ARBITRARY PARAMETER EXTRACTION WITH STATIONARY PHASE MIGRATION

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ABSTRACT

Reflectivity is not the only specular-ray related parameter that can be obtained through prestack migration. Surprisingly, parameters such as the source and receiver coordinates, the traveltime, the incidence angles, departure angles from the source points, emergence angles at the receiver points and the reflector normals can be obtained by prestack migration. Such parameters have many potential applications in stationary phase migration, traveltime inversion, migration velocity analysis, and AVO studies. Numerical tests on synthetic and field seismic data show that such parameters can be accurately extracted by stationary phase migration.

INTRODUCTION

Bleistein (1987) and Schleicher et. al. (1993), used the stationary phase method to derive true amplitude migration weighting functions to compensate for geometric spreading and transmission losses in order to recover reflection coefficients along reflectors. Their formulae showed that the contributions to the reflector images are dominated by the so-called specular ray traces which satisfy Snell’s law at the reflectors. The traces far from the vicinity of the specular raypaths introduce aliasing artifacts into the migration images.

Bleistein (1987) showed how to obtained the incidence angle for the specular rays in migration. As a generalization, I showed that many other parameters that are
related to specular rays can be obtained from migration. These features include source and receiver coordinates, reflection traveltimes, incidence angles, departure angles from the sources, emergence angles at the receivers as well as reflectors normals. These parameters have many potential applications in seismic data processing once they are obtained.

One application is to use the source and receiver coordinates to exclude the artifacts energies from the migration such that the migrated images have better horizon continuity and remarkably fewer migration artifacts. Such a migration scheme is called stationary phase migration (Chen, 1999a).

Another application is found in tomographic migration velocity analysis (Chen, 1999b). In tomographic velocity analysis, the depth residual moveout picked in the migrated common image gathers needs to be converted to traveltime residuals and backprojected along the raypaths. The incidence angle information extracted from migration can be used to convert depth residuals to travelt ime residuals. The source and receiver coordinates, along with the reflection coordinates, can be used to do 2-point ray tracing to find the raypaths for the specular rays along which the travelt ime residuals will be backprojected.

In this report, I show how to use the stationary phase theory to obtain the above parameters from prestack migration. Synthetic and field data examples are used to validate this procedure.

**CALCULATING SPECULAR RAY RELATED FEATURES IN PRESTACK MIGRATION**

In this section I show how to compute arbitrary parameters of specular ray during the migration process with stationary phase method. Stationary phase method was employed by Bleistein (1987) and Schleicher et al. (1993) to derive true amplitude migration operators.

The prestack migration operator, in Schleicher’s (1993) notation, is given by

\[ V(M) = \int_{\Omega} d\xi_{1} d\xi_{2} w(\xi, M) \hat{U}(\xi, \tau_{D}(\xi, M)), \quad (6.1) \]

where \( V(M) \) is the migrated image at point \( M \), \( w(\xi, M) \) is the migration weighting function and \( \hat{U}(\xi, \tau_{D}(\xi, M)) \) is the time-differentiated seismic data, and

\[ \tau_{D}(\xi, M) = \tau(S(\xi), M) + \tau(M, G(\xi)), \]

is the travelt ime from source point \( S \) to image point \( M \) and to receiver point \( G \). Here \( \xi \) is a parameter describing the source and receiver locations:

\[ S = S(\xi), \quad \text{and} \quad G = G(\xi). \]

The integration goes over the data domain \( \Omega \) which contains all of the seismic traces available, i.e. \( \xi \in \Omega \).
Applying the stationary phase theory to the above integral (Bleistein, 1987; Schleicher, et al, 1993) yields

\[ V(M = R) = w(\xi^*, M) \dot{U}(\xi^*, \tau_D(\xi^*, M)) C(\xi^*, M), \] 

where \( R \) is the reflector and \( C(\xi^*, M) \) is a function of the source-receiver coordinate parameter \( \xi \) and image coordinate \( M \), and is evaluated at \( \xi^* \) and \( M = R \):

\[ C(\xi^*, M) = e^{-i \frac{\pi}{2} [1 - \text{sgn}(\det(H_F))]} \left| \frac{1}{\sqrt{\text{det}(H_F)}} \right| \bigg|_{\xi = \xi^*}. \] 

The detail of the derivation and the definition of \( H_F \) can be found in Schleicher (1993) and is not important to our derivation. Similar derivation with different notations can also be found in Bleistein (1987).

