ABSTRACT

I present the equations for inverting and migrating crosscorrelograms, which are an extension of the known equations for migrating autocorrelograms. Crosscorrelating two traces is partly equivalent to taking the difference between their phases in the frequency domain, so crosscorrelogram migration is a form of interferometric waveform imaging. The benefit of crosscorrelogram migration, like autocorrelogram migration, is that it does not require knowledge of the source wavelet or source initiation time, which is the case, e.g., in IVSP while drilling experiments. In addition, it has the potential of mitigating problems with source and receiver static errors. For example, if there is a source static error due to a near-surface inhomogeneity then crosscorrelation between two traces in the same shot gather will mitigate this source static errors. Similar considerations are true for a receiver static error in a common receiver gather. The main problem with crosscorrelogram migration is the presence of events that are unphysical, i.e., correlations between reflections. It is hoped that such events will cancel with stacking, as shown in an earlier report on autocorrelation migration.

INTRODUCTION

Schuster and Zhou (1999) suggested the use of interferometric imaging using traveltimes. The potential benefits included mitigation with source timing errors, static errors and source location problems. Here I present the companion paper which explores the possible uses of waveform interferometric imaging, with potential benefits similar to that in travelt ime interferometry. The special case we treat is that of
crosscorrelogram migration, which has the potential of reconstructing the reflectivity distribution from reflection data. Similar to the companion paper, this report only treats the theoretical details of crosscorrelogram imaging, and does not consider the practical application to field data.

**THEORY OF CROSSCORRELOGRAM MIGRATION**

The derivation of the crosscorrelogram migration algorithm will follow that for a waveform inversion algorithm. That is, a crosscorrelogram misfit function will be defined, the gradient of the misfit function will be derived, and the first iterate solution to a steepest descent method will yield the equations for waveform interferometric imaging. The first iterate of the steepest descent method is crosscorrelogram migration. These steps are explained below.

Assume harmonic energy emanating from one source at \( \mathbf{r}_s \) and recorded by geophones at \( \mathbf{r}_g \) to yield the seismogram denoted by \( P(\mathbf{r}_g; \mathbf{r}_s) \). The observed spectrum of the crosscorrelogram associated with one shot gather is denoted by

\[
\tilde{\Phi}_{gg'}^{obs} = \tilde{P}(\mathbf{r}_{g'}; \mathbf{r}_s)^* \cdot \tilde{P}(\mathbf{r}_g; \mathbf{r}_s),
\]  

(15.1)

where the unknown source history has the spectrum denoted by \( W(\omega) \). The notation for angular frequency \( \omega \) will be usually suppressed, and a tilde denotes a function in the frequency domain.

1. The misfit function for the crosscorrelograms in a single shot gather is given by:

\[
\epsilon = \frac{1}{2} \sum_{g'} |\tilde{\Phi}_{gg'} - \tilde{\Phi}_{gg'}^{obs}|^2,
\]  

(15.2)

where \( \tilde{\Phi}_{gg'} \) is the predicted spectrum for the synthetic crosscorrelogram. The summation is over the \( g' \) geophone index, so all of the traces have been correlated with the \( gth \) trace in the shot gather.

2. The slowness model \( s(\mathbf{r})^i \) at the \( i^{th} \) iterate is updated by

\[
s(\mathbf{r})^{i+1} = s(\mathbf{r})^i - \kappa \gamma(\mathbf{r}),
\]  

(15.3)

where \( \kappa \) is the step length and \( \gamma(\mathbf{r}) \) is the misfit gradient defined as:

\[
\gamma(\mathbf{r}) = \frac{\delta \epsilon}{\delta s(\mathbf{r})},
\]

\[
= Re\left( [\tilde{\Phi}_{gg'} - \tilde{\Phi}_{gg'}^{obs}]^* \frac{\partial \tilde{\Phi}_{gg'}}{\partial s(\mathbf{r})} \right).
\]  

