Report 4

Wavepath Migration versus Kirchhoff Migration: Theory and Poststack Examples

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ABSTRACT

3-D prestack Kirchhoff migration (KM) is too computationally intensive for iterative velocity analysis. This is partly because each time sample in a trace must be smeared along a quasi-ellipsoid in the model. As an alternative, we show that the stationary phase approximation to the KM integral restricts the smearing of the time sample along a small Fresnel zone portion of the quasi-ellipsoid. This is equivalent to smearing the time samples in a trace along a 1.5-D fat ray (i.e., wavepath), so we call this wavepath migration (WM). This compares to standard KM which smears the energy in a trace along a 3-D volume of quasi-concentric ellipsoids. In principle, WM has a computational count of $O(N^{5.5})$ compared to KM which has a computational count of $O(N^7)$, where $N$ is the number of grid points along one side of a cubic velocity model. Our results with poststack synthetic and poststack field data show that WM provides an image that is in some places less in quality than KM. However, the computation time of WM is less than 1/3 that of KM.

INTRODUCTION

Standard 3-D diffraction-stack migration (French, 1974) takes each time sample in a trace and smears that amplitude along a quasi-ellipsoid in the velocity model. The summation of all such smears for all of the prestack traces yields the 3-D prestack migrated image. For an $NxNxN$ grid of velocity cells, and $N^4$ traces, the computational count for 3-D prestack migration can be as high as $O(N^7)$. For large velocity models
this computational expense is too large, especially for iterative migration velocity analysis. To partially overcome this expense, we suggest a degraded form of KM we denote as WM. Briefly, WM smears a trace’s energy along fat rays or wavepaths rather than a volume of quasi-concentric ellipsoids. The dimension of a fat ray is about 1.5-D, so it is 1.5 fewer dimensions than a 3-D volume of ellipsoids. Thus, the computational cost of WM could, in principle, be considerably less than that of KM.

The idea of WM is not new. It is closely related to time-map migration (Sheriff and Geldart, 1985) which maps the reflection times along their corresponding skinny (rather than fat) rays to their place of origin in the velocity model. Relaxing the high frequency approximations allows the use of fat rays, whose width (Harlan, 1990) corresponds to the size of a Fresnel zone. These fat rays can be exploited to reduce the computational burden of prestack migration (Schleicher et al., 1997; Hill, 1990). Moreover, amplitudes from neighboring traces are smeared along the same wavepath, further reducing CPU time.

**STATIONARY PHASE GREENS FUNCTION FOR ZERO-OFFSET MIGRATION**

This section will derive the WM integral. The starting point will be the diffraction stack integral in the frequency domain, where a stationary phase approximation will be applied, and then an integration over a limited band of frequencies will be used to get the final form of the migration equation. The derivation will be for the 2-D case of a single shot gather, where the extension to the 3-D case is straightforward.

Assume geophones on the surface at \((x_g, 0)\), and a buried single reflector such that a common shot gather of traces \(\hat{D}(x_g, 0)\), for a harmonic point source at \((x_s, 0)\) is expressed as:

\[
\hat{D}(x_g, 0) = \hat{W}(\omega)A(x_s, x_g)e^{i\omega\tau_{sg}},
\]

where \(A(x_s, x_g)\) contains geometric spreading information, \(\hat{W}(\omega)\) is the spectrum of the source wavelet, and \(\tau_{sg}\) is the reflection traveltimes for primary reflections from a single subsurface reflector (see Figure 4.1). Here the data are recorded over a line of length \(D\) for a monochromatic source of angular frequency \(\omega\).

