Report 1

AVO Possibilities With Least Squares Migration

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ABSTRACT

Application of least-squares Kirchhoff migration (LSM) to synthetic reverse vertical seismic profile data (RVSP) demonstrates that LSM reduces the migration artifacts, improves the image resolution and produces a more accurate image than the standard migration, but cannot completely recover the model reflectivity for AVO analysis because of the limited data coverage. The influence of incomplete data on the recovery of reflectivity is discussed and a method using comparable image points as the basis for AVO migration/imaging is proposed.

INTRODUCTION

Amplitude variation with offset (AVO) can provide direct detection of hydrocarbon reservoirs. There are various means for extracting AVO information from seismic data. One approach is AVO migration (de Bruin and Berkhout, 1992; Mosher, etc., 1996). The goal of AVO migration is to provide migrated images with amplitudes estimating the reflector reflectivities.

Least squares migration (Nemeth, 1996; Schuster, 1992 and 1997) is capable of reducing migration artifacts and therefore produces better migrated images. In this progress report, we test the possibility of obtaining true reflectivity images by applying least squares migration to synthetic seismic data.

The standard Kirchhoff migration operator is the adjoint of the forward modeling operator. The image obtained by standard Kirchhoff migration is a filtered version of the true reflectivity model and contains migration artifacts. Least-squares migration (LSM) tries to find a model that minimizes the data misfit function in an iterative way. It is believed that LSM can reduce migration artifacts, and therefore make better reflectivity estimates.
LEAST SQUARES KIRCHHOFF MIGRATION

Nemeth (1996) and Schuster (1992 and 1997) discuss least-squares migration extensively. Standard seismic migration operators represent the adjoint of a seismic forward modeling operator (Claerbout, 1992). Although the adjoint operator is a useful approximation to the inverse of the forward modeling operator and is widely used in the industry, it is not the exact inverse.

In general, the seismic forward modeling operator is given by

\[ \mathbf{d} = \mathbf{L} \mathbf{m}, \] (1.1)

where \( \mathbf{d} \) is the vector of modeled seismic data, \( \mathbf{m} \) is the reflectivity model vector and \( \mathbf{L} \) is a linear forward modeling operator. \( \mathbf{L} \) is assumed to simulate wave propagation in the earth media, which is not exactly correct.

Standard migration uses the adjoint of the forward modeling operator to give

\[ \mathbf{m}_{\text{mig}} = \mathbf{L}^T \mathbf{d}, \] (1.2)

where \( \mathbf{m}_{\text{mig}} \) is the standard migrated section. Substituting equation (1.1) into this expression yields

\[ \mathbf{m}_{\text{mig}} = \mathbf{L}^T \mathbf{L} \mathbf{m}. \] (1.3)

Equation (1.3) shows that the standard migrated model vector \( \mathbf{m}_{\text{mig}} \) is an \( \mathbf{L}^T \mathbf{L} \)-filtered version of \( \mathbf{m} \). The filter \( \mathbf{L}^T \mathbf{L} \) has the following characteristics:

(a) The diagonal elements of \( \mathbf{L}^T \mathbf{L} \) have different values. This is because geometric spreading is not compensated for during migration. When proper migration weights are used the diagonal elements can have similar values, which facilitates recovering the true model parameters.

(b) The off-diagonal elements of \( \mathbf{L}^T \mathbf{L} \) are non-zero and represent the migration artifacts in \( \mathbf{m}_{\text{mig}} \). The presence of non-zero off-diagonal elements makes it difficult to recover the true reflectivity.

