Multi-Component Separation of P-P and P-SV Waves by a Least-Squares Migration Method: Synthetic Tests on CDP Data

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ABSTRACT

Separating P-P and P-SV waves can sometimes be a difficult task for multicomponent data. Simple FK filtering methods do not perform suitably if the moveout trajectory is similar for different events. Here the least-squares migration filtering (LSMF) method is modified to separate P-P and P-SV waves recorded in multicomponent CDP data set. We exploit the property that different wave types have different moveout trajectories as well as particle motion directions. Based on such information, modeling operators are derived for multi-component data and the LSMF filtering operator is designed to separate P-P from P-SV waves. A four-layer velocity model and a salt body velocity model are used to test the method. Results show that P-P and P-SV can be separated by LSMF method.

INTRODUCTION

Nemeth (1996) developed the least-squares migration filtering (LSMF) method to separate coherent signals and coherent noise in seismic data. His method decomposed the modeling operator into a sum of signal and coherent noise modeling operators, and used these operators to separate signal and coherent noise in a least squares sense. The modeling operators were assigned to be the forward modeling operators and their inverses to be the least-squares migration operators. His study showed that
the success in separability was sensitive to the similarity between the observed signal and noise. Wang (1996) applied LSMF method to a multi-component crosswell data set, and the results showed that the particle motion information can help separate the P-P and S-S waves.

P-SV data can provide additional information to P-P data. In the paper of Sollid et al. (1996), P-SV data was used to image a reservoir zone while the P-P data could not map this zone because of gas leaking to the overburden sediments. However, the P-SV converted waves in the horizontal component may be mixed with P-P waves to confuse the interpretation. The FK filtering and stacking process can attenuate the P-P waves, but it can also damage the P-SV waves because the overlapping of P-P and P-SV energies in FK domain. To remedy this problem, this report describes my results in using LSMF method to separate P-P and P-SV waves from seismic data. Results show that the P-P and P-SV can be separated by LSMF method.

METHOD

**LSM filtering of coherent noise**

The basic equations of the LSMF algorithm are now derived. Assume that the data are a composite of signal and coherent noise components

\[ d \sim \mathbf{L} \sim \mathbf{m} = (\mathbf{L}_{z} \cdot \mathbf{L}_{s}) \begin{pmatrix} \mathbf{m}_{p} \\ \mathbf{m}_{s} \end{pmatrix} = \mathbf{L}_{z} \cdot \mathbf{m}_{p} + \mathbf{L}_{z} \cdot \mathbf{m}_{s}, \]  

(12.1)

where \( \mathbf{m} \) is the model vector and \( \mathbf{L}_{z} \) is a linear forward modeling operator. The model vector consists of two parts, the model describing the signal (\( \mathbf{m}_{p} \)) and the model describing the coherent noise (\( \mathbf{m}_{s} \)). Similarly, the modeling vector also consists of the two modeling operators, \( \mathbf{L}_{z} \cdot \mathbf{L}_{p} \) and \( \mathbf{L}_{z} \cdot \mathbf{L}_{s} \). To solve for \( \mathbf{m} \) the following parametric functional is formed:

\[ P(\mathbf{m}) = \| \mathbf{L}_{z} \mathbf{m} - d \|^{2}. \]  

(12.2)

The model that minimizes equation (12.2) is

\[ \mathbf{m} = (\mathbf{L}_{z}^{T} \mathbf{L}_{z})^{-1} \mathbf{L}_{z}^{T} d, \]  

(12.3)

or componentwise

\[ \begin{pmatrix} \mathbf{m}_{p} \\ \mathbf{m}_{s} \end{pmatrix} = (\mathbf{L}_{z}^{T} \mathbf{L}_{z} \cdot \mathbf{L}_{z}^{T} \cdot \mathbf{L}_{z}^{T} \cdot \mathbf{L}_{z}^{T} \cdot \mathbf{L}_{z}^{T})^{-1} \left( \begin{array}{c} \mathbf{L}_{z}^{T} \\ \mathbf{L}_{z}^{T} \end{array} \right) d, \]  

(12.4)
Figure 12.1: Rays associated with P-P and P-SV diffractions from a point scatterer model. The particle-motion direction of the P-P wave is parallel to the wave propagation direction, while the particle-motion direction of the P-SV wave is perpendicular to the wave propagation direction.
LSM filtering of P-P and P-SV waves

