below which is a point scatterer embedded in a homogenous medium (see Figure 2.1). This is equivalent to saying that the source and the receiver distributions are separable, i.e., the 4-D sampling function $h'(\mathbf{r}_s, \mathbf{r}_g)$ can be decomposed into a concatenation of 2 2-D sampling functions $h'(\mathbf{r}_s, \mathbf{r}_g) = h(\mathbf{r}_s)h(\mathbf{r}_g)$. Let $n(\mathbf{r})$ and $e(\mathbf{r})$ represent the north-south and east-west 1-D sampling combs, so that the 2-D sampling functions can be described by $h(\mathbf{r}_s) = \int n(\mathbf{r}_s - \mathbf{r}')e(\mathbf{r}')d\mathbf{r}'$ and $h(\mathbf{r}_g) = \int n(\mathbf{r}_g - \mathbf{r}')e(\mathbf{r}')d\mathbf{r}'$. Here, e.g., the north-south comb is represented by

$$n(\mathbf{r}) = \delta(x)\sum_k (y - kdy).$$  \hspace{1cm} (2.7)

Under the farfield approximation, the migrated image of a point scatterer is given by:

$$m(\mathbf{r}, \omega) = A \int \int h'(\mathbf{r}_s, \mathbf{r}_g)e^{-i\mathbf{r}_s \cdot \mathbf{k}_s + \mathbf{r}_g \cdot \mathbf{k}_g}d\mathbf{r}_sd\mathbf{r}_g,$$

$$= A \int h(\mathbf{r}_s)e^{-i\mathbf{r}_s \cdot \mathbf{k}_s}d\mathbf{r}_s \int h(\mathbf{r}_g)e^{-i\mathbf{r}_g \cdot \mathbf{k}_g}d\mathbf{r}_g,$$

$$\approx \tilde{N}(\mathbf{k}_s)\tilde{E}(\mathbf{k}_s)\tilde{N}(\mathbf{k}_g)\tilde{E}(\mathbf{k}_g),$$  \hspace{1cm} (2.8)
where \( m(\mathbf{r}, \omega) \) is the migration image, \( k_s = k_g = k(\mathbf{r} - \mathbf{r}_0)/r \), and \( k \) is the wavenumber (see Figure 2.1). The Fourier transforms of \( n(\mathbf{r}) \) and \( e(\mathbf{r}) \) are represented by \( \hat{N}(\mathbf{k}) \) and \( \hat{E}(\mathbf{k}) \), respectively.

The above theorem is a simple application of the convolution theorem. Equation 2.8 shows that just two 2-D FFT's are needed to compute the migration point scatterer response of a general sampling function that is separable. For the orthogonal sampling geometry shown in Figure 2.2, the acquisition footprint can be computed by a multiplication of four 1-D functions (Oppenheim and Wilsky, 1983), i.e.,

\[
\begin{align*}
|\hat{N}(\mathbf{k}_g)| &= \left| \frac{\sin(k_{s y} L_y / 2)}{\sin(k_{g y} d_{y g} / 2)} \right|, \\
|\hat{E}(\mathbf{k}_g)| &= \left| \frac{\sin(k_{s x} L_x / 2)}{\sin(k_{g x} d_{x g} / 2)} \right|, \\
|\hat{N}(\mathbf{k}_s)| &= \left| \frac{\sin(k_{s y} L_y / 2)}{\sin(k_{s y} d_{y s} / 2)} \right|, \\
|\hat{E}(\mathbf{k}_s)| &= \left| \frac{\sin(k_{s x} L_x / 2)}{\sin(k_{s x} d_{x s} / 2)} \right|,
\end{align*}
\]

(2.9)

where the wavenumbers are defined in equation 2.5, \( dx_s \) and \( dy_s \) are the source sampling intervals in the \( x \) and \( y \) directions, and \( dx_g \) and \( dy_g \) are the receiver sampling intervals in the \( x \) and \( y \) directions.

This Seismic Array theorem (SAT) can be adjusted to accommodate geophone arrays that vary with shot position. For a marine seismic survey, the receiver locations are source dependent so that the recording cables are always shifted with any shift of the source group. Therefore, the 4-D sampling function can be decomposed as \( h'(\mathbf{r}_s, \mathbf{r}_g) = h(\mathbf{r}_s) h(\mathbf{r}_g - \mathbf{r}_s) \) and the resulting formula for the point scatterer migration response is given as:

\[
m(\mathbf{r}, \omega) \approx \hat{N}(2\mathbf{k}_s) \hat{E}(2\mathbf{k}_s) \hat{N}(\mathbf{k}_g) \hat{E}(\mathbf{k}_g),
\]

(2.10)

where an orthogonal geophone grid is assumed. Thus, the point scatterer migration response associated with an orthogonal marine recording geometry can be computed by a trivial multiplication of four analytical expressions.

### 2.3 Numerical Verification

This section verifies that the acquisition footprint calculated by the SAT is a useful approximation to the point scatterer response under the farfield condition.
Figure 2.2. (Top) Graphical representation of 1-D sampling brushes along the north-south and east-west axes. (Bottom) Convoluting these two functions with one another yields the 2-D sampling brush shown at the bottom.
2.3.1 Example 1: Zero-offset Survey

To verify the SAT, a ray tracing scheme was used to generate a uniform distribution of zero-offset traces over a 1667 X 1667 m$^2$ area, with a station interval of 80 m. The model was a point scatterer embedded in a homogeneous earth at a depth of 6667 m beneath the lower left corner of the data grid. The source wavelength 33 m. These data were migrated using an equation similar to equation 2.4 except the integration is 2-D because the data are zero-offset traces.

Figures 2.3 and 2.4 show slices of the migrated image at the depth of the point scatterer, as calculated by the diffraction stack migration and the SAT, respectively. Figure 2.5 shows the magnitude of both images in the X-direction. The SAT image closely resembles the Kirchhoff image, but it required only 6 CPU seconds for a SUN ULTRA 140 to generate compared to over 5 CPU hours for the calculation of the diffraction-stack image.

2.3.2 Example 2: Source-dependent Receiver Survey

If the receiver pattern is moved with the shot location, the acquisition footprint can be also calculated by the SAT. As an example, a seismic survey is defined over a 1667 X 1667 m$^2$ area, with a shot station interval of 83 m. The receiver pattern covers a 667 X 667 m$^2$ area and is centered about the shot, with a receiver station interval of 167 m. The model is a point scatterer embedded in a homogeneous earth at a depth of 8333 m beneath the lower left corner of the data grid. The wavelength of the seismic wave is 50 m.