Least squares migration gives a model that minimizes the objective function

\[ P(\mathbf{m}) = ||\mathbf{L} \mathbf{m} - \mathbf{d}||^2, \] (1.4)

which yields

\[ (\mathbf{L}^T \mathbf{L}) \mathbf{m} = \mathbf{L}^T \mathbf{d}, \] (1.5)

or

\[ \mathbf{m} = (\mathbf{L}^T \mathbf{L})^{-1} \mathbf{L}^T \mathbf{d}. \] (1.6)

Equation (1.6) shows that if the inverse of \( \mathbf{L}^T \mathbf{L} \) exists, the true reflectivity model \( \mathbf{m} \) can be recovered from the data \( \mathbf{d} \). The LSM algorithm is implemented with a conjugate gradient algorithm discussed in Nemeth (1996) and Schuster (1992 and 1997).
SYNTHETIC DATA TESTS

The LSM method is applied to three simple models to test its ability to produce true reflectivity images. The first model is a point scatterer model, the second model is a one layer model and the third model is a more realistic multi layer model suggested by Sen Chen of Exxon.

Point Scatterer Model

In the point scatterer model shown in Figure 1.1, three point scatterers with different reflectivities are buried in an homogeneous medium. Table 1 shows the coordinates and reflectivities of the point scatterers, where \( x_p \) is the horizontal distance from the left side and \( y_p \) is the depth. The reflectivity distribution is also shown in Figure 1.2. The model scale is 5,000 ft long in the horizontal direction and is 7,000 ft deep. Figure 1.1 shows the survey geometry for the point scatterer model. The shot point is on the free surface and is 500 ft away from the well which is at the left side of the model. There are 70 geophones placed along the well, and the geophone spacing is 30 ft. The geophones are distributed between the depth of 1,000 ft and 3,000 ft. The vertical component records of one shot gather with 70 traces are generated and migrated, where the Kirchhoff modeling operator is used. The source wavelet is a zero-phase Ricker wavelet with a central frequency of 40 Hz.

We expect that after least squares migration, the relative amplitudes of the point scatterers in the migrated image will be proportional to their reflectivities.

Table 1. Point scatterer coordinates and reflectivities.

<table>
<thead>
<tr>
<th>Pt. Scatterer</th>
<th>( x_p ) (ft)</th>
<th>( y_p ) (ft)</th>
<th>Reflectivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,000</td>
<td>2,000</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>3,000</td>
<td>800</td>
<td>0.75</td>
</tr>
<tr>
<td>C</td>
<td>1,000</td>
<td>4,000</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The seismogram is obtained using the following forward modeling operator

\[
d(r, s, t) = \int \cos \theta \frac{\cos \theta}{r_{sx}r_{xr}} w(t - \tau_{sx} - \tau_{xr}) m(x) dx
\]

where \( d(r, s, t) \) represents the data and \( m(x) \) denotes the reflectivity model; \( w(t) \) is the source wavelet and \( \tau_{sx} \) and \( \tau_{xr} \) are the traveltimes for energy to propagate from the source point \( s \) to the model point \( x \), and from the model point \( x \) to the receiver point \( r \), respectively; \( r_{sx} \) and \( r_{xr} \) are the geometric spreading terms; and \( \theta \) is the angle between the ray direction and the vertical direction at the receiver point.

The forward modeling operator accounts for geometric spreading and reflectivity. The modeled seismograms are shown in Figure 1.3. This operator is also used as forward modeling operator \( L \) in the least squares migration scheme. Therefore \( L \) fully describes the wave propagation properties in the modeled seismograms. There is no unmodeled physics in the seismograms.
Figure 1.1: The point scatterer model.
Figure 1.2: The reflectivity distribution for the point scatterer model.
Figure 1.3: The RVSP seismograms predicted with the forward modeling operator and the point scatterer model.
Figure 1.4: The standard migration image for the point scatterer model. The three point scatterers were imaged at the correct locations, but the relative reflectivity of the point scatterers are not correctly recovered

The migration operator used in the LSM iterations is the adjoint of the forward modeling operator:

\[ m(x) = \int \int \frac{\cos \theta}{r_{sx} r_{xt}} w(t - \tau_{sx} - \tau_{xt}) d(r, s, t) dr ds. \] (1.8)

Figure 1.4 shows the standard migration image which is also used as the starting image for the LSM iterations. The LSM migrated image after 300 iterations is shown in Figure 1.5. In the true model, point scatterer A has a reflectivity value of 1.00; point B has a value of 0.75 and point C has a value of 0.50. In the standard migration image, the point scatterer C has the largest amplitude and point B has the smallest amplitude. The amplitudes of the point scatterers in the standard migration image do not indicate the actual reflectivity distribution shown in Figure 1.2.