The prestack migrated section can be computed from substituting equation 4.1 into the diffraction stack formula:

\[
\hat{M}(\Phi, \omega) = \int_{0}^{D} \hat{D}(x_g, 0)\omega B(x_s, \Phi, x_g)e^{-i\omega(\tau_{gr}+\tau_{sr})}dx_g,
\]

\[
\hat{M}(\Phi, \omega) = \int_{0}^{D} \hat{W}(\omega)A(x_s, x_g)B(x_s, \Phi, x_g)e^{-i\omega(\tau_{gr}+\tau_{sr}-\tau_{sg})}dx_g,
\]

where \(B(x_s, \Phi, x_g)\) contains geometric spreading terms; and \(\tau_{gr}\) and \(\tau_{sr}\) correspond to the traveltimes for energy to propagate from the geophone to the subsurface interrogation point at \(\Phi\) and from the source to the interrogation point, respectively.
Figure 4.1: Dipping reflector embedded in a homogeneous halfspace below a receiver array of width $D$. The specular ”direct” ray is depicted as emanating from the image source at $(x_s, z_s)$ below the reflection plane; here the image source is along the perpendicular projection of the source below the reflection plane. The usual KM algorithm (equation 4.8) smears reflection energy along the fat ellipsoid shown above.

Defining $\phi(x_g) = \tau_{sg} - \tau_{gr} - \tau_{sr}$ where the $x_s$ and $\mathbf{r}$ notation is suppressed, setting $C(x_g, \mathbf{r}, x_s) = A(x_s, x_g)B(x_s, \mathbf{r}, x_g)$ in equation 4.2, and assuming a sufficiently high source frequency yields the stationary phase approximation (Bleistein, 1984) to the migration equation:

$$
\tilde{M}(\mathbf{r}, \omega) = \tilde{W}(\omega) \int_{0}^{D} C(x_g, \mathbf{r}, x_s) e^{i\omega\phi(x_g)} dx_g
\approx e^{i\omega\phi(x_g^*) + \mu\pi/4} \tilde{W}(\omega) C(x_g^*, \mathbf{r}, x_s) \sqrt{\frac{2\pi}{\omega|\phi(x_g^*)'|}},
$$

(4.3)

where $x_g^*$ is the stationary point of $\phi(x_g)$, $\mu = \text{sign}(\phi(x_g)^{''})$, and $\phi(x_g)^{''}$ is the second derivative of $\phi$ with respect to $x_g$. Here we assume a first-order stationary point that is interior to the integration range. The formula for the case of an exterior stationary point is given in Bleistein (1984).

To understand the physical meaning of equation 4.3 let us examine the special case of a reflector buried in a homogeneous medium.
Case of a Reflector in a Homogeneous Medium

Let the model be a buried reflector in a homogeneous background medium. Setting the gradient of the phase function in equation 4.3 to zero yields the stationary point condition:

$$\frac{\partial \phi(x_g)}{\partial x_g} = \frac{\partial \tau_{gr}}{\partial x_g} - \frac{\partial \tau_{gs}}{\partial x_g},$$

which implies

$$\frac{\partial \tau_{gr}(x_g^s)}{\partial x_g} = \frac{\partial \tau_{gs}(x_g^s)}{\partial x_g}. \quad (4.5)$$

Recognizing that $\frac{\partial \tau_{gs}(x_g^s)}{\partial x_g}$ is proportional to the incidence angle of the specular reflection ray at the free surface, equation 4.5 says that the stationary point at $(x_g^s, 0)$ demands that the migration interrogation point $\mathbf{r}$ be somewhere along the upgoing specular reflection ray that intersects the free surface at $x_g^s$ (see Figure 4.1).

Integrating equation 4.3 over a finite band of frequencies results in the raypath KM formula:

$$m(\mathbf{r}) = w_{mod}(\tau_{gs} - \tau_{gr} - \tau_{sr}) \mathbf{r} \in \text{raypath}, \quad (4.6)$$

where

$$w_{mod}(t) = 2\pi C(x_g^s, \mathbf{r}, x_s)[1/\sqrt{t}] \int_{-\infty}^{\infty} \tilde{W}(\omega)e^{i\omega t + i\mu x/4} d\omega] / \sqrt{|\phi(x_g^s)|}, \quad (4.7)$$

and $\otimes$ denotes temporal convolution.