The variable \( \xi^* \) is called the stationary point for \( M \) and is the solution of the following system of equations:

\[ \nabla_S [\tau(S, R) - \tau(S, M)] \cdot \frac{dS}{d\xi} + \nabla_G [\tau(G, R) - \tau(G, M)] \cdot \frac{dG}{d\xi} = 0, \]  

and

\[ \nabla_M [\tau(S, M) + \tau(G, M)] = 0, \]  

where \( \nabla_S \tau(S, M) \) is the gradient of \( \tau(S, M) \) with respect to the source coordinate \( S \). \( \nabla_G \tau(G, M) \) and \( \nabla_M \tau(S, M) \) are defined similarly. The first equation requires that the image point \( M \) must be on the reflector \( R \), and the second equation is a form of Snell’s law. A ray satisfying the above two equations is called the specular ray for image point \( M \).

The following conditions must be satisfied for the stationary phase method to be applied to migration integral equation 6.1 (Bleistein 1984, 1987; Schleicher, et al., 1993):

1. Stationary point \( \xi^* \) satisfying equations 6.4 and 6.5 exists for a given image point \( M \);

2. There exists only one such stationary point \( \xi^* \) in \( \Omega \), the domain of integration;

3. The integrand \( w(\xi, M) \dot{U}(\xi, \tau_D) \) smoothly vanishes away from the stationary point.

The integral in equation 6.1 won’t reduce to the analytic formula in equation 6.2 unless all three conditions are satisfied.

The significance of equation 6.2 is that all the quantities on the right hand side are only evaluated at the stationary point \( \xi^* \) and reflector point \( R \). Equation 6.2 indicates that if a stationary point \( \xi^* \) exists for an image point on the reflector \( (M = R) \), then the integration at this point \( M \) is determined by the evaluation of the seismic trace.
amplitude $\hat{U}(\xi, \tau_D(\xi, M))$, the weighting function $w(\xi, M)$, and a function $C(\xi, M)$ at the stationary point $\xi^*$.

Bleistein (Bleistein, 1987; Bleistein et al., 1987) pointed out that the opening angle of the specular ray can be obtained with the stationary phase method too. Liu (1995) used the stationary phase method to compute the derivative of depth residual moveout with respect to the velocity parameters. Here this idea is generalized to arbitrary parameters that are related to the stationary points and specular rays.

Assume $F(\xi, M)$ is an arbitrary quantity that is a function of the image point $M$ and source-receiver parameter $\xi$. Besides the image volume $V(M)$ obtained in migration, another volume can also be produced by plugging $F(\xi, M)$ into the integral of equation (6.1) as an extra weighting function:

$$V_F(M) = \int \int \Omega d\xi_1 d\xi_2 w(\xi, M) \hat{U}(\xi, \tau_D(\xi, M)) \cdot F(\xi, M).$$

Let’s now examine the integral when the image point $M$ is on the reflector. According to the stationary phase theory, if an point $\xi^*$ satisfying equations (6.4) and (6.5) exists for image point $M = R$, equation (6.6) reduces to

$$V_F(M = R) = w(\xi^*, M) \hat{U}(\xi^*, \tau_D(\xi^*, M)) C(\xi^*, M) F(\xi^*, M).$$

Notice that equation (6.7) is identical to equation (6.2) except a factor of $F(\xi^*, M)$. Therefore $F(\xi^*, M)$ can be obtained by the division between $V_F(M)$ and $V(M)$:

$$F(\xi^*, M) = \frac{V_F(M)}{V(M)}.$$  \hspace{1cm} (6.8)

Equation (6.8) shows that by producing one more image volume $V_F(M)$ during migration, the quantity $F(\xi^*, M)$ associated with the specular rays can be computed. There are many parameters which can be extracted from migration and have many potential applications in seismic data processing. In the following subsections some parameters are listed along with their potential applications.

**Source and receiver coordinates**

$F(\xi, M)$ can be chosen either to be the source point coordinate $S(\xi)$, the receiver point coordinates $G(\xi)$, or the source-receiver midpoint coordinates $X_m$:

$$F(\xi, M) = S(\xi)$$

$$F(\xi, M) = G(\xi)$$

$$F(\xi, M) = X_m(\xi)$$

$$= (S(\xi) + G(\xi))/2$$

Once obtained from migration, the source and receiver coordinates have several applications. First, the ray paths connecting the source, the reflection point, and
the receiver can be traced with a 2-point ray tracer and then used in traveltime
tomography and velocity analysis. Second, they can be used to judge if a trace
falls within the Fresnel zone of the specular ray and if it should be migrated or not.
The migration carried out in such a way is called stationary phase migration (Chen,
1999a). By excluding the non-specular ray trace energies, stationary phase migration
is able to produce fewer alias artifacts.

