ABSTRACT

Time migration is widely used in practice because of its lower cost and relatively insensitive to the errors in the migration velocity model. Seismic-attribute analysis is mainly based on time migration traces, so time-migration quality will influence the final attribute interpretation. However, time migration produces artifacts in the migration image, which can be reduced by time-migration deconvolution. To validate this statement, migration deconvolution is applied to poststack-time-migration images of the SEG overthrust model and a marine data set from the North Sea. The results indicate that migration deconvolution is able to attenuate the influence of the migration Green’s function in a migrated image, reduce related artifacts in the migration image, and improve spatial resolution and image quality. The next steps will be to test the time-migration deconvolution algorithm on 2-D and 3-D prestack data.

INTRODUCTION

Time migration is widely used in practice because of its efficiency and relative insensitivity to errors in the velocity model. Time migration is similar to depth migration except it is implemented in the time domain. Analogous to depth migration, the time-migration image can also be represented as an integration of the quasi-reflectivity distribution weighted by a time-migration Green’s functions domain. Thus the migration image is the quasi-reflectivity distribution smeared by the migration Green's function.

Migration deconvolution (Hu (1998); Hu and Schuster 1998, 1999a) is an effective technique for reducing the influence of the migration Green’s function in the migration
image, and it has been successfully applied to both post and prestack-depth migration images (Hu and Valasek, 1999). In this paper, the theory of depth-migration deconvolution is adjusted for time-migration deconvolution. The time-migration deconvolution method is tested on synthetic traces associated with a 2-D SEG/EAGE overthrust model, and a 2-D field marine data set.

**TIME MIGRATION DECONVOLUTION THEORY**

The time migrated image associated with primary-relection data can be represented (Schuster and Hu, 2000) by:

\[
m(x, y, t) = \int_{\Omega} G(x, y, t | x_o, y_o, t_o) R(x_o, y_o, t_o) dx_o dy_o dt_o,
\]

where \(m(x, y, t)\) is the migrated image, \(G(x, y, t | x_o, y_o, t_o)\) is the migration Green’s function (Schuster and Hu, 2000), \(R(x_o, y_o, t_o)\) is the reflectivity strength at \((x_o, y_o, t_o)\), and \(\Omega\) is the volume associated with the model space in \((x, y, t)\).

We assume that the time-migration-Green’s function is shift invariant in the horizontal coordinates, and apply a 2-D FFT to equation 2.1 to give

\[
\widehat{M}(k_x, k_y, t) = \int_{t_{min}}^{t_{max}} \widehat{G}(k_x, k_y, t | x_{ref}, y_{ref}, t_o) \widehat{R}(k_x, k_y, t_o) dt_o,
\]

where \((x_{ref}, y_{ref})\) is the reference position of the migration-Green’s function; and \(t_{min}\) and \(t_{max}\) defines the lower and upper limits of the time integration, respectively.

Assuming \(G(x, y, t | x_o, y_o, t_o)\) is localized and its influence is negligible beyond some critical time interval \(D\) above or below \((x_o, y_o, t_o)\), then equation 2.2 can be approximated by the following summation:

\[
\widehat{M}(k_x, k_y, t) \approx \sum_{i=-N}^{N} \widehat{G}(k_x, k_y, t | x_{ref}, y_{ref}, t_i) \widehat{R}(k_x, k_y, t_i) t_o \delta t;
\]

where \(t_i = t_o + i \cdot \delta t\), \(\widehat{R}(k_x, k_y, t_i) t_o\) is the deconvolved image for the \(2N + 1\) layers centered at the time \(t_o\).

Typically \(D\) is about 1 period of the source wavelet and \(N\) ranges between 4 or 6. Here we implicitly assume that the migrated image \(\widehat{M}(k_x, k_y, t)\) at \(t \in (t_i; i = -N, \ldots, N)\) is primarily influenced by the reflectivity distribution between the time levels \(t_o + N \cdot \delta t\) and \(t_o - N \cdot \delta t\). We replace \(\widehat{G}\) in equation 2.3 by \(| \widehat{G} |\), and solve for \(\widehat{R}(k_x, k_y, t_i) t_o\) by matrix inversion for each time level and all wavenumbers. The result is averaged for all time levels and inverse transformed to give the reflectivity.
\( R(x, y, t) \). In the algorithm, the migration Green’s function is approximated to be zero phase, and a 5-7 point median and 5-7 averaging filter is used to smooth the spectrum of the migration-Green’s function.