(15.4)
Here the summation notation is dropped because a single crosscorrelogram is now assumed for pedagogical simplicity. The Frechet derivative is defined as (see Appendix):

\[
\frac{\partial \Phi_{gg'}}{\partial s(\mathbf{r})} = [\hat{P}(\mathbf{r}_g', \mathbf{r}_s)]^* \frac{\partial \hat{P}(\mathbf{r}_g, \mathbf{r}_s)}{\partial s(\mathbf{r})} + [\hat{P}(\mathbf{r}_g, \mathbf{r}_s)]^* \frac{\partial \hat{P}(\mathbf{r}_g', \mathbf{r}_s)}{\partial s(\mathbf{r})},
\]

\[
= 2s(\mathbf{r})\omega^2 |\tilde{W}(\omega)|^2 \tilde{G}(\mathbf{r}_g | \mathbf{r}_s)^* \tilde{G}(\mathbf{r}_s | \mathbf{r}_g)
+ \tilde{G}(\mathbf{r}_g | \mathbf{r}_s) \tilde{G}(\mathbf{r}_s | \mathbf{r}_g)^* \tilde{G}(\mathbf{r}_g | \mathbf{r}_s)^*,
\]

(15.5)

where \(\tilde{G}(\mathbf{r}_s | \mathbf{r}_g)^*\) is the Green's function for a source at \(\mathbf{r}_s\) and an observer at \(\mathbf{r}_g\). Note that the Frechet derivative does not require knowledge about the phase spectrum of the source wavelet, it only requires an estimate of the magnitude spectrum of the source wavelet. The magnitude of the source spectrum \(|\tilde{W}(\omega)|\) can sometimes be estimated by taking the Fourier transform of the observed autocorrelogram out to the second zero crossing.

3. Substituting equation 15.5 into 15.4 yields the crosscorrelogram migration equation in the \(\omega\) domain.

Equation 15.4 can be physically interpreted by using the high-frequency Green's function for the special case of a single source and single geophone.

Case 1: Prestack Crosscorrelogram Migration.

Let

\[
\tilde{G}(\mathbf{r}_s | \mathbf{r}_g)^* = \frac{e^{-i\omega \tau_{gr}}}{r},
\]

(15.6)

where \(\tau_{sr}\) is the time for energy to propagate from the source at \(\mathbf{r}_s\) to the interrogation point at \(\mathbf{r}_s\), and \(1/r\) is the geometrical spreading term. Assuming a homogeneous model with a buried scatterer, and substituting equation 15.6 into equation 15.5 yields

\[
\frac{\partial \Phi_{gg'}}{\partial s(\mathbf{r})} = 2s(\mathbf{r})\omega^2 |\tilde{W}(\omega)|^2 [e^{i\omega(\tau_{g's'} - \tau_{g'r})}/(r_{sr}r_{gr}r_{g's'})
+ e^{-i\omega(\tau_{gs} - \tau_{g'r} - \tau_{sr})}/(r_{sr}r_{gr}r_{gs})]
\]

(15.7)

The composite migrated image is obtained by summing equation 15.7 over all frequencies to yield:

\[
\frac{\partial \Phi_{gg'}}{\partial s(\mathbf{r})} = -2s(\mathbf{r})[\tilde{\phi}(\tau_{gs} - \tau_{gr} - \tau_{sr})/(r_{sr}r_{gr}r_{gs})
+ \tilde{\phi}(\tau_{g's'} - \tau_{g'r} - \tau_{sr})/(r_{sr}r_{gr}r_{g's'})],
\]

(15.8)
where \( \ddot{\phi} \) is the second time derivative of the wavelet’s autocorrelogram function.

Inserting equation 15.8 into equation 15.4 and summing over all frequencies yields the crosscorrelogram migration equation:

\[
\gamma(\mathbf{r}) \approx \left( \Phi_{gg'} - \Phi^{obs}_{gg'} \right) \bigotimes \left[ \ddot{\phi}(\tau) \bigg|_{\tau=\tau_g+\tau_g'+\tau_{sr}} + \ddot{\phi}(\tau) \bigg|_{\tau=\tau_{g'}+\tau_{gr}-\tau_{sr}} \right]
\]

(15.9)

where \( \Phi_{gg'} - \Phi^{obs}_{gg'} \) is the crosscorrelogram of the scattered pressure field. Here we have neglected the multiplicative terms associated with geometrical spreading, constants, and \( s(\mathbf{r}) \).