Figure 2.6 shows the slice of the migrated image at the depth of the point scatterer, as calculated by the diffraction stack migration. This computation required more than a 40 CPU hours on SUN ULTRA 140 to migrate these 10,000 traces, while the SAT image in Figure 2.7 was computed in 10 CPU seconds. Figure 2.8 shows the magnitude of both images along the X-direction. The close agreement shows that the SAT calculation is a good approximation to the diffraction stack migration.
Figure 2.3. Point scatterer response for zero-offset survey calculated by the diffraction stack migration. The both sides of the rectangular survey are 1667 m long, and the station interval is 80 m. The point scatterer is located at \((x_0, y_0, z_0) = (0, 0, 6667 \text{ m})\), and the source wavelength is 33 m.
Figure 2.4. Point scatterer response computed by the SAT for the same survey in Figure 2.3. This slice of the SAT image is close to that of the diffraction stack migration image, but it requires only 6 CPU seconds to compute on a SUN ULTRA1 140 while the other one requires 5 CPU hours.
Figure 2.5. Comparison of the SAT image (solid line) and diffraction stack image (dashed line). The close agreement shows that, under the far field approximation, the SAT is an accurate approximation to the diffraction stack migration.
Figure 2.6. Point scatterer response for prestack survey calculated by the diffraction stack migration. The source stations are distributed over a 1667 X 1667 m$^2$ area with a station interval of 83 m; the receivers cover a 667 X 667 m$^2$ area centered about each activated source with an interval of 167 m. The point scatterer is located at $(x_0, y_0, z_0) = (0, 0, 8333 \text{ m})$, the wavelength is 50 m.
Figure 2.7. Point scatterer response calculated by the SAT for the same survey in Figure 2.6. This slice of the SAT image is close to that of the diffraction stack migration image, but it requires only 10 CPU seconds to compute on a SUN ULTRA1 140 while the other one requires 40 CPU hours.
Figure 2.8. Comparison of the SAT image (solid line) and the diffraction stack image (dashed line) for prestack migration. The close agreement shows that, under the farfield approximation, the SAT is an accurate approximation to the diffraction stack migration.
2.4 Extension of SAT to Transient Sources and Irregular Scatterers

Equations 2.8 and 2.10 predict the migration images for a monochromatic source calculated by the SAT. Considering the phase factor, the migration image can be given by:

\[ m(\mathbf{r}, \omega) = |m(\mathbf{r}, \omega)|e^{i2k|\mathbf{r} - \mathbf{r}_0|}, \]

(2.11)

where \( k \) is the wave number, \( \mathbf{r} \) denotes the position of the image point, \( \mathbf{r}_0 \) is the location of the point scatterer, and \( |m(\mathbf{r}, \omega)| \) is the magnitude of the migration image calculated by the SAT. Based on this equation, we can use the SAT to rapidly calculate the migration image for a transient source and irregular scatterers by

\[ m(\mathbf{r}) = \sum_{\mathbf{r}_0} \sum_{\omega} W(\omega)|m(\mathbf{r}, \omega)|e^{i2k|\mathbf{r} - \mathbf{r}_0|}, \]

(2.12)

where \( W(\omega) \) is the spectrum of the transient source.

In this section, I use this equation to compute the point scatterer response for a transient source and the monochromatic migration image of a meandering river.

2.4.1 Point Scatterer Response for a Transient Source

If the spectrum of a transient source wavelet is given by \( W(\omega) \), equation 2.12 can be used to compute the point scatterer response of a transient source as

\[ m(\mathbf{r}) = \sum_{\omega} W(\omega)|m(\mathbf{r}, \omega)|e^{i2k|\mathbf{r} - \mathbf{r}_0|}, \]

(2.13)

where \( |m(\mathbf{r}, \omega)| \) is the magnitude of the migration image which can be rapidly calculated by the SAT.

Figures 2.9, 2.10 and 2.11 show the point scatterer responses for a monochromatic source, a Ricker wavelet source and an impulsive source. All three migration images are calculated by the SAT and equation 2.13. The aperture area of the survey is 667 X 667 m², the source spacing is 33 m in both the X- and Y- directions, and the receiver spacing is 13 m in both the X- and Y- directions. The point scatterer is located at a depth of 3333 m beneath the center of the survey area. Here the number of frequency samples for both the Ricker wavelet and delta function are 4096 points.
Figure 2.9. Point scatterer migration response calculated by the SAT, where there is a 51 X 51 grid of uniformly-spaced receivers and a 21 X 21 grid of uniformly-spaced sources. The frequency of the source is 10000 Hz. The point scatterer is located at (333 m, 333 m, 3333 m) and the image plane is at the depth of 3333 m.
Figure 2.10. Point scatterer response calculated by SAT for the Ricker wavelet with the peak frequency of 10000 Hz is used for the source. The number of frequency samples is 4096. The recording survey is the same as in Figure 2.9.
Figure 2.11. Point scatterer response calculated by SAT for a delta function as the source time history. The recording survey is the same as in Figure 2.9.
Figure 2.12 shows the point scatterer migration response for a 10000 Hz Ricker wavelet source obtained by Kirchhoff migration (Hu, 1997). Figure 2.13 shows the difference between the SAT and Kirchhoff migration images. The SAT image, which is the summation of 4096 monochromatic images is in close agreement with the Kirchhoff migration image, but it is computationally cheaper to obtain.

Figure 2.14 shows the point scatterer response for an impulsive source obtained by Kirchhoff migration. Figure 2.15 displays the difference between the SAT and Kirchhoff migration images and suggests that the SAT provides an useful approximation to the Kirchhoff migration image.

2.4.2 Migration Image for Irregular Scatterers

If there are a set of point scatterers located at the same depth beneath the seismic survey, equation 2.12 can be rewritten to compute the monochromatic migration image which is the summation of the responses for all the scatterers, i.e.,

\[ m(\mathbf{r}, \omega) = \sum_{\mathbf{r}_0} |m(\mathbf{r}, \mathbf{r}_0, \omega)| e^{2\pi i (\mathbf{r} - \mathbf{r}_0) / \lambda} \]  

(2.14)

where \(|m(\mathbf{r}, \mathbf{r}_0, \omega)|\) is the magnitude of the migration image for a point scatterer at \(\mathbf{r}_0\) which can be rapidly calculated by the SAT. Equation 2.14 is now used to compute of the migrated image of the stream channel model shown in Figure 2.16. Here 101 point scatterers are used to create the model.