Equation 4.6 says that, for the trace at $(x_g, 0)$, the reflection amplitude at time $\tau_{gs}$ should be relocated in model space to the specular reflection point. This compares to a typical migration formula for a single trace

$$m(\mathbf{r}) = w(\tau_{gs} - \tau_{gr} - \tau_{sr}) \mathbf{r} \in \text{model volume}, \quad (4.8)$$

which says that, for the single trace, the reflection amplitude at time $\tau_{gs}$ should be smeared in model space along the fat ellipsoid shown in Figure 4.1. Here an ellipse is described by $\delta(\tau_{gs} - \tau_{gr} - \tau_{sr})$, and $w(t)$ is the source wavelet function.

Equation 4.6 can be re-expressed as

$$m(\mathbf{r}) = w_{mod}(\tau_{gs} - \tau_{gr} - \tau_{sr}) \delta(\tau_{gr} + \tau_{s^*r} - \tau_{sg}), \quad (4.9)$$

where $\tau_{s^*r}$ is the traveltime for energy to propagate from the image source point (denoted as $s^*$) to the interrogation point at $\mathbf{r}$. Thus $\delta(\tau_{gr} + \tau_{s^*r} - \tau_{sg})$ describes the "direct" ray that starts as a solid line from the image source in Figure 4.1.
The asymptotic analysis restricted the migration formula to high frequencies, and so the reflection energy was backprojected onto raypaths. But the reflection energy really propagated along wavepaths (Woodward and Rocca, 1988; Luo and Schuster, 1991; Quintus-Bosz, 1992), and so we should backproject such energy along "wavepaths" not raypaths.

A wavepath is a "fat ray" defined by the Fresnel volume (Harlan, 1990), i.e., the region defined by the rays that start from the source point, end up at the receiver, and differ in arrival time by no more than a half period from the arrival time of the specular ray. Figure 4.2 depicts a wavepath, which can be described as approximating the raypath term in equation 4.9 by a bandlimited ray:

$$\delta(\tau_gr + \tau_s*r - \tau_sg) = \int_{-\infty}^{\infty} e^{i\omega(\tau_gr + \tau_s*r - \tau_sg)} d\omega$$

$$\approx \int_{-\infty}^{\infty} \tilde{W}(\omega)e^{i\omega(\tau_gr + \tau_s*r - \tau_sg)} d\omega$$

$$= W(\tau_gr + \tau_s*r - \tau_sg),$$

(4.10)

where \(\tilde{W}(\omega)\) limits the bandwidth of the spectrum. Substituting equation 4.10 into equation 4.9 yields the WM formula:

$$m(\xi) = w_{\text{mod}}(\tau_gr - \tau_gr - \tau_sr)w(\tau_gr + \tau_s*r - \tau_sg).$$

(4.11)

The above equation says that the reflectivity energy should be backprojected onto that part of the fat ray (i.e., wavepath described by \(w(\tau_gr + \tau_s*r - \tau_sg)\)) that intersects the fat ellipsoid described by \(w_{\text{mod}}(\tau_gr - \tau_gr - \tau_sr)\), as shown by the hatched region in Figure 4.2. This hatched area is much less than the area of the fat ellipsoid in Figure 4.1 and so should lead to reduced computation time, especially for 3-D migration.

It is noted that reciprocity demands a symmetrical smearing of reflection energy around and below the specular reflection point shown in Figure 4.2. This symmetry condition is accommodated by also smearing energy along the appropriate fat ray that emanates from \((x_s, 0)\) and connects to the image "geophone" position.