Figure 12.1 shows a point scatterer model with the particle motion vectors for the associated P-P and P-SV waves. The particle motion direction of the P-waves are parallel to the wave propagation direction, while the particle motion direction of the SV-waves are perpendicular to the wave propagation direction. Consequently the particle velocity components for the P-P and P-SV waves can be constructed as:

\[
V_{x}^{p-p} = V^{p-p}\cos\theta, \quad (12.5)
\]
\[
V_{z}^{p-p} = V^{p-p}\sin\theta, \quad (12.6)
\]
\[
V_{x}^{p-su} = V^{p-su}\cos\alpha, \quad (12.7)
\]
\[
V_{z}^{p-su} = -V^{p-su}\sin\alpha, \quad (12.8)
\]

where the P-P and P-SV particle velocity seismograms are denoted by \((V_{x}^{p-p}, V_{z}^{p-p})\) and \((V_{x}^{p-su}, V_{z}^{p-su})\), respectively, and the subscripts x and y denote the directions of the particle velocity motion. \(\theta\) denotes the angle between the P-P particle-motion and horizontal directions, \(\alpha\) denotes the angle between the P-SV particle-motion and horizontal directions (Figure 12.1), and:

\[
\|V^{p-p}\| = \sqrt{(V_{x}^{p-p})^2 + (V_{z}^{p-p})^2}, \quad (12.9)
\]
\[
\|V^{p-su}\| = \sqrt{(V_{x}^{p-su})^2 + (V_{z}^{p-su})^2}, \quad (12.10)
\]

The Kirchhoff integral is used as the forward modeling operator, i.e.,

\[
V^{p-p}(r, s, t) = \int \frac{1}{r_{sx}r_{xr}}w(t - \tau_{sx}^{p} - \tau_{xr}^{p})m^{p}(x)dx, \quad (12.11)
\]
\[
V^{p-su}(r, s, t) = \int \frac{1}{r_{sx}r_{xr}}w(t - \tau_{sx}^{p} - \tau_{xr}^{s})m^{s}(x)dx, \quad (12.12)
\]

where \(w(t)\) is the source wavelet, \(\tau_{sx}\) is the travelttime from the source point \(s\) to model point \(x\), and \(\tau_{xr}\) is the travelttime from the model point \(x\) to the receiver point \(r\), \(r_{sx}\) and \(r_{xr}\) are the geometric spreading terms, the superscripts \(p\) and \(s\) denote traveltimes for P-P and P-SV waves, and \(m^{p}(x)\) and \(m^{s}(x)\) represent the reflectivity distributions for P-P and P-SV waves, respectively. In this report the traveltimes and angles are calculated using an eikonal solver. Then the x- and z- component of P-P and P-SV waves can be calculated by using equations (12.5), (12.6), (12.7) and
The P-P and P-SV migration operators are the transposes of the forward modeling operators:

\[
m_p(x) = \int \frac{1}{r_{sz} T_{zz}} w(t - \tau_{sz}^p - \tau_{xz}^p) (V_{z}^{p-p}(r, s, t) \cos \theta + V_{z}^{p-p}(r, s, t) \sin \theta) dt (12.13)
\]

\[
m_s(x) = \int \frac{1}{r_{sz} T_{zz}} w(t - \tau_{sz}^p - \tau_{xz}^s) (V_{z}^{s-s}(r, s, t) \cos \alpha - V_{z}^{s-s}(r, s, t) \sin \alpha) dt (12.14)
\]

where \(\theta\) denotes the angle between the P-P particle-motion and horizontal directions, \(\alpha\) denotes the angle between the P-SV particle-motion and horizontal directions (Figure 12.1).

The P-P and P-SV moveout curves have similar slope at the apex location, but the P and S-wave particle motion directions are perpendicular. This feature is expected to aid in separating the two waves types.

### SYNTHETIC TEST

#### 4-layer velocity model

Figure 12.2 shows an inhomogeneous velocity model. Wang et al. (1996) used this model to demonstrate the feasibility of the P-SV velocity analysis. This model consists of four layers with depth dependent velocities (\(V_p\) and \(V_s\)) and velocity ratio. It also consists of a symmetric dipping structure with a dip angle of 30 degrees. The model is discretized into 200x140 grid. Using an eikonal solver, a synthetic seismic data set was generated. The prestack data set consists of 75 P-wave shots with a shot interval of 40m. Each shot shots into 200 receivers with a receiver interval of 20 m. Figures 12.3a and 12.3b show one common shot gather (shot 1) for the x- and z-components, respectively. Figures 12.4a and 12.4b show one common offset gather (offset = 400 m) for the x- and z-components, respectively. The P-P and P-SV wave arrivals appear in both the x and z components of the data.