Figure 2.17 shows the SAT migration image of the meandering stream channel. The number of traces is about 1000000, and the computation of this image required about 30 CPU minutes on a SUN ULTRA-1 140. It is difficult to calculate the migrated image by Kirchhoff migration because it would take more than one month to compute on the same computer. This example demonstrates that, for a large seismic survey, the SAT, may be the only cost effective means to compute the point scatterer migration response.
Figure 2.12. Point scatterer response computed by a Kirchhoff migration formula. The source wavelet and recording survey are the same as in Figure 2.10.
Figure 2.13. Difference between SAT image and Kirchhoff migration image for the Ricker wavelet source. The misfit values are at the level of $10^{-3}$. 
Figure 2.14. Point scatterer response for delta function computed by a Kirchhoff migration formula. The recording survey is the same as in Figure 2.11.
The Difference between the Seismic Array Theorem Image and Kirchhoff Migration Image

Figure 2.15. Difference between SAT image and Kirchhoff migration image for the impulsive source. The misfit values are also at the level of $10^{-3}$. 
Figure 2.16. Meandering stream channel model consist of 101 point scatterers, where the depth of the stream channel is 5000 m and the reflectivity of the stream channel is 1.0.
Figure 2.17. Migration image of a meandering river channel computed by the SAT for 1147041 traces recorded by an acquisition array, where parameters are listed above. Here the imaging depth is the same as the depth of the river at 5000 m, and the sinusoidal features represents the location of the river channel.
CHAPTER 3
SURVEY DESIGN BY TRIAL AND ERROR METHOD

3.1 Introduction

Using the SAT, the point scatterer response can be computed for different surveys, and the optimal survey should be the one with the weakest alias energy. Since the ideal image of a point scatterer response is a delta function (see equation 2.6), we can define the alias energy $E$ as

$$E(dx_s,dy_s,dx_g,dy_g,\omega) = \sum_x \sum_y (m(\xi,\omega) - \delta(\xi))^2,$$

where $(dx_s,dy_s)$ and $(dx_g,dy_g)$ are the source and receiver sampling intervals, respectively.

I define the best survey that introduces the weakest acquisition footprint into the migration section. For several recording geometries, we can compute the point scatterer response for each survey and identify the best survey which has the weakest alias energy. This iterative method is called a trial and error method. We can practically implement this method because the SAT can be used to rapidly compute the point scatterer responses for different seismic surveys.

In this chapter, I illustrate this trial and error method for three different surveys: a West Texas survey geometry similar to one designed by Phillips Oil Co., a land survey and a marine survey designed by ARCO.

3.2 Example I: West Texas Survey

An example of a recent W. Texas survey designed by Phillips Oil Co. is described by a regular grid of sources distributed over a 7920 X 3520 m$^2$ area, while the regular receiver distribution covers a 7920 X 7920 m$^2$ area. For the SAT I assume a homogeneous velocity medium with a point scatterer at a depth of 5000 m beneath the lower left corner of the
data area, and the source wavelength is 33 m. Table 3.1 shows the recording parameters and the associated alias energies for four different geometries. In this case, the SAT is used to compute the alias energy of the migrated point scatterer image along the horizontal plane at the depth of the scatterer.

Figures 3.1 and 3.2 show slices of the monochromatic migrated images at the depth of 5000 m for the four surveys, where the source wavelength is taken to be 33 m. For almost the same number of traces, the ”regular” grid data provides a migrated image with less than five times the alias energy of the ”uniform” survey.

Why did the ”uniform” grid produce stronger migration artifacts than the ”regular” grid? Heuristically speaking, the source-receiver configuration of the ”regular” grid possesses less spatial periodicity than the ”uniform” grid. A greater spatial periodicity in the sampling function will tend to amplify the grating lobe energy in the migration section.

To rigorously illustrate this last statement, consider the 3-D prestack migration formula associated with an orthogonal source-receiver array (Schuster, 1997):

\[
m(x, y, z_0) \approx \frac{\sin(k_{sx}L_{sx}/2)}{\sin(k_{sx}d_{sx}/2)} \frac{\sin(k_{sy}L_{sy}/2)}{\sin(k_{sy}d_{sy}/2)} \frac{\sin(k_{gx}L_{gx}/2)}{\sin(k_{gx}d_{gx}/2)} \frac{\sin(k_{gy}L_{gy}/2)}{\sin(k_{gy}d_{gy}/2)},
\]

and

\[
k_{sx} = k_{gx} = k(x - x_0)/r \quad ; \quad k_{sy} = k_{gy} = k(y - y_0)/r,
\]

where \(k\) is the wavenumber; \((x_0, y_0, z_0)\) is the point scatterer location; \((x, y, z_0)\) is the point at which the migrated image is evaluated; \(L_{sx}, L_{sy}, L_{gx}, L_{gy}\) are the side lengths of the rectangular source and receiver apertures; and \(d_{sx}, d_{sy}, d_{gx}, d_{gy}\) are the

<table>
<thead>
<tr>
<th>Array</th>
<th>(dx_s)</th>
<th>(dy_s)</th>
<th>(dx_g)</th>
<th>(dy_g)</th>
<th># of Traces</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular grid</td>
<td>440 m</td>
<td>110 m</td>
<td>73 m</td>
<td>440 m</td>
<td>1298517</td>
<td>84.6495</td>
</tr>
<tr>
<td>Uniform grid</td>
<td>193 m</td>
<td>193 m</td>
<td>193 m</td>
<td>193 m</td>
<td>1309499</td>
<td>453.7248</td>
</tr>
<tr>
<td>1/4 sampling</td>
<td>440 m</td>
<td>110 m</td>
<td>293 m</td>
<td>440 m</td>
<td>333564</td>
<td>125.9711</td>
</tr>
<tr>
<td>regular grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4 sampling</td>
<td>277 m</td>
<td>377 m</td>
<td>277 m</td>
<td>277 m</td>
<td>317057</td>
<td>756.9169</td>
</tr>
<tr>
<td>uniform grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1. Recording parameters for four W. Texas surveys.
Figure 3.1. The SAT image for the W. Texas survey, where the survey parameters are shown in Table 3.1. The number of traces is about 1300000, and the alias energy for the (top) regular survey is much less than that for the (bottom) uniform survey.
Figure 3.2. Same as Figure 3.1 except 1/4 subsampled data. The alias energy of acquisition footprint for the (top) 1/4 regular survey is not only much less than that for the (bottom) 1/4 uniform survey, but also less than that for the uniform survey in Figure 3.1.
sampling intervals for the sources and receivers. This product can be disastrous if the grating lobe locations of the sinc-like functions coincide with one another to produce amplified grating lobes. Strong grating lobes are undesirable because they introduce strong aliasing artifacts into the migrated section. For example, a really bad regular geometry is the orthogonal uniform survey where the sampling interval is the same for both the geophones and sources. In this case, the arguments of the sinc functions are the same so the grating lobes for all four sinc-like functions will amplify one another.