In migration deconvolution, the basic assumption is that the migration Green’s function invariant in the lateral coordinates. This implies a 1-D velocity model with an infinite recording geometry. However, for a laterally-variant velocity and a finite-recording geometry, the migration Green’s function is laterally variant. To account for lateral velocity variations, we divide the 2-D migration image into a number of vertical strips. Each strip is deconvolved with a separate migration-Green’s function. Finally, the entire deconvolved image is constructed by concatenating all the strips. In this way, migration deconvolution is able to partly account for the lateral velocity variations and attenuate far-field artifacts in the migrated image.

**NUMERICAL RESULTS**

Migration deconvolution was applied to two different migration images: 1) a 2D poststack Kirchhoff time-migration image associated with data from the SEG/EAGE Overthrust model, and 2) a 2D poststack Kirchhoff time-migration image of a North Sea data set.

**SEG/EAGE Overthrust Model**

Migration deconvolution was applied to a 2-D poststack Kirchhoff time-migration image associated with the SEG/EAGE overthrust model data. The starting reflectivity image was generated by applying Kirchhoff time migration to the stacked data to give the poststack time migration image shown at the top of Figure 2.1. Time-migration deconvolution was applied to the migrated image to give the result shown at the bottom of Figure 2.1. In order to account for the lateral velocity variations, the whole model is divided into 5 equi-sized vertical strips. Each strip is 128 gridpoints or traces wide and the reference position of the Green’s function for each strip is at the center of the strip. The waveform period increases with time, so the deconvolution filter length should also increase with time. Here a nine-layer migration deconvolution filter was used above 1.1 s, an eleven-layer migration deconvolution filter was used between 1.1 to 1.5 s, and a thirteen-layer migration-deconvolution filter was used below 1.5 s.

The zoom views of the boxed areas in the Figure 2.1 are shown at the top of Figure 2.2. Compared to the original time-migration image (leftside images), the deconvolved images (rightside images) are noticeably improved with better artifact attenuation.

To see how time-migration deconvolution suppresses artifacts caused by spatial aliasing, we constructed a sub-sampled data set consisting of every other trace in the original zero-offset data set. The half-sampled data were migrated to give the severely aliased image at the top of Figure 2.3. This image was deconvolved with a 9+11+13 variable-layer filter to give the result shown at the bottom of Figure 2.3. Compared
Figure 2.1: (Top) The 2-D poststack Kirchhoff time-migration image for the SEG/EAGE overthrust model, where all 312 traces were used to construct the image at any point. (Bottom) Image after applying time-migration deconvolution to the top image. The filter consisted of 9 layers for the [0, 1.1] second; 11 layers for the [1.1, 1.5] second, and 13 layers for the [1.5, 2.8] second. Compared to the original migration image, migration deconvolution noticeably attenuates some of the migration artifacts and noise, and improves the quality of the migration image.
Figure 2.2: The zoom views of the boxed areas in the image at the (top left) top and (top right) bottom of Figure 2.1. Bottom row of figures are the same as the top row except the 1/2 sampled data were used to induce aliasing artifacts. It is clear that migration deconvolution partly attenuates migration artifacts and noise, and improves the quality of the migration image.
Figure 2.3: (Top) The 2-D poststack Kirchhoff time migration image of the half-sampled zero-offset data. The footprint noise due to spatial aliasing is quite pronounced. (Bottom) Result of applying migration deconvolution to the migrated image in the top, where the noise is noticeably decreased and image quality is improved.
to the time-migration image at the top of Figure 2.3, there are fewer aliasing artifacts and better amplitude recovery in the deconvolved migration image at the bottom of Figure 2.3.

The zoom views of the boxed areas in Figure 2.3 are shown at the bottom of Figure 2.2. The left one is the original migration image, and the right one is the deconvolved image. It is clear that the spatial aliasing artifacts are partly attenuated by migration deconvolution.