**Reflection Tomography without Interface Parameterization**

The WM algorithm can be adjusted to backproject reflection traveltimes along the wavepath without having to parameterize the reflection interface. This idea is a straightforward extension of wavepath traveltime tomography presented in Schuster and Quintus-Bosz (1993). The incidence angle of the upcoming ray at the receiver is computed by determining the \(\partial \tau_{gs}/\partial x\) from the CSG traces, and the incidence angle of the downgoing source ray is computed by determining the \(\partial \tau_{gs}/\partial x\) from the CRG. The traveltime residual is then smeared all along the associated wavepath to update...
Figure 4.2: Same as previous figure except a fat ray (i.e., wavepath) is depicted, where the intersection of the fat ray and the fat ellipsoid is where the reflection energy is smeared. This "smearing" area is much less than in the previous figure so should lead to reduced computation time.

the slowness.

IMPLEMENTATION OF WAVEPATH MIGRATION ALGORITHM

WM is not going to migrate the trace energy throughout a fat ellipsoid, but to a limited portion of the ellipsoid. Therefore, the key goal of WM is to determine this limited portion cheaply and accurately. The implementation of WM algorithm can be summarized as below:

Traveltime Picking

We start the algorithm by letting the computer automatically pick the reflection traveltimes. Prior to the traveltime picking, automatic gain control (AGC) should be applied to the original data. AGC is necessary for emphasizing the later reflections such that their amplitudes can surpass the background noise. This automatic picking works well even when the seismic events cross with each other. Only those events with very strong energy can be picked.
Incidence Angle

The most important step in WM is to find an efficient and accurate means for finding the incidence angles from the data. High accuracy for the incidence angle is required by this algorithm, because a small change of the incidence angle will result in a big change in the raypath orientation, especially when the velocity model is complex. In practice, the incidence angle is determined by performing a local slant stack over a window of traces centered at the sample of interest. The incidence angle is determined by choosing the slant angle associated with maximum coherence. In order to increase the accuracy, the seismograms should be interpolated.

Partial Trace Summation

A ray is computed for one trace, and the surrounding 6 traces are slant stacked according to the incidence angle. The stacked energy is backprojected along the ray. Another ray is computed except this is for the traces not used in the former slant stack, and the slant stack procedure is repeated for the new ray.

Shooting Ray Tracing

After finding the incidence angles, rays are shot from the geophones, and the raypaths are recorded. In our algorithm, the 2-D shooting ray tracer was provided by R. T. Langan of Chevron.

Fresnel Zone

Once the raypath has been found, the reflection energy can be migrated to the image point. But the reflection energy in the trace also originated from the surrounding image points which fill the Fresnel zone. Therefore, event energy will be smeared along the Fresnel zone of the reflector. In our algorithm, we apply a straight line approximation to determine the Fresnel zone, which is perpendicular to the plane defined by the incident and reflected rays at the image point.

Weighted Migration

Though all of the image points within the Fresnel zone contribute to the event energy in the trace, their contributions are not equal. Thus in the migration, different image points should be given different energy portions. In our algorithm, we project the event energy to the Fresnel zone weighted by the source wavelet (see equation 4.11).
NUMERICAL RESULTS

The WM algorithm is now tested on three poststack data sets: 2-D synthetic data associated with a point scatterer, 2-D synthetic data associated with the SEG Overthrust model, and 2-D marine field data collected by Mobil in the North Sea. The resulting migration images are then compared to the KM images.

2-D Point Scatterer Model

The model is a point scatterer in a homogeneous medium where 201 shot stations are uniformly distributed along the free surface with an interval of 20 m. The source is a 50 Hz Ricker wavelet, the medium velocity is homogeneous with \( c = 5000 \) m/s, the time sampling interval is 1.0 ms, and the model is represented by a 201 x 201 grid with a horizontal and vertical spacing of 20 m. Thus the total number of reflectivity values will be equal to 201 x 201 = 40401. A 2-D diffraction stack forward modeling algorithm is used to generate the synthetic seismograms, where the geometric spreading is applied. Figure 4.3a shows the forward modeled zero offset data.