In contrast, the regular grid survey, where the sampling intervals are not all the same, has grating lobes reinforcement at fewer locations. This is because the arguments of the numerical sinc-like functions, e.g., \( \frac{\sin(k_{gx}L_{gx}/2)}{\sin(k_{gx}d_{gx}/2)} \) and \( \frac{\sin(k_{gy}L_{gy}/2)}{\sin(k_{gy}d_{gy}/2)} \), are not the same.

### 3.3 Example II: ARCO’S Land Seismic Survey

In this example, I analysis three different land surveys similar to the one designed by ARCO. Figure 3.3 shows the source and receiver geometries, respectively, for the three different surveys, and the parameters of the acquisition are list in Table 3.2. In these surveys, all receivers are activated for each shot. The recording area of each survey is 2333 \( \times \) 2333 m\(^2\), and the geophone and source sampling intervals are given by \((d_{xg}, dy_g)\) and \((dx_s, dy_s)\), respectively. In survey I, \( dx_g = dy_s = 80 \text{ m}, \ dy_g = dx_s = 13 \text{ m}\); and in survey II, \( dx_g = dy_s = 40 \text{ m}, \ dy_g = dx_s = 13 \text{ m}\); and in survey III, \( dx_s = dy_s = 40 \text{ m}, \ dy_g = dx_s = 7 \text{ m}\). So, about 28000000 traces are computed for each of the three surveys. The source wavelength is taken to be 50 m, and the point scatterer is located at \((0,0,z_0 = 1333 \text{ m})\).

Figure 3.4 shows the monochromatic migrated images of a buried point scatterer, i.e., the acquisition footprint of the surveys calculated by the SAT, where the acquisition footprint for survey II has weaker artifacts than the others. Artifacts are defined as migration energy located outside the main pulse centered at \((0,0,1333 \text{ m})\). For this

<table>
<thead>
<tr>
<th>Survey</th>
<th>( dx_s )</th>
<th>( dy_s )</th>
<th>( dx_g )</th>
<th>( dy_g )</th>
<th># of Traces</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>13 m</td>
<td>80 m</td>
<td>80 m</td>
<td>13 m</td>
<td>27878400</td>
<td>971</td>
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<tr>
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<td>40 m</td>
<td>40 m</td>
<td>13 m</td>
<td>29160000</td>
<td>192</td>
</tr>
<tr>
<td>III</td>
<td>7 m</td>
<td>40 m</td>
<td>40 m</td>
<td>7 m</td>
<td>29160000</td>
<td>735</td>
</tr>
</tbody>
</table>
The source and receiver geometries for the three different land surveys. The survey parameters are described in Table 3.2, where there are about 5100 source and receiver stations for each survey, respectively.
Figure 3.4. Acquisition footprint magnitude of the three surveys evaluated at depth of 1333 m. The survey II image has the fewest migration artifacts so that it is a better survey than the others.
frequency, survey II should produce the least acquisition footprint noise than the other surveys.

Why did survey I produce the most migration artifacts while survey II produced the fewest migration artifacts? Heuristically speaking, the source-receiver configuration of survey II possessed the least regularity while survey I had the most regularity. A greater regularity in the sampling function will tend to amplify the grating lobe energy in the migration section.

For the somewhat regular geometry of survey I, the geophone sampling interval in the y direction is one sixth of the source sampling interval in the y direction. Thus the 6th harmonic grating lobe of the source will be coincident with the first grating lobe of the receiver, leading to an amplification of every 6th grating lobe for the source. Moreover, the first source grating lobe may be coincident with strong side lobes of the receiver. Such amplification does not happen so readily in the survey II geometry because it is less regular than that of survey I where the migrated section is no longer a concatenation of four sinc functions. Instead, it is a concatenation of functions with noncoincident grating lobes. Such noncoincidence prevents grating lobe amplification.

### 3.4 Example III: ARCO’S Marine Seismic Survey

Figures 3.5 and 3.6 show four different marine surveys designed by ARCO with ◯ and + representing the source and the receiver locations, respectively. Table 3.3 contains the recording parameters and alias energies for these four surveys. I assume a point scatterer at a depth of 10000 m beneath the center of the data area, and the seismic wavelength is 10 m.

**Table 3.3.** Array parameters for four marine surveys.

<table>
<thead>
<tr>
<th>Array</th>
<th>$dx_s$ (m)</th>
<th>$dy_s$ (m)</th>
<th>$dx_g$ (m)</th>
<th>$dy_g$ (m)</th>
<th>Cable Length (m)</th>
<th># of Cable</th>
<th># of Traces</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single streamer</td>
<td>75</td>
<td>25</td>
<td>25</td>
<td>3000</td>
<td>1</td>
<td>977161</td>
<td>31.1</td>
<td></td>
</tr>
<tr>
<td>Multi-streamer</td>
<td>72.48</td>
<td>289.9</td>
<td>24.16</td>
<td>96.74</td>
<td>2899.2</td>
<td>6</td>
<td>1022208</td>
<td>47.8</td>
</tr>
<tr>
<td>Inline swath</td>
<td>183.17</td>
<td>61.06</td>
<td>30.53</td>
<td>30.53</td>
<td>3663.6</td>
<td>8</td>
<td>994136</td>
<td>19.1</td>
</tr>
<tr>
<td>Narrow brick</td>
<td>119.97</td>
<td>89.28</td>
<td>29.76</td>
<td>89.28</td>
<td>3571.2</td>
<td>8</td>
<td>1036728</td>
<td>19.1</td>
</tr>
</tbody>
</table>
Figure 3.5. Two source-receiver geometries for ARCO’s marine seismic surveys: (top) single streamer and (bottom) multistreamer. The receiver arrays are shifted by the same amount as the shift in the active shot arrays. The survey parameters are presented in Table 3.3.
**Figure 3.6.** Two source-receiver geometries designed for ARCO’s marine seismic surveys: (top) inline swath and (bottom) narrow brick. These receiver arrays are shifted by the same amount as the shift in the shot arrays.
Figures 3.7 and 3.8 show the acquisition footprint images for the four seismic arrays. Arrays III and IV have the weakest sidelobe energies and, therefore, are preferable to the first two arrays. It is surprising that the Array I 2-D survey (single streamer) has less alias energy than the Array II 3-D survey (multistreamer). Apparently, the finely sampled 2-D survey provides fewer aliasing artifacts than the coarsely sampled 3-D survey! Does this mean the 2-D survey has better resolutions than the 3-D survey? According to the Projection-Slice theorem for continuous surveys this is not true, but this theorem does not account for discrete arrays.
Figure 3.7. The acquisition footprints (i.e., the point scatterer migration response evaluated at the scatterer depth) for ARCO surveys I and II in Figure 3.5. The point scatterer is located at a depth of 10 km beneath the center of the surveys.
Figure 3.8. The acquisition footprints (i.e., the point scatterer migration response evaluated at the scatterer depth) for ARCO surveys III and IV in Figure 3.6. The point scatterer is located at a depth of 10 km beneath the center of the surveys.
CHAPTER 4