Equation 15.9 says that the crosscorrelogram of the scattered field is crosscorrelated with the autocorrelogram of the wavelet and backprojected into the medium to give the migration image. Compare equation 15.9 to that for migrating reflection seismograms:

\[
\gamma(\mathbf{r}) \approx p(\mathbf{r}_g, \mathbf{r}_s, \tau) \bigotimes \tilde{W}(\tau) \bigg|_{\tau=\tau_{sr}+\tau_{gr}},
\]

(15.10)

which says that the scattered data are crosscorrelated with the wavelet’s second derivative and backprojected into the medium.

The crosscorrelogram imaging condition codified in the argument, say, \( \tau_{sr} + \tau_{gr} - \tau_{sg} \) is slightly different than the usual one for reflected waves. To see this, assume a wideband source (i.e., \( \ddot{\phi}(\tau) = \delta(\tau) \)), a homogeneous background medium (\( \Phi_{gg'} - \Phi^{obs}_{gg'} = \Phi^{scattered}_{gg'} \)), and a multi-trace shot gather so that the first term on the RHS of equation 15.9 reduces to

\[
\gamma(\mathbf{r}) \approx \sum_g \tilde{\Phi}^{scattered}_{gg'}(\tau_{sg} - \tau_{sr} - \tau_{gr}),
\]

(15.11)

which says that the image at point \( \mathbf{r} \) is obtained by summing scattered trace energy along the curve defined by \( \tau = \tau_{sr} + \tau_{gr} - \tau_{sg} \). Here the direct wave traveltimet \( \tau_{sg} \) at the \( g' \) trace is subtracted from the two-way scattered traveltimetime \( \tau_{sr} + \tau_{gr} \) to give the shifted curves shown on the RHS of Figure 15.1. The migrated crosscorrelogram image is obtained by summing energy along shifted hyperbola curves in the common shot crosscorrelogram. The direct wave traveltimetime from the source to the \( g' \) trace is used to downward slide the time scale of the graph.

Note that each seismogram on the LHS of Figure 15.1 contains only two "physical" events, a direct wave and a primary reflection. If this shot gather is crosscorrelated with the direct wave at the \( g' \) trace then there will only be two "physical" events in the resulting trace trace: the events corresponding to the correlation of the direct wave at the \( g' \) position with the direct wave and the primary reflection in the shot gather. If the primary reflection in the \( g \) trace is also allowed to participate in the correlation, then it will produce two extra events: the primary reflection at the \( g' \) position correlating with both direct waves and primary reflections. These latter two correlations are not shown in the RHS figure and are neglected. It is these "virtual
Figure 15.1: Left figure depicts a shot gather containing reflected (dotted) and direct (dashed) waves, and right figure depicts the associated crosscorrelograms after crosscorrelating this shot gather with the direct arrival in the trace at the g’ position. Seismograms are migrated by summing energy along hyperbolas (dotted curve in left figure), while crosscorrelograms are migrated by summing energy along shifted hyperbolas (dotted curve in right figure). The hyperbola is shifted by subtracting the direct arrival time at g’ position from the reflected arrival times.
multiples" that will contaminate the migrated crosscorrelogram image. Just like physical multiples, it is hoped that the "virtual multiples" will be attenuated with prestack migration of many shot gathers.