Figure 4.3b shows the KM image, where the point scatterer is clearly resolved, but the migration artifacts are very strong. In contrast, the WM image in Figure 4.3c shows fewer migration artifacts, and the point scatterer is as well resolved as the KM image. Figure 4.3d shows the calculated incidence angles versus their theoretical values, and demonstrates that the two curves match well, except for the beginning traces and the ending traces where the coherence information is insufficient. This test demonstrates that with clean data and simple models the incidence angle of reflection events can be accurately searched by a local slant stack method.

SEG Overthrust Model

The SEG/EAGE Overthrust velocity model is shown in Figure 4.4. The corresponding common shot gather (CSG) data were generated by a finite-difference solution to the acoustic wave equation. Prior to migration, direct waves were first removed, and the trace energy after 2.5 s was muted because they were mostly polluted by multiples. The processed zero offset data are shown in Figure 4.5, and the related model parameters are given in Table 1.

Table 1. The parameters for the phase A data associated with the SEG/EAGE 3-D Overthrust model. Here the model is gridded into a \( N_x \times N_z \) grid, with grid intervals of \( (dx \times dz) \); a time sampling interval \( dt \); a source interval of \( ds \); the number of traces is equal to \( N_s \); and the source and receiver depths of \( sp \) and \( gp \), respectively.
The poststack KM image is shown in Figure 4.6. In the migration, all of the traces were used to construct the image at any subsurface point.

The first step in WM algorithm is to pick the event traveltimes on the time-offset section with AGC applied. The zero offset data after AGC are shown in Figure 4.7, where the later reflections are emphasized, but the random noise is exaggerated too. Figure 4.8 shows the seismic events which were picked automatically. The picked events in Figure 4.8 match the reflections in Figure 4.7 quite well. Once the events have been picked, the incidence angles associated with these events are found by searching for the maximum local coherence of a local slant stack among the trace samples in Figure 4.7. The local slant stack extends to the 3 neighboring traces on either side of the trace of interest.

Figure 4.9 shows the WM image. Compared to the KM image in Figure 4.6, WM image contains fewer migration artifacts, and is about as well resolved as the KM image. This can be clearly seen in box A, where the migration artifacts are as strong as the true reflectivities in the KM image, but much weaker in the WM image. But in box B, where the associated velocity model is very complex, KM delineates the structure somewhat better than WM. I believe that the complex structure introduced complexities into the data, which confused the traveltime picker. This problem can be somewhat alleviated for prestack data. In boxes C and D, where the velocity model is simple, WM shows fewer artifacts than in KM image, but generates the image with the same resolution of reflector interfaces. To eliminate some of the artifacts, migration deconvolution (Hu, 1998) was applied to the Figure 4.9 image. The results are shown in Figure 4.10 and show improved clarity of the reflections, especially in the deep layers beneath box B.

The computational costs for KM and WM are very similar, because the data are 2-D and the migration velocity model is complex. The computational cost for WM can be lowered by migrating fewer seismic events, by applying a narrower Fresnel zone, or by slant stacking the neighboring traces in the seismic section so that the neighboring traces are not used again.

**Mobil North Sea Data**

Robert Keys of Mobil provided us with 2-D marine data collected in the North Sea. The data parameters and model parameters are given in Table 2.

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$N_x$</th>
<th>$N_z$</th>
<th>$ds$</th>
<th>$dx$</th>
<th>$dz$</th>
<th>$dt$</th>
<th>Wavelet</th>
<th>Frequency</th>
<th>$sp$</th>
<th>$gp$</th>
</tr>
</thead>
<tbody>
<tr>
<td>311</td>
<td>621</td>
<td>187</td>
<td>50 m</td>
<td>25 m</td>
<td>25 m</td>
<td>8 ms</td>
<td>Ricker</td>
<td>15 Hz</td>
<td>-25 m</td>
<td>-25 m</td>
</tr>
</tbody>
</table>

Here the model is gridded into a $N_x \times N_z$ grid, with grid intervals of ($dx \times dz$); a time sampling interval of $dt$; a recording length of $N_t$; the number of traces is equal
to \( Ns \); and the source interval of \( ds \).