SURVEY DESIGN BY OPTIMIZATION ALGORITHM

4.1 Introduction

The acquisition footprint is the numerical noise introduced into the migration section by the discrete sampling of the wavefields measured over a finite-width aperture. Such noise can be seriously degrade the interpretation of the 3-D seismic images, especially for AVO studies that interpret horizontal slices of 3-D migration sections. The optimal seismic survey should be the one that introduces the least acquisition footprint to the migration images. As shown in the last chapter, the trial and error method can be used to determine the best survey from a small set of survey options. But the trial and error procedure can be tedious and inefficient if there are many survey options. A more productive approach in designing the optimal survey might be to use nonlinear optimization.

A typical optimization problem might be posed in the following way: suppose the survey is distributed over the $L_x \times L_y$ area, and the number of traces is a constant, how can we find the optimal station intervals for the source and receiver?

If I define the cost function as the alias energy $E$, i.e.,

$$ E(dx_s, dy_s, dx_g, dy_g, \omega) = \sum_x \sum_y (m(x, \omega) - \delta(x))^2, \quad (4.1) $$

where $(dx_s, dy_s)$ and $(dx_g, dy_g)$ are the source and receiver sampling intervals, respectively. Since the point scatterer response $m(x, \omega)$ can be rapidly computed by the SAT, an optimization algorithm can be used to find the survey parameters which minimize the cost function. Mathematically, this optimization problem can be written as:

$$ \min \ E(dx_s, dy_s, dx_g, dy_g, \omega), \quad (4.2) $$
subject to

\begin{align*}
0 & \leq dx_s \leq L_{sx}, \\
0 & \leq dy_s \leq L_{sy}, \\
0 & \leq dx_g \leq L_{gx}, \\
0 & \leq dy_g \leq L_{gy}, \\
N_{\text{trace}} & = \text{const}. 
\end{align*}

Here \( N_{\text{trace}} \) is the number of traces; \( L_{sx} \) and \( L_{sy} \) are the side lengths of the rectangular aperture for the source geometry in the X- and Y- directions, respectively; \( L_{gx} \) and \( L_{gy} \) are the side lengths of the rectangular aperture for the receiver geometry in the X- and Y- directions, respectively; \( dx_s, dy_s \) are the source intervals in the X- and Y- directions; and \( dx_g, dy_g \) are the geophone intervals in the X- and Y- directions.

### 4.2 Constraint Optimization Algorithm

To solve the above problem, we consider a method that can be used to solve a general class of constrained optimization problems. The general problem (Rao, 1978) is stated as

Minimize \( f(\mathbf{X}) \) subject to constraints

\begin{align*}
g_j(\mathbf{X}) & \leq 0, \quad j = 1, 2, \ldots, m, \\
l_j(\mathbf{X}) & = 0, \quad j = 1, 2, \ldots, p. 
\end{align*}

(4.4)

where \( g_j \) and \( h_j \) are constraints for the parameters \( \mathbf{X} \). This problem can be converted into an unconstrained minimization problem by constructing a functional of the form

\begin{align*}
\phi_k & = \phi(\mathbf{X}, r_k) \\
& = f(\mathbf{X}) + r_k \sum_{j=1}^{m} G_j[g_j(\mathbf{X})] + H(r_k) \sum_{j=1}^{p} l_j^2(\mathbf{X}), 
\end{align*}

(4.5)

where \( G_j \) is some function of the constraint \( g_j \) that tends to infinity as the constraint boundary is approached; \( H(r_k) \) is some function of the parameter \( r_k \) tending to infinity as \( r_k \) tends to zero, and \( r_k \) is a positive constant known as the penalty parameter (Fiacco and McCormick, 1968). The motivation for the third term in equation (4.5) is that as
$H(r_k) \to \infty$, the quantity $\sum_{j=1}^{p} l_j^2(X)$ must tend to zero. If $\sum_{j=1}^{p} l_j^2(X)$ does not tend to zero, $\phi_k$ would tend to infinity, and this cannot happen in a sequential minimization process if the problem has a solution.

Fiacco and McCormick (1968) used the following form of equation 4.5:

$$\phi_k = \phi(X, r_k) = f(X) - r_k \sum_{j=1}^{m} \frac{1}{g_j(X)} + \frac{1}{\sqrt{r_k}} \sum_{j=1}^{p} l_j^2(X), \quad (4.6)$$

and they proved that if $\phi_k$ is minimized for a decreasing sequence of values $r_k$, the unconstrained minima $X_k^{\ast}$ will converge to the solution $X^{\ast}$ of the original problem stated in Equation (4.4). After this conversion, the Davidon-Fletcher-Powell conjugate gradient method (Fletcher and Powell, 1963) can be used to solve this problem.

The next section will use this optimization procedure to determine the survey geometry with a minimal acquisition footprint noise.

### 4.3 Numerical Examples

To solve our survey design problem, I apply this constraint optimization algorithm to solve equation 4.2 with the constraints in equation 4.3. I use four examples to illustrate the effectiveness of this constraint optimization method.