**Case 2: Source-Receiver Static Error Elimination.**

Assume a CDP shooting geometry and that the source and receiver static time shifts are denoted by $\delta t^s$ and $\delta t^q$, respectively. Here, the subscript denotes the geophone number and surface consistency is assumed for the statics (i.e., all rays are vertically incident through the near-surface inhomogeneity that gives rise to the static shift). Also assume a flat reflector at depth and the corresponding reflections are denoted as the "master" reflections (see Figure 2). For traces in an unprimed shot gather, the observed master reflection traveltime $T_i$ at the $i$th geophone is given by

$$T_i = t_i + \delta t^s + \delta t^q,$$  \hspace{1cm} (15.12)

where $t_i$ is the actual reflection traveltime if the inhomogeneity is replaced by sediment with the background velocity. We will assume that this actual reflection traveltime $t_i$ can be approximately calculated by tracing rays through some background velocity. A residual $\Delta T_{ij}$ in the unprimed shot gather can now be calculated so that the source static is eliminated:

$$\Delta T_{ij} = T_i - T_j$$

$$= t_i - t_j + \delta t^q - \delta t^q.$$  \hspace{1cm} (15.13)

Note, the source static is eliminated but there is a coupled receiver static denoted by $\delta t^q_i - \delta t^q_j$. To eliminate the coupled receiver static we form residuals that honor the phase closure principle (Schuster and Zhou, 1999):

$$\gamma_{ijk} = \Delta T_{ij} + \Delta T_{ki} + \Delta T_{jk},$$

$$= t_i - t_j + t'_k - t'_i + t''_j - t''_k,$$  \hspace{1cm} (15.14)

where the ' and " notation denotes traveltimes measured from two separate shot gathers different from the unprimed shot gather. This equation only contains the static-free master reflection traveltimes and is devoid of any source or receiver static errors. Thus, the $i$th trace from the unprimed shot gather has a master reflection arriving at the $i$th trace at time $T_i$, but this reflection can be time shifted to the correct time given by

$$\gamma_{ijk} - [ -t_j + t'_k - t'_i + t''_j - t''_k ] = t_i.$$  \hspace{1cm} (15.15)

Here, the value of $\gamma_{ijk}$ is measured from the master reflection traveltimes and the $t_i$’s are calculated from some background velocity model.
The above time correction can be implemented in many ways. One way is to measure the master reflection travel times, compute the phase closure corrections, and apply a manual time shift to the traces. A more complicated way is to mute all but the master reflections, crosscorrelate these master reflections with the original traces in the manner prescribed by the phase closure method, and perform crosscorrelogram migration.

**SUMMARY**

Equations are derived for both inverting and migrating crosscorrelograms. The crosscorrelograms are crosscorrelated with the autocorrelogram of the wavelet, and then migrated using the an imaging condition for crosscorrelograms. These migration equations can be applied to data where the origin time and/or the source wavelet is unknown. I also outlined a possible application of the phase closure principle to eliminate source and receiver statics. The next step will be to apply this method to both synthetic and field data to test its effectiveness and deepen our understanding of its limitations and merits.

**REFERENCES**


**APPENDIX**

The derivation of the Frechet derivative is now presented. For the 3-D Helmholtz equation we have:

\[
(\nabla^2 + \omega^2 s(|\mathbf{r}'|^2) \tilde{P}(\mathbf{r}'|\mathbf{r}_s)) = \tilde{F}(\mathbf{r}', \mathbf{r}_s)
\]

(15.16)
Src-Rec Static Elimination by Phase Closure

\[ \Delta t'_{31} = (t'_3 - t'_1) \]
\[ \Delta t'_{23} = (t'_2 - t'_3) \]
\[ \Delta t_{12} = (t_1 - t_2) \]

Figure 15.2: Rays associated with three shot gathers, where a ray terminates at the 1, 2, or 3 geophone, and the residuals where the source static is eliminated. The rays reflect off the "master" reflector and give rise to the "master" reflections. Variables associated with the 3 different shot gathers are denoted in unprimed, single primed or double primed notation.
where \( \tilde{P}(\mathbf{x}, \mathbf{z}) \) is the source term associated with a harmonically oscillating source at \( \mathbf{x}_s \), and \( \tilde{P}(\mathbf{x}'|\mathbf{x}_s) \) is the associated pressure field. Perturbing this equation with respect to perturbations in the slowness field, multiplying the result by the Helmholtz Green’s function \( \tilde{G}(\mathbf{x}'|\mathbf{x}_g) \) for the background slowness field \( s(\mathbf{x}') \), rearranging terms and integrating over all space \( V' \) yields:

\[
\int_{V'} \tilde{G}(\mathbf{x}'|\mathbf{x}_g)(\nabla^2 + \omega^2 s(\mathbf{x}')^2) \delta \tilde{P}(\mathbf{x}'|\mathbf{x}_s) dx' dy' dz' =
-2\omega^2 \int_{V'} s(\mathbf{x}') \tilde{P}(\mathbf{x}'|\mathbf{x}_s) \tilde{G}(\mathbf{x}'|\mathbf{x}_g) \delta s(\mathbf{x}') dx' dy' dz'.
\] (15.17)

Integrating the LHS of this equation by parts, invoking Green’s theorem, and using radiation boundary conditions at infinity gives:

\[
\delta \tilde{P}(\mathbf{x}_g|\mathbf{x}_s) = 2\omega^2 \int_{V'} s(\mathbf{x}') \tilde{P}(\mathbf{x}'|\mathbf{x}_s) \tilde{G}(\mathbf{x}'|\mathbf{x}_g) \delta s(\mathbf{x}') dx' dy' dz'.
\] (15.18)

Setting \( \delta s(\mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}') \delta s(\mathbf{x}) \) in the above equation yields:

\[
\delta \tilde{P}(\mathbf{x}_g|\mathbf{x}_s) = 2\omega^2 s(\mathbf{x}) \tilde{P}(\mathbf{x}'|\mathbf{x}_s) \tilde{G}(\mathbf{x}'|\mathbf{x}_g) \delta s(\mathbf{x}).
\] (15.19)

Dividing both sides by \( \delta s(\mathbf{x}) \) and replacing \( \delta \) by \( \partial \) gives the Frechet derivative for the pressure field:

\[
\frac{\partial \tilde{P}(\mathbf{x}_g|\mathbf{x}_s)}{\partial s(\mathbf{x})} = 2\omega^2 s(\mathbf{x}) \tilde{P}(\mathbf{x}'|\mathbf{x}_s) \tilde{G}(\mathbf{x}'|\mathbf{x}_g).
\] (15.20)

The crosscorrelogram spectrum of the pressure field is represented in the frequency domain by

\[ \tilde{\Phi}_{gg'} = \tilde{P}(\mathbf{x}_g|\mathbf{x}_s) \tilde{P}(\mathbf{x}_g'|\mathbf{x}_s)*, \]

so that the Frechet derivative of \( \tilde{\Phi}_{gg'} \) is given by:

\[
\frac{\partial \tilde{\Phi}_{gg'}}{\partial s(\mathbf{x})} = [\tilde{P}(\mathbf{x}_g'|\mathbf{x}_s)* \frac{\partial \tilde{P}(\mathbf{x}_g|\mathbf{x}_s)}{\partial s(\mathbf{x})}] + [\tilde{P}(\mathbf{x}_g|\mathbf{x}_s) \frac{\partial \tilde{P}(\mathbf{x}_g'|\mathbf{x}_s)*}{\partial s(\mathbf{x})}],
\]

\[
= 2\omega^2 |\tilde{W}(\omega)|^2 s(\mathbf{x}) |\tilde{G}(\mathbf{x}_g'|\mathbf{x}_s)*\tilde{G}(\mathbf{x}_g|\mathbf{x}_s) \tilde{G}(\mathbf{x}_g|\mathbf{x}_g) + \tilde{G}(\mathbf{x}_g|\mathbf{x}_s) \tilde{G}(\mathbf{x}_g|\mathbf{x}_s)* \tilde{G}(\mathbf{x}_g|\mathbf{x}_g)\]
\] (15.21)

where \( \tilde{W}(\omega) \) is the magnitude spectrum of the source wavelet, and the last step assumes \( \tilde{P}(\mathbf{x}_g|\mathbf{x}_s) = \tilde{W}(\omega) \tilde{G}(\mathbf{x}_g|\mathbf{x}_s) \).