After 300 iterations of least squares migration, the migration artifacts are reduced. As we expected, point A has the largest amplitude. However, point B has a smaller amplitude than does point C. In the true model, point B has a larger reflectivity than does the point C. The LSM image is more accurate than the standard migration image, but it still is not a good estimate of the true reflectivity distribution. The
amplitudes of point scatterers A, B, and C are measured from both the standard migration image and the LSM image and are plotted in Figure 1.6. The reflectivities of the model are also plotted in Figure 1.6 for comparison. It shows that LSM gives much better estimates of the reflectivities for point scatterer A, B and C than does the standard migration.

The point scatterer test shows that, for the point scatterer model and the RVSP geometry, the LSM reduces the migration artifacts, improves the image resolution and provides better reflectivity estimations. But the LSM can not completely recover the relative reflectivities of the three point scatterers.

**One Layer Model**

This model simulates a reverse vertical seismic profile (RVSP) shooting geometry for a one layer over an half space model. The model is 1,000 ft long in the horizontal direction and 5,000 ft in depth extent. The depth of the horizontal layer interface is 4,000 ft. We assume that the reflectivity takes the value of one along the reflector and zero elsewhere. Figure 1.7 shows the reflectivity distribution.

There are 10 shot points at the free surface with a shot spacing of 50 ft. The minimum shot offset is 500 ft from the well. There are 50 geophones placed in the
Figure 1.6: Comparison of the reflection coefficients. Circle represents the model reflection coefficients of the reflectors at the normal incidence. ”+” represents the reflection coefficients estimated from the standard migration image and ”∗” represents the reflection coefficients estimated from the least squares migration image. LSM gives much better estimates of the reflectivities for point scatterer A, B and C than does the standard migration.
well with a geophone spacing of 30 ft, and the geophone depths range from 1,000 ft to 2,500 ft. A 40 Hz Ricker wavelet is used to generate the seismogram.

This geometry covers the horizontal reflector at offsets from 140 ft to 410 ft from the well. An ideal image is that the amplitude is one at the depth of 4,000 ft with an offset from 140 ft to 410 ft along the reflector, and zero elsewhere.

Figure 1.8 shows the standard migration image and Figure 1.9 shows the LSM image after 300 iterations. The LSM image has fewer migration artifacts and better spatial resolution than the standard migration image. It suggests that LSM will provide more accurate AVO estimates.

**Multi Layer Model**

Table 2 shows the parameters for the multi-layer model suggested by Sen Chen of Exxon. This model has 10 layers and some of the layer interfaces slightly dip. The dipping angles range from 1 to 5 degrees. The P velocities increase from 4,333 ft/s in the first layer to 19,353 ft/s, while the S velocities change from 2,000 ft/s to 10,186 ft/s. The attenuation factors $Q_p$ and $Q_s$ are 100 and 80, respectively for each layer. The first layer represents a shallow low velocity zone and the 10th layer is a low velocity layer. The model has a scale of 4,000 ft in the horizontal direction and 6,000 ft in the depth direction. Figure 1.10 shows the P velocity distribution.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>P Velocity (ft/s)</th>
<th>S Velocity (ft/s)</th>
<th>Density (g/cm$^3$)</th>
<th>Dip Angle (Deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>143</td>
<td>4333</td>
<td>2000</td>
<td>2.4</td>
<td>0</td>
</tr>
<tr>
<td>572</td>
<td>7000</td>
<td>3400</td>
<td>2.7</td>
<td>0</td>
</tr>
<tr>
<td>1113</td>
<td>9001</td>
<td>4600</td>
<td>2.7</td>
<td>1</td>
</tr>
<tr>
<td>1913</td>
<td>9987</td>
<td>4700</td>
<td>2.7</td>
<td>2</td>
</tr>
<tr>
<td>2646</td>
<td>12278</td>
<td>6200</td>
<td>2.7</td>
<td>3</td>
</tr>
<tr>
<td>3466</td>
<td>13409</td>
<td>7057</td>
<td>2.7</td>
<td>4</td>
</tr>
<tr>
<td>4052</td>
<td>14627</td>
<td>7698</td>
<td>2.7</td>
<td>5</td>
</tr>
<tr>
<td>5093</td>
<td>17321</td>
<td>9116</td>
<td>2.7</td>
<td>0</td>
</tr>
<tr>
<td>5871</td>
<td>19353</td>
<td>10186</td>
<td>2.7</td>
<td>0</td>
</tr>
<tr>
<td>6000</td>
<td>18279</td>
<td>9621</td>
<td>2.7</td>
<td>0</td>
</tr>
</tbody>
</table>