**North Sea Field Data**

The time-migration deconvolution algorithm is applied to a poststack-time migration image computed from a 2-D North Sea data set (courtesy of Robert Keys, Mobil). Conventional data processing is applied to the data, and the NMO velocity model and the poststack-time-migration section are shown in Figures 2.4 and 2.5, respectively, where all traces are used to construct the image at any point. In these images, the number of CDP points is 2142, the image gridpoint spacing is \( dx = 12.5m, dt = 4ms \), and the number of time samples for each trace is 1500.

The 9+11 variable-layer time-migration-deconvolution filter was applied to the time-migration section and the result is shown at the bottom of Figure 2.5, where a nine-layer filter was applied above one second, and an eleven-layer filter was applied below one second. In order to account for the lateral-velocity variations, the whole image is divided into three strips and a different reference velocity profile was assumed at the center of each strip (indicated by the arrows in Figure 2.4).

The zoom views of the boxed areas in Figure 2.5 are shown in Figures 2.6 and 2.8. Compared to the original migration image at the top of Figure 2.5, the deconvolved-migration image at the bottom of Figure 2.5 is noticeably improved in resolution, migration artifact attenuation and reflectivity amplitude recovery. The zoom views of the boxed areas in Figure 2.5 are shown in Figures 2.6 to 2.8. In order to show the difference more clearly, Figure 2.9 is the zoom view of Figure 2.6. Compared to the zoomed images at the top of Figures 2.6 to 2.8, especially in Figure 2.9, the deconvolved migration image reveals more geological details and has the best overall quality.

In order to compare the MD image with the conventional processing results, an SU spike deconvolution with a band-pass filter (with bandpass 5-10-210-250 Hz) and a whitening filter are applied to the original time-migration image, and the zoom-view images are shown at the bottom of Figures 2.6 to 2.9, respectively. The frequency-domain response of the whitening filter is shown in Figure 2.10. Compared with the predictive-deconvolution image and whitening filter results, migration deconvolution is noticeably better in improving resolution and amplitude recovery.

A note of caution - we have not tested other post-migration filtering methods, so we can not claim that migration deconvolution is the optimal post-migration filter. However, it’s good performance on these data and the underlying physics used in it’s
Figure 2.4: The NMO-velocity model of this marine data set. This NMO velocity is used in time migration and migration deconvolution. The arrows at the top of this figure indicates a strip’s reference position for the three different Green’s function used in migration deconvolution.
Figure 2.5: (Top) A Kirchhoff poststack time migration image of this data set. (Bottom) The time-migration deconvolution result for a 9+11 variable-layer filter. The deconvolved image shows noticeable improvements in resolution, migration artifact, and noise attenuation, and reflection amplitude enhancement.
Figure 2.6: (Top row) The zoom views of the boxed area A in the images in Figure 2.5. (Bottom row) The zoom views of the conventional processed images in the same area, the left one is the spike-deconvolution image and the right one is the whitening-filter image. Compared with these four images, it is clear that migration deconvolution provides the image with higher spatial resolution and better overall quality.
Figure 2.7: (Top) Same as previous figure except box B in Figure 2.5 is enlarged. The time-migration deconvolution result is judged to have the best quality.
Figure 2.8: (Top) Same as previous figure except box C is enlarged.
Figure 2.9: The zoom views of Figure 2.6 highlight the differences between the migration deconvolution image and the other results. Compared with these four images, it is clear that migration deconvolution provides better resolution, and reveals more geological details.
Figure 2.10: The frequency response of the whitening filter.
design, make it a compelling choice.

CONCLUSIONS

The application of migration deconvolution to time-migration images indicates migration deconvolution can be used to partly attenuate migration noise, improve resolution, and enhance migration amplitudes. In principle, migration deconvolution is also able to lessen the dependence of the migration image on the source-receiver geometry, which is very important for seismic-image interpretation and seismic-attribute analysis. Using the subdivision method, migration deconvolution can partly account for lateral-velocity variations, and even attenuate some far-field artifacts in the migration image. Migration deconvolution can be applied to any part of the migrated image, so it is easy to implement this method in target-oriented processing and analysis. A good strategy might be to apply a cheap single filter to the entire image, and apply multiple deconvolution filters to the areas of interest. Future work will apply this method to 2-D and 3-D prestack time migration images.

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REFERENCES