The LSMF method was applied to these data in order to separate the P-P and P-SV waves. Figures 12.5a and 12.5b show the modeled and LSMF reconstructed z-component P-P waves for one shot gather, respectively. Figures 12.6a and 12.6b show the modeled and LSM filtered x-component of the P-SV arrivals for one shot gather. It is clear that the P-P and P-SV arrivals are well separated.

#### Velocity model with a salt body

The above LSMF method was also tested on a realistic velocity model, which is shown in Figure 12.7 (P-velocity). The velocity model is 5 km by 3 km and contains...
Figure 12.2: The 4-layer model. There are 76 sources distributed along the surface from 500 m to 3500 m with a shot interval of 40 m, and there are 200 receivers for each shot with an interval of 20 m.
Figure 12.3: Shot gather 1 for the four layer model in previous figure. The P-P and P-SV arrivals appear in both the (a) x-component data and (b) z-component data.
Figure 12.4: Common offset gather with an offset equal to 400 m for the four layer model in Figure 2. The P-P and P-SV waves appear in both the (a) x-component data and (b) z-component data.
Figure 12.5: Z-component data of (a) modeled and (b) LSM filtered P-P waves. The P-P waves are well separated from the P-SV waves.
Figure 12.6: X-component data of (a) modeled and (b) LSM filtered P-SV waves. The P-SV waves are well separated from the P-P waves.
Figure 12.7: The salt model with layer velocities (from top to bottom): 1480 m/s, 2200 m/s, 2600 m/s, 2800 m/s, 3000 m/s, 3200 m/s, 3400 m/s, 3600 m/s; and the salt velocity is 3500 m/s.
Figure 12.8: Synthetic shot gather for the salt model. (a) Horizontal component. (b) Vertical component. The P-SV converted waves in the horizontal component are mixed with strong P-P waves.
Figure 12.9: F-K spectrums for (a) P-P and (b) P-SV waves. The P-P and P-SV energies overlap each other in the F-K domain so that a dip filter cannot separate them.
several layers and a salt body. A constant $V_p/V_s$ ratio 2 is used to calculate the S-velocity model. Using this velocity model, a time table was calculated using an eikonal solver and synthetic seismograms were generated using the kirchhoff forward modeling method. Here the velocity contrast between the salt body and the sediment around is mild. According to Nemeth (1995), a wave equation solver is needed to compute correct traveltimes for a larger velocity contrast. Figures 12.8a and 12.8b show the horizontal and vertical component of one synthetic shot gather, with the shot position at 1500 m and the 201 geophones located on the surface with sampling interval of 25 m. The P-SV converted waves in the horizontal component are mixed with strong P-P waves. Figures 12.9a and 12.9b show the F-K spectrums for P-P and P-SV waves, respectively. The P-P and P-SV arrivals overlap each other in the F-K domain so that a dip filter cannot separate it except for the shallow part. The LSMF method is applied to separate the P-P and P-SV waves. Figures 12.10a and 12.10b show the modeled and LSM filtered x-components of the P-P waves, respectively. Figures 12.11a and 12.11b show the modeled and LSM filtered x-components of the P-SV waves, respectively. Although there exist artifacts for the early arrivals, most of the P-P and P-SV waves are correctly separated.

CONCLUSION

The LSMF method for a multi-component data set was applied to separate P-P and P-SV waves. The synthetic tests show that the P-P and P-SV waves appear in both the horizontal and vertical seismograms, and they can be separated by the LSMF method. Future tests will apply this method to multi-component CDP data, and extend it to S-P and S-S filtering.

REFERENCE


Wang, Y. and Nemeth, T., 1996, Multi-component separation of PP and SS by least-squares migration method: Synthetic and field tests: Univ. of Utah To-
Figure 12.10: Comparison between (a) modeled and (b) LSM filtered x-component P-P waves. The P-P waves are well reconstructed from the horizontal shot gather in Figure 8.
Figure 12.11: Comparison between (a) modeled and (b) LSM filtered x-component P-SV waves. The P-SV waves are well reconstructed from the horizontal shot gather in Figure 8 except some artifacts appear at early times.

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