In this model a reverse vertical seismic profile (RVSP) survey geometry is implemented. A well is located at the left side of the model area. 14 shot stations are spaced along the free surface with a shot interval of 150 ft. The minimum shot offset from the well is zero and the maximum offset is 1,950 ft; and 97 geophones are placed along the well from 1,000 ft to 3,900 ft.

A 2-D visco-elastic forward modeling scheme is used to generate the synthetic seismograms for the RVSP shooting geometry. A 40 Hz zero-phase Ricker wavelet is used as the source wavelet. The seismogram contains direct P and S waves, reflected
Figure 1.7: One layer model. The reflectivity is one along the reflector and zero elsewhere.
Figure 1.8: The standard migration image.
Figure 1.9: The LSM image after 300 iterations.
P and converted S waves and multiples from the shallow low velocity zone. An $f-k$ domain dip filter is applied to these traces to remove the strong direct waves. Figure 1.11 shows one common shot gather after dip filtering.

The data were first migrated using the standard Kirchhoff migration to give the standard migration image shown in Figure 1.12. The standard migration image was used as the starting model for the least squares migration. The forward modeling operator and the migration operator in the least squares migration scheme is designed to account for the P-P reflections, while converted P-SV waves and the multiples are not accounted at this time. Also the attenuation was accounted at this time. Figure 1.13 shows the least squares migration image after 50 iterations.

The reflector at the 5,800 ft depth cannot be seen in the standard migration image but it can be found in the least squares migration image.

The amplitudes of the reflector at 5,000 ft are stronger in the least squares migration image than in the standard migration image.

Figures 1.14 and 1.15 are the expanded view of the migration images with offsets between 100 ft and 300 ft for both the standard migration image and the LSM image. Figures 1.14 and 1.15 show that the least squares migration image has a better vertical image resolution than the standard migration image because the images of the reflectors are sharper in the least squares migration image.

The amplitudes of reflectors B, C, D, E, F and G at the offset of 200 ft are measured from the standard and the least squares migration images and plotted in Figure 1.16. To identify which method provided better estimation of the true reflectivity, the model reflection coefficients at normal incidence for each reflector are also plotted in Figure 1.16. Figure 1.16 shows that both methods provide acceptable reflectivity estimations for reflectors B, C, D and E. For reflector F, the least squares migration provides a much closer estimate of the reflection coefficient than the standard migration. Although least squares migration gives a smaller reflectivity estimate of reflector G, standard migration even fails to image this reflector.

This example shows that although least squares migration can not completely recover the reflectivity model, it can provide more accurate estimations than can standard migration.

INVERTIBILITY AND INCOMPLETE DATA

The point scatterer and layer model examples show that least squares migration reduces migration artifacts, improves the image resolution and produces more accurate reflectivity amplitudes. But it cannot completely recover the true reflectivities.