#### 4.3.1 Example I: A Two-parameter Problem

To simplify the survey design problem, I assume that the number of traces and the receiver stations to be fixed, and we seek the optimal source interval for a given recording aperture. So, this optimization problem can be written mathematically as:

$$\min \ E(dx_s, dy_s, \omega), \quad (4.7)$$

subject to

$$0 \leq dx_s \leq L_x,$$

$$0 \leq dy_s \leq L_y,$$

$$N_s = \text{const.} \quad (4.8)$$

Here $N_s$ is the number of sources.
Suppose the survey aperture is over a 1667 X 1667 m² area, with 400 source stations, and 25 receiver stations are distributed on a 667 X 667 m² orthogonal grid about each shot position. How can we find the optimal in-line and cross-line source spacing \( dx_s \) and \( dy_s \)? To answer this question I first show in Figure 4.1 a contour plot of the associated cost function \( E(dx_s, dy_s, \omega) \). The solid curve in this plot represents the combinations of \( dx_s \) and \( dy_s \) that satisfies the constraint conditions in equation 4.6. There are several local minima in this area, so, we cannot get a unique solution if we directly minimize \( E(dx_s, dy_s, \omega) \).

To avoid local minima we rewrite equation 4.8 as

\[
-dx_s \leq 0, \\
\frac{dx_s - L_x}{dx_s} \leq 0, \\
-dy_s \leq 0, \\
\frac{dy_s - L_y}{dy_s} \leq 0, \\
\frac{L_{sx}}{dx_s} \cdot \frac{L_{sy}}{dy_s} - N_s = 0. 
\]

This optimization problem can be solved by using the interior penalty function method with equation 4.6. Thus, we will minimize the constraint cost function

\[
\phi_k = \phi(dx_s, dy_s, r_k) \\
= E(dx_s, dy_s) - r_k \left( \frac{1}{-dx_s} + \frac{1}{dx_s - L_x} + \frac{1}{-dy_s} + \frac{1}{dy_s - L_y} \right) \\
+ \frac{1}{\sqrt{r_k}} \left( \frac{L_{sx}}{dx_s} \cdot \frac{L_{sy}}{dy_s} - N_s \right)^2, 
\]

for decreasing value of \( r_k = 0.11, 0.11 \times 0.2, 0.11 \times 0.2^2, \ldots \). The optimal solution for minimizing \( \phi_k \) will converge to the optimal solution which minimizes the cost function \( E \) under the constraints 4.9.

Figure 4.2 shows the contours of the constraint cost function \( \phi_k \) (equation 4.6) for \( r_k = 0.11 \). Note there is no so much local minima shown in Figure 4.1.

Figure 4.3 shows the variation of the cost function and the constraint cost function \( \phi_k \) vs. the number of iterations. This result shows that after 250 steps, the constraint cost function tends to the unconstrained cost, and so the optimal solution of the unconstrained problem converges to the optimal solution of the original constraint problem.

Figures 4.4, 4.5 and 4.6 show the footprint images for the survey with the starting parameters, several intermediate parameters and the final parameters; and the optimal
Figure 4.1. The contour of the cost function $E(dx_s, dy_s, \omega)$ for the survey geometry described in the text. The solid curve represents the value of $dx_s$ and $dy_s$ which satisfy the constraint conditions in equation 4.6. Note there are many local minima in this contour plot.
The contour plot of constraint cost function $\phi_k$ when $r_k = 0.11$. There are fewer local minima compared to the original cost function shown in Figure 4.1. The optimal solutions $X_k^*$ to minimize this constraint cost function can be found by the DFP algorithm.

Figure 4.2. The contour of the constraint cost function $\phi_k$ when $r_k = 0.11$. There are fewer local minima compared to the original cost function shown in Figure 4.1. The optimal solutions $X_k^*$ to minimize this constraint cost function can be found by the DFP algorithm.
Figure 4.3. The variation of the (bottom) cost function and (top) constraint cost function vs. the iteration number. The cost function and constraint cost function are defined in equations 3.1 and 4.10, respectively.
Figure 4.4. The acquisition footprint images for the surveys with (top) the initial parameters \((dx_s, dy_s) = (333 \text{ m}, 333 \text{ m})\) and (bottom) the intermediate parameters \((dx_s, dy_s) = (195 \text{ m}, 195 \text{ m})\) after 15 iterations. Note that the alias energy is much reduced in the bottom figure.
Footprint for Intermediate Parameters after 30 Iterations: Alias Energy = 0.119

Footprint for Intermediate Parameters after 90 Iterations: Alias Energy = 0.0355

Figure 4.5. The acquisition footprint images for the surveys with (top) the intermediate parameters $(dx_s, dy_s)=(187 \text{ m}, 187 \text{ m})$ after 30 iterations and (bottom) the intermediate parameters $(dx_s, dy_s)=(103 \text{ m}, 103 \text{ m})$ after 90 iterations. Note the alias energy in the top figure is larger than that in the bottom figure in Figure 4.4. This suggests that there are many local minima in the cost function.
Figure 4.6. The acquisition footprint images for the surveys with (top) the intermediate parameters \((dx_s, dy_s) = (104 \text{ m}, 66 \text{ m})\) after 200 iterations and (bottom) the final parameters \((dx_s, dy_s) = (104 \text{ m}, 67 \text{ m})\) after 250 iterations. Note the alias energy for the optimal survey is reduced by 90% compared to the starting parameters.
parameters, \(dx_s = 104.2\) m and \(dy_s = 67.7\) m, are computed after 250 iterations. The footprint alias energy for the survey with optimal parameters is only one-tenth of that for the survey with the starting parameters. Table 4.1 shows the parameters and the associated alias energy during this procedure.

### 4.3.2 Example II: A Four-parameter Problem

Now I want to find the optimal source and receiver sampling intervals that minimize the footprint noise. Suppose the sources and receivers are regularly distributed in a square area with an aperture area of \(1667 \times 1667\) m\(^2\), and the total number of traces is \(10^6\). I assume a point scatterer located 5 km beneath the lower-left corner of the survey, and the source wavelength is taken to be 8 m. The goal is to find the \((dx_s, dy_s)\) and \((dg_x, dg_y)\) parameters that minimize the footprint energy, where the imaging area is at the same depth as the point scatterer.

Figure 4.7 shows the variation of the cost function and the constraint cost function \(\phi_k\) vs. the number of iterations. The irregular behavior of the cost function suggests that local minima are encountered in the search for the optimal parameters. This means that the constraint optimization method bypasses the local minima, and identifies the recording parameters that minimize alias energy.