<table>
<thead>
<tr>
<th>( Ns )</th>
<th>( ds )</th>
<th>( dt )</th>
<th>( Nt )</th>
<th>( Nx )</th>
<th>( Nz )</th>
<th>( dx )</th>
<th>( dz )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>12.5 m</td>
<td>4 ms</td>
<td>1500</td>
<td>500</td>
<td>301</td>
<td>12.5 m</td>
<td>12.5 m</td>
</tr>
</tbody>
</table>

The poststack data along a 2-D line are shown in Figure 4.11, and the migration velocity is shown in Figure 4.12 (Courtesy of J. Hu, UTAM). Figure 4.13 (left) shows the KM image. In the migration, all of the traces were used to construct the image at any point in the model space. Figure 4.13 (right) shows the WM image. Compared to the KM image in Figure 4.13 (left), the WM image contains fewer migration artifacts but the reflection events are weaker, and is about as well resolved as the KM image.

**DISCUSSION**

The WM formula is derived under the stationary phase approximation for prestack data. Instead of smearing the reflection energy along a quasi-ellipsoid, WM smears reflection energy along a Fresnel zone centered about the specular reflection point. The wavepath is approximately a 1.5-D geometric entity compared to the concentric sequence of quasi-ellipsoids that fills a 3-D volume. Thus WM, in principle, can be more than an order of magnitude less expensive than standard migration.

Numerical tests on 2-D poststack synthetic data and 2-D poststack field data show that WM can generate reflectivity images of acceptable quality. Compared to KM, WM images contain fewer migration artifacts, and are about as well resolved as the KM images, but the overall quality is somewhat less than the KM images. The computational costs for KM and WM for the SEG/EAGE and North Sea data were very similar, mostly because the data are 2-D poststack data and did not backproject slant stacked traces. Nevertheless, the computational efficiency of WM could become significant for 2-D prestack data sets, especially for huge 3-D prestack data sets.

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**REFERENCES**


Figure 4.3: (a) Zero offset data associated with the 2-D point scatterer model. 201 traces were generated by a 50 Hz Ricker wavelet source; (b) KM image. The point scatterer is centered below the geophone configuration at a depth of 2000 m. All of the 201 traces were used to construct this image; (c) same as (b) except the WM image, where fewer migration artifacts are created; and (d) comparison of the computed incidence angles with their theoretical values. The smooth curve shows the theoretical values.
Figure 4.4: Overthrust velocity model associated with the Phase A data set (SEG/EAGE 3-D Modeling Series No.1).

Figure 4.5: Zero offset Phase A data set associated with the SEG/EAGE Overthrust model.
Figure 4.6: KM image of the zero offset data shown in Figure 4.5. The associated migration velocity is shown in Figure 4.4.

Figure 4.7: Same as Figure 4.5 except with AGC applied. On this section, the seismic events will be picked, and the incidence angles will be computed.
Figure 4.8: Picked seismic events from the AGC data in Figure 4.7. Almost all of the reflections have been picked, and only these picked events will be used to construct the image.

Figure 4.9: WM image. Compared to the KM image in Figure 4.6, this image contains fewer migration artifacts (see box A, C, and D), but the complex structure (see box B) is not as well resolved as the KM image.
Figure 4.10: Same as previous figure except with migration deconvolution applied (Courtesy of J. Hu, UTAM). The migration deconvolution have removed some of the migration artifacts and improved the image continuity.

Figure 4.11: Poststack traces for Mobil’s North Sea data (courtesy of R. Keys).
Figure 4.12: Migration velocity model associated with the zero offset data in Figure 4.11 (Courtesy of J. Hu, UTAM).

Figure 4.13: (left) Poststack KM image of the zero offset data shown in Figure 4.11. All of the traces were used to construct the image at any point in the model space. The migration velocity is shown in Figure 4.12; (right) same as the left figure except the WM image, where fewer migration artifacts are involved, but the events are weaker.