The capability of imaging reflectivity depends on the invertibility of the filter operator $L_T L_z$ in equation (1.5). Theoretically, if the inverse $(L_T L_z)^{-1}$ exists, the reflectivity model $m$ can be correctly reconstructed and used for AVO analysis. The existence of $(L_T L_z)^{-1}$ requires a complete and continuous data distribution along a closed surface which surrounds the model $m$. But this is not practical because no
Figure 1.10: The multi-layer P velocity model. The unit is \( ft/s \).
Figure 1.11: Seismograms with direct waves removed with $f - k$ dip filter.
Figure 1.12: Standard migration image.
Figure 1.13: Least squares migration image after 50 iterations.
Figure 1.14: Standard migration image.
Figure 1.15: Least squares migration image after 50 iterations.
Figure 1.16: Comparison of the reflection coefficients. Circle represents the model reflection coefficients of the reflectors at the normal incidence. "+" represents the reflection coefficients estimated from the standard migration image and "*" represents the reflection coefficients estimated from the least squares migration image. Both methods give acceptable reflectivity estimates for reflectors B, C, D and E, while LSM gives much better reflectivity estimates for reflectors F and G than does the standard migration.
seismic survey meets this requirement. Hence, with a incomplete data, no migration technique, including the amplitude-preserving migration techniques, can exactly reconstruct the true reflectivity model.

The amplitude-preserving migration techniques differ from the standard Kirchhoff migration in that they apply proper weights in the diffraction stack (Hanitzsch, 1997) which remove influences on amplitudes (e.g., geometric spreading). But in case of coarse and incomplete data coverage, the image amplitudes, although compensated for geometric spreading to give true reflectivity estimates, are different for two image points with the same reflectivities because they have different data coverage. There also exist migration artifacts in the amplitude-preserved migration images which make it difficult to recover true reflectivity.

Least squares migration finds models that minimize the objective, or data misfit functional. For an incomplete data set, there are many models that minimize the objective functional with a given precision.

The following test demonstrate that if the data coverage is wider, the LSM gives a more accurate image. In this test the point scatterer model is used, while the geophones are placed around the image area, including the left side, the right side, the top and the bottom of the image area. In all, there are 480 traces recorded. The least squares migration image after 250 iterations is shown in Figure 1.17. The image is better than the image in Figure 1.5 which is obtained by migrating only 70 traces in the left side of the image area. The relative amplitudes of point scatterers A, B and C show that the least squares migration almost gives the exact estimates of the their relative reflectivities.

**COMPARABLE IMAGE POINTS**

What kind of images do we need for AVO analysis? The ideal image is one in which amplitude is the exact reflectivity or is proportional to the reflectivity at every image point. Due to incomplete data coverage such images are not available. What is the best approximation to such an image? In industry there have been many successes in extracting AVO information from migrated images. But there also have been some failures. This seems to contradict my claim that without complete data coverage true reflectivity images are not available. My explanation to this contradiction is that under some conditions AVO informations can be extracted from the migrated images. Here a concept of comparable image points is proposed to explain why AVO succeeds in some cases while fails in other cases.

The seismic forward modeling operator can be written as

\[ d(r, s, t) = \int \frac{w(t - \tau_{xx} - \tau_{xx})}{A_{sx}A_{xr}} \cos \theta m(x) dx, \]  
(1.9)

where \( d(r, s, t) \) represents the data, \( m(x) \) denotes the reflectivity model, \( w(t) \) is the source wavelet and \( A_{sx}, A_{xr} \) are the amplitude terms which account for geometric
Figure 1.17: The LSM image after 250 iterations by migrating traces with wider coverage. The least squares migration almost gives the exact estimates of the relative reflectivities of points A, B and C.
spreading losses, and \( \theta \) is the angle between the ray direction and the vertical direction at the receiver point.