Figures 4.8, 4.9 and 4.10 show the footprint images for the surveys with the starting parameters, several intermediate parameters and the final parameters where the alias energy has been reduced by 75% after 250 iterations. The final image show that the

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>(dx_s)</th>
<th>(dy_s)</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>333 m</td>
<td>333 m</td>
<td>0.312</td>
</tr>
<tr>
<td>15</td>
<td>195 m</td>
<td>195 m</td>
<td>0.0589</td>
</tr>
<tr>
<td>30</td>
<td>187 m</td>
<td>187 m</td>
<td>0.119</td>
</tr>
<tr>
<td>90</td>
<td>103 m</td>
<td>103 m</td>
<td>0.0355</td>
</tr>
<tr>
<td>200</td>
<td>104 m</td>
<td>66 m</td>
<td>0.033453</td>
</tr>
<tr>
<td>250</td>
<td>104 m</td>
<td>67 m</td>
<td>0.033426</td>
</tr>
</tbody>
</table>
Figure 4.7. The variation of (bottom) the cost function and (top) constraint cost function vs. the iteration number. The cost and constraint cost functions are defined as equations 3.1 and 4.10, respectively. The constraint cost function has fewer local minima than the original cost function.
Figure 4.8. The acquisition footprint images for the surveys with (top) the initial parameters, $dx_s = 105 \text{ m}$, $dy_s = 15 \text{ m}$, $dx_g = 53 \text{ m}$, and $dy_g = 105 \text{ m}$, and (bottom) the intermediate parameters, $dx_s = 128 \text{ m}$, $dy_s = 201 \text{ m}$, $dx_g = 90 \text{ m}$, and $dy_g = 118 \text{ m}$, after 25 iterations.
Figure 4.9. The acquisition footprint images for the surveys with (top) the intermediate parameters, $dx_s = 125$ m, $dy_s = 183$ m, $dx_g = 89$ m, and $dy_g = 106$ m, after 60 iterations; and (bottom) the intermediate parameters $dx_s = 111$ m, $dy_s = 169$ m, $dx_g = 75$ m, and $dy_g = 83$ m, after 90 iterations.
Figure 4.10. The acquisition footprint images for the surveys with (top) the intermediate parameters, $dx_s = 75$ m, $dy_s = 188$ m, $dx_g = 31$ m, and $dy_g = 63$ m, after 150 iterations; and (bottom) the final parameters, $dx_s = 67$ m, $dy_s = 166$ m, $dx_g = 11$ m, and $dy_g = 62$ m, after 250 iterations. Note the alias energy is reduced by 75% compared to the starting model.
optimal parameters are $dx_s = 67$ m, $dy_s = 166$ m, $dx_g = 11$ m, and $dy_g = 62$ m. Table 4.2 shows the parameters and the associated alias energy.

In this example, the numbers of traces for the starting survey and the final optimal survey are both about $10^{6}$. The fact that the alias energy in the migration image for the optimal survey is much weaker than that for starting survey means that this constraint optimization algorithm is useful for designing an optimal survey.

4.3.3 Example III: The Optimization Algorithm for Transient Sources

In this and the next sections, I apply the optimization algorithm to data generated by transient sources and data generated for a meandering stream model.

Assume the source is a Ricker wavelet, we want to find the optimal survey which satisfy the given constraint conditions. In this case, the cost function should be changed to

$$E(dx_s, dy_s, dx_g, dy_g) = \sum_x \sum_y (m(\mathbf{x}) - \delta(\mathbf{x}))^2; \quad (4.11)$$

where $m(\mathbf{x})$ is the point scatterer response calculated by equation 2.13.

To simplify the solution of this problem, I assume that the receiver geometry, which is shifted with each shot position, is distributed over a 667 X 667 m$^2$ area with a 167 m receiver interval. The source survey aperture is over a 1667 X 1667 m$^2$ area with 1000 source stations, and the source is a Ricker wavelet with a peak frequency of 100 Hz. The point scatterer is located at the depth of 8333 m beneath the center of the survey. This procedure uses the initial parameters $(dx_s, dy_s) = (333$ m, 333 m). Notes these parameters do not satisfy the constraint, i.e., the number of source stations is 1000. Figure 4.11

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>$dx_s$</th>
<th>$dy_s$</th>
<th>$dx_g$</th>
<th>$dy_g$</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>105 m</td>
<td>15 m</td>
<td>53 m</td>
<td>105 m</td>
<td>406</td>
</tr>
<tr>
<td>25</td>
<td>128 m</td>
<td>201 m</td>
<td>90 m</td>
<td>118 m</td>
<td>225</td>
</tr>
<tr>
<td>60</td>
<td>125 m</td>
<td>183 m</td>
<td>89 m</td>
<td>106 m</td>
<td>104</td>
</tr>
<tr>
<td>90</td>
<td>111 m</td>
<td>167 m</td>
<td>75 m</td>
<td>83 m</td>
<td>472</td>
</tr>
<tr>
<td>150</td>
<td>75 m</td>
<td>188 m</td>
<td>31 m</td>
<td>63 m</td>
<td>101</td>
</tr>
<tr>
<td>250</td>
<td>67 m</td>
<td>166 m</td>
<td>11 m</td>
<td>62 m</td>
<td>97</td>
</tr>
</tbody>
</table>
Figure 4.11. The acquisition footprint images for the surveys with (top) the initial parameters \((dx_s, dy_s) = (333 \text{ m}, 333 \text{ m})\), and (bottom) the intermediate parameters \((dx_s, dy_s) = (193 \text{ m}, 190 \text{ m})\) after 25 iterations. Note the alias energy is reduced by almost 90% after only 25 iterations.
shows the acquisition footprint migrated from the data generated by this survey. The associated alias energy in this migration image is 0.246, and after 286 iterations, the optimal survey parameters \((dx_s, dy_s)\)=(115.1 m, 24.1 m) are obtained with the associated alias energy reduced by 95%.

Figures 4.11, 4.12 and 4.13 show the footprint images migrated by the data generated from the initial, optimal and several intermediate geometry surveys. Table 4.3 shows the parameters and the associated alias energy. The alias energy quickly decreases at the early iterations and slow down after about 100 iterations. This is because the number of source stations is increasing to match the constraint, and after that only the source intervals are updated under the constraint that the number of source stations is a constant.

### 4.3.4 Example IV: Meandering Stream Channel Model

If the subsurface reflector is not a point scatterer but a complex structure such as meandering stream channel, can we still use this method to find the optimal survey?

To answer this question, suppose the source and receiver apertures are both over an area of 1667 X 1667 m\(^2\) and the receiver sampling interval is 167 m. The wavelength of the source is 3 m. and the meandering stream channel is located at the depth of 5000 m. Figure 4.14 shows this meandering stream channel model which consists of 101 point scatterers. What is the optimal distribution of 400 shot stations in this area?