Traveltimes

\[ F(\xi, M) = \tau_D(\xi, m) = \tau(S(\xi), M) + \tau(G(\xi), M). \quad (6.10) \]

\( \tau_D(\xi^*, M) \) is the traveltime for the specular ray from the source to the reflection
point and to the receiver. According to the migration imaging condition, \( \tau_D(\xi^*, M) \) is
equal to the observed traveltime of the primary reflection event at the corresponding
trace at \( \xi^* \). Instead of being picked from data space manually, the observed traveltime
associated with certain reflectors can be extracted through the migration procedure
and then used in traveltime inversion. \( \tau_D(\xi^*, M) \) can also be used to reject the traces
which do not fall within the vicinity of the specular rays in stationary phase migration.

Incidence angles

\[ F(\xi, M) = \alpha_I(\xi, M) \]
\[ = \sin^{-1} \left[ \frac{\left| \nabla_M \tau(S, M) - \nabla_M \tau(M, G) \right|}{2 \left| \nabla_M \tau(S, M) \right|} \right], \quad (6.11) \]

where \( \alpha_I(\xi, M) \) is the half-opening angle at \( M \) for the rays \( SM \) and \( GM \). When \( M \) is
on the reflector \( (M = R) \), \( \alpha_I(\xi^*, M = R) \) extracted from the migration image volume
is the incidence angle of the corresponding specular ray at the reflector point \( R \).

The extracted incidence angle \( \alpha_I(\xi^*, R) \) has several possible applications. First, it
can be used to convert the migrated images from the offset domain directly into the
incidence angle domain which can then be analyzed for AVO information. Second,
it can be used in migration velocity analysis to convert the depth residual moveouts
picked in common image gathers into traveltime residuals. Third, it can be used to
mute the image amplitudes with large incidence angles in the common image gathers
after prestack migration. The common image gathers with large incidence angle
amplitudes muted will yield a better stacked image.

Departure angles at source points

\[ F(\xi, M) = \alpha_S(\xi, M) \]
\[ = \cos^{-1} \left[ \frac{\vec{n} \cdot \nabla_S \tau(S, M)}{\left| \nabla_S \tau(S, M) \right|} \right], \quad (6.12) \]
where \( \hat{n} \) is the unit normal at source \( S \). The parameter \( \alpha_S(\xi^*, R) \) extracted from the migration image volume is the ray departure angle at the source point \( S \) for the corresponding specular ray. It can be used to shoot rays from the source points down to the subsurface.

**Emergence angles at receiver points**

Similarly, the emergence angle of the specular ray at the receiver point \( G \) can also be extracted from migration:

\[
F(\xi, M) = \alpha_G(\xi, M) = \cos^{-1}\left( \frac{\hat{n} \cdot \nabla \tau(G, M)}{|\nabla \tau(G, M)|} \right),
\]

where \( \alpha_G(\xi^*, R) \) can also be used to shoot rays back to the subsurface from the receiver point.

**Reflector normals**

\[
F(\xi, M) = \alpha_n(\xi, M) = - (\nabla M \tau(S, M) + \nabla M \tau(G, M)),
\]

where \( \alpha_n(\xi, M) \) is the direction that bisects the rays \( SM \) and \( GM \), while \( \alpha_n(\xi^*, R) \) is the reflector normal at point \( R \), according to the Snell’s law.

Because the traveltime table \( \tau(S, M) \) is calculated during migration and is available, the above quantities \( \alpha_I, \alpha_S, \alpha_G \) and \( \alpha_n \) can be calculated from \( \tau(S, M) \) and \( \tau(G, M) \) at little cost.

In the following sections, some of the above mentioned parameters are extracted and displayed after prestack Kirchhoff migration is applied to either the synthetic or the field data.

**SYNTHETIC DATA EXAMPLES**

Common offset gathers were generated from the SEG/EAEG overthrust velocity model and were migrated to obtain both the image volume and the parameter volumes. The parameter volumes obtained include traveltimes, incidence angles, and source-receiver midpoint coordinates. To honor the stationary phase condition, each time one common offset gather is migrated. By migrating a common offset gather (or a common shot gather, a common receiver gather) each time, the third condition for the stationary phase approximation to the migration diffraction stack to be valid is guaranteed.
Figure 6.1a shows the traveltimes obtained by migrating a common offset gather (offset = 100m), and shows many wild values. The wild values occur where there is no reflector and therefore the stationary phase conditions are not satisfied. These wild values can be filtered out by a median filter. Figure 6.1b shows that, after median filtering, the traveltimes are quite reasonable and reliable.