The Kirchhoff migration (i.e., adjoint modeling) equation can be written as

\[
m_{\text{mig}}(x) = \int \int \int \frac{w(t - \tau_{sx} - \tau_{xr})}{A_{sx}A_{xr}} \cos \theta' d(r,s,t) dtdrds = \int \int \int \frac{w(t - \tau_{sx} - \tau_{xr})}{A_{sx}A_{xr}} \cos \theta' \frac{\cos \theta}{A_{sx'}A_{x'r}} \cos \theta m(x')dx'dtdrds
\]

or in vector-matrix notation,

\[
\hat{m}_{\text{mig}} = \mathbf{K} \hat{m},
\]

where \( \mathbf{K} \) is the filter or the Hessian matrix.

We rewrite equation (1.10) as

\[
m_{\text{mig}}(x) = K(x,x)m(x) + \left[ \int K(x,x')m(x')dx' - K(x,x)m(x) \right].
\]

Equation (1.13) separates the contribution of \( m(x) \) into two parts. The second part denotes the migration artifacts.

If the second term is zero or negligible, i.e.,

\[
\int K(x,x')m(x')dx' - K(x,x)m(x) = 0,
\]

which means that either there are no migration artifacts in the migrated image profile or that they are eliminated by some means such as LSM. For the later case, equation (1.13) becomes

\[
m_{\text{mig}}(x) = K(x,x)m(x).
\]

For an image point \( x_j \),

\[
m_{\text{mig}}(x_j) = K(x_j,x_j)m(x_j).
\]

The ratio of the amplitudes at image points \( x_i \) and \( x_j \) is given by

\[
\frac{m_{\text{mig}}(x_i)}{m_{\text{mig}}(x_j)} = \frac{K(x_i,x_i)m(x_i)}{K(x_j,x_j)m(x_j)}.
\]

When

\[
\frac{K(x_i,x_i)}{K(x_j,x_j)} = 1,
\]

\[
\frac{m_{\text{mig}}(x_i)}{m_{\text{mig}}(x_j)} = 1,
\]
we have

\[
\frac{m_{\text{mig}}(x_i)}{m_{\text{mig}}(x_j)} = \frac{m(x_i)}{m(x_j)}.
\] (1.19)

This means that the ratio of amplitudes at image points \(x_i\) and \(x_j\) is proportional to the ratio of their reflectivities. With this condition, the relative amplitudes at \(x_i\) and \(x_j\) represent the relative reflectivities. Points fulfilling this condition are called comparable image points. AVO analysis is only valid for comparable image points.

Equation (1.18) shows that for two image points to be comparable, they must have the same or roughly the same data coverage such that the influences of geometric spreading on these two points are the same. In Figure 1.18, for example, for the CDP survey geometry, Point A and B have the same data coverage and therefore are comparable, while point C has a different data coverage and cannot be compared with points A and B.

For the CDP survey geometry it is relatively easy to find comparable image points in the migrated section. Typically in the central part of the migrated image the image points have roughly the same data coverage and therefore are comparable for reflectivities. This can explain why AVO analysis succeeds in some cases and fails in other cases. AVO analysis succeeds where the comparable image points exist and fails where image points have different data coverage.

Velocity variation is another factor affecting comparability of image points. Even if two image points have the same data coverage, their traveltimes and amplitude losses will be different because the rays associated with these two points travel through different velocity regions.

Equation (1.14) holds when the diagonal elements in the Hessian matrix are dominant and the off-diagonal elements are negligible. This can be achieved by either increasing the data coverage and density, or using least-squares migration to eliminate the artifacts.

**FUTURE WORK**

In the future I will first test the ability of amplitude-preserving migration techniques to recover the true reflectivity in case of coarse and incomplete data coverage for various survey geometries. I want to confirm that for incomplete data even the amplitude-preserving migration cannot completely recover the true reflectivity. I will also test the concept of comparable image points by generating and migrating seismograms which have same data coverage for some image points while have different coverage for other points. I will explore the possibility of extracting AVO information from seismic data by waveform inversion.
Figure 1.18: The CDP survey geometry showing that point A and B have same data coverage while point C has different data coverage.
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REFERENCES