The cost function (see equation 4.1) for this problem is modified, i.e., the migration image \(m(\mathbf{x})\) is computed by equation 2.14. The procedure is started from the initial parameters \((dx_s, dy_s)\)=(333 m,333 m) with the initial footprint alias energy 0.2391. After 223 iterations, the parameters converge to the optimal solution \((dx_s, dy_s)\)=(114.6 m,60.6 m) with the footprint alias energy reduced by 99.8%.

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>(dx_s)</th>
<th>(dy_s)</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>333 m</td>
<td>333 m</td>
<td>0.246</td>
</tr>
<tr>
<td>25</td>
<td>193 m</td>
<td>190 m</td>
<td>0.0348</td>
</tr>
<tr>
<td>80</td>
<td>141 m</td>
<td>103 m</td>
<td>0.0292</td>
</tr>
<tr>
<td>200</td>
<td>116 m</td>
<td>24 m</td>
<td>0.0069</td>
</tr>
<tr>
<td>240</td>
<td>115.3 m</td>
<td>24.1 m</td>
<td>0.0064</td>
</tr>
<tr>
<td>286</td>
<td>115.1 m</td>
<td>24.1 m</td>
<td>0.00125</td>
</tr>
</tbody>
</table>
Figure 4.12. The acquisition footprint images for the surveys with (top) the intermediate parameters \((dx_s, dy_s) = (141 \text{ m}, 103 \text{ m})\) after 80 iterations; and (bottom) the intermediate parameters \((dx_s, dy_s) = (116 \text{ m}, 24 \text{ m})\), after 200 iterations.
Footprint for Intermediate Parameters After 240 Iterations: Alias Energy = 0.064

Footprint for Final Parameters After 286 Iteration: Alias Energy = 0.00125

Figure 4.13. The acquisition footprint images for the surveys with (top) the intermediate parameters \((dx_s, dy_s) = (115.3 \text{ m}, 24.1 \text{ m})\), after 240 iterations; and (bottom) the final parameters \((dx_s, dy_s) = (115.1 \text{ m}, 24.1 \text{ m})\) after 286 iterations.
Figure 4.14. Meandering stream channel model, which consists of 101 point scatterers. This river is buried at a depth of 5000 m, and the reflectivity of the stream channel is 1.0.
Figures 4.15, 4.16 and 4.17 show the footprint images for the initial survey, optimal survey and several intermediate surveys, and Table 4.4 shows the survey parameters and the associated alias energy. This example shows that the constraints optimization method is also useful to design the optimal survey for complex subsurface model. But if the number of the point scatterers is large, it might take a large computing time to achieve the solution.

**Table 4.4.** Recording parameters and associated alias energy vs. iteration number for the monochromatic meandering stream channel problem.

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>$dx_s$</th>
<th>$dy_s$</th>
<th>Alias Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>333 m</td>
<td>333 m</td>
<td>0.2391</td>
</tr>
<tr>
<td>10</td>
<td>247.0 m</td>
<td>233.6 m</td>
<td>0.0504</td>
</tr>
<tr>
<td>45</td>
<td>116.3 m</td>
<td>111.3 m</td>
<td>0.0324</td>
</tr>
<tr>
<td>100</td>
<td>113.9 m</td>
<td>58.4 m</td>
<td>2.2e-3</td>
</tr>
<tr>
<td>180</td>
<td>115.0 m</td>
<td>60.4 m</td>
<td>5.19e-4</td>
</tr>
<tr>
<td>223</td>
<td>114.6 m</td>
<td>60.6 m</td>
<td>5.137e-4</td>
</tr>
</tbody>
</table>
Figure 4.15. The acquisition footprint images for the surveys with (top) the initial parameters \((dx_s, dy_s) = (333 \text{ m}, 333 \text{ m})\), and (bottom) the intermediate parameters \((dx_s, dy_s) = (247.0 \text{ m}, 233.6 \text{ m})\) after 10 iterations. Note the alias energy is reduced by almost 90\% after only 10 iterations.
Footprint for Intermediate Parameters After 45 Iterations: Alias Energy = 0.0324

Footprint for Intermediate Parameters After 100 Iterations: Alias Energy = 2.20e−3

Figure 4.16. The acquisition footprint images for the surveys with (top) the intermediate parameters \((dx_s, dy_s) = (116.3 \text{ m}, 111.3 \text{ m})\) after 45 iterations; and (bottom) the intermediate parameters \((dx_s, dy_s) = (113.9 \text{ m}, 58.4 \text{ m})\) after 100 iterations.
Footprint for Intermediate Parameters After 180 iterations: Alias Energy = 5.19e−4

Footprint for Final Parameters After 223 iterations: Alias Energy = 5.137e−4

Figure 4.17. The acquisition footprint images for the surveys with (top) the intermediate parameters \((dx_s, dy_s) = (115.0 \text{ m}, 60.4 \text{ m})\) after 180 iterations; and (bottom) the final parameters \((dx_s, dy_s) = (114.6 \text{ m}, 60.6 \text{ m})\) after 223 iterations.
CHAPTER 5

CONCLUSION AND DISCUSSION

The acquisition footprint noise in the migrated sections consists of migration artifacts associated with a discrete recording geometry. Such noise can corrupt the interpretation of seismic sections, especially for AVO studies. In this thesis, I showed how to efficiently evaluate the footprint noise and how to design the survey with the least footprint noise. A simple formula deduced from the diffraction stack migration formulae shows that the acquisition footprint is proportional to the stretched Fourier transform of the source and geophone sampling pattern. Similar to the Array theorem used by Optical/Electrical Engineers, I presented the Seismic Array theorem to rapidly compute this Fourier transform by a product of 1-D analytical functions. The SAT can be used to compute the migration image for both a fixed source-receiver survey and a roll-along survey, so it can be used to analyze the acquisition footprint of marine surveys as well as land surveys.

I showed that the point scatterer response of the farfield Kirchhoff migration operator is proportional to the stretched Fourier transform of the source and geophone sampling functions. For an orthogonal sampling grid, this Fourier transform reduces to a product of four analytical functions. Thus, 3-D prestack migration images of point scatterers can be rapidly computed to identify the recording geometry with the weakest "acquisition footprint." As examples, I computed the point scatterer responses for different recording geometries, including ARCO and Phillips 3-D surveys. Images of these recording footprints clearly distinguished the "good" footprints from the "bad" footprints. I also showed that the optimal survey geometry could be computed by a nonlinear optimization method. Results show a fairly rapid convergence for realistic survey constraints.

I conclude that the SAT can be used as a practical tool to design 3-D seismic surveys, where the "best" survey might be the one with the weakest acquisition footprint. This is the first time such a cost-effective tool is available for survey design.
REFERENCES