Figure 6.2 shows the corresponding incidence angles and the midpoint coordinates. In fact, other parameters such as the reflector normals, departing and emerging angles can also be obtained.

The image volume and the parameter volumes can be resorted into common surface point gathers after all common offset gathers are migrated. Figure 6.3 shows the traveltimes, midpoint, incidence angle and the image gathers at one CDP point.

**Verification of the accuracy and reliability of the parameters extracted**

To verify the accuracy and the reliability of the parameters extracted, a common offset gather was migrated to produce both the image and the parameter volumes. Figure 6.4 shows the migrated image. The positions of the reflector in the windowed area was measured. The midpoint coordinates, traveltimes, and incidence angles for each image point were extracted from the parameter volumes accordingly, and plotted in Figure 6.5. The traces associated with the specular rays for each image point in that windowed area were found using the midpoint coordinates and plotted in Figure 6.6. The specular ray traveltimes associated with each image point were also plotted in Figure 6.6. According to the stationary phase theory, the traveltimes extracted should match the primary reflection events which were migrated to the reflector. Figure 6.6 shows that the traveltimes do match the events. Of course, the parameters extracted are not exact but accurate enough to be used in later applications. The accuracy and reliability are also confirmed by its successful applications in stationary phase migration (Chen, 1999a) and tomographic velocity analysis (Chen, 1999b).

**FIELD DATA EXAMPLES**

The feature extraction migration was also applied to the Husky data, which is a 2-D land data. Each time one common offset gather was migrated. Figure 6.7 shows the midpoint coordinates before and after median filtering. The incidence angles and traveltimes extracted from the migration of the same common offset gather is shown in Figure 6.8. The parameters extracted have smooth variations after median filtering and are reliable.
Figure 6.1: The traveltimes (a), before and (b), after median filtering. The units are seconds. The traveltimes were obtained by migrating a common offset gather with an offset of 100 m. The wild values were removed by the median filter.
Figure 6.2: (a), The incidence angles and (b), the source-receiver midpoint coordinates extracted from migrating a common offset gather with an offset of 100 m for the SEG overthrust data. The units are in degrees for the incidence angles and in meters for the midpoint coordinates, respectively. A median filter was applied to remove the wild values.
Figure 6.3: The migrated image and associated parameters at a surface location. (a) traveltime gather; (b) midpoint gather; (c) incidence angle gather and (d) common image gather. The vertical axis is depth and the horizontal axis is offset.
Figure 6.4: A migrated common offset gather. The specular-ray related source-receiver midpoint coordinates, the traveltimes, and the incidence angles are extracted from the parameter volumes for the reflector inside the windowed area, and is shown in Figure 6.5.
Figure 6.5: (a) The specular-ray related source-receiver midpoint coordinates, (b) the traveltimes, and (c) the incidence angles along the reflector shown in the windowed area in Figure 6.4.
Figure 6.6: The stationary phase traces for the reflector in the windowed area in Figure 6.4. The traces were found using the specular-ray related source and receiver coordinates. The white line is the specular-ray related traveltime. The traveltimes match the primary reflection events in the common offset gathers which were migrated to the reflector in the windowed area in Figure 6.4.
Figure 6.7: The source-receiver midpoint coordinates (a), before and (b), after median filtering, obtained by migrating a common offset gather of the Husky data. The wild values were removed by the median filter.
Figure 6.8: (a) Incidence angle and (b) traveltimes obtained from the migration of a common offset gather of the Husky data.
CONCLUSIONS

I showed that arbitrary parameters associated with specular rays can be extracted from prestack migration by generating extra image volumes. These parameters include the source and receiver coordinates, the traveltimes, the incidence angles, departure angles from the source points, emergence angles at the receiver points and the reflector normals. Such parameters have many potential applications such as in stationary phase migration, travelt ime inversion, migration velocity analysis, and AVO. Numerical tests on synthetic and field seismic data show that after median filtering, these parameters are quite reliable. Although served as the first step in the stationary phase migration scheme whose second step is to use the extracted parameters to suppress the migration artifacts, the feature extraction migration could be carried out separately.

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REFERENCES


