Report 1

Greens Functions for 3-D Seismic Migration

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ABSTRACT

The space-time Greens functions for 3-D poststack and 3-D prestack migration in a homogeneous medium are derived under the far-field approximation. These Greens functions can be used to practically compute synthetic migrated images. Examples are given for the recording footprint noise associated with a point scatterer and a meandering river channel. The Greens functions also provide fundamental insights into how migration artifacts are influenced by different recording geometries. As an example, we see that the strong artifacts in the migration image are perpendicular to the resonant directions of the recording array.

INTRODUCTION

It is very important for seismic explorationists to understand the noisy influence of the source-receiver geometry on the final migration image. This noisy influence is sometimes referred to as the "recording footprint noise" in the migration image, and will be defined as the quadrature errors that result from evaluating the Kirchhoff migration integral on a coarse grid of traces. This problem is especially acute for 3-D acquisition geometries where the recording footprint can severely distort the geological interpretation of the migrated image.

The influence of the recording footprint noise can be understood by migrating synthetic data for different recording geometries. That is, prestack synthetic seismic data can be generated for a specified recording geometry and velocity model, the synthetic traces are migrated and the migrated image is compared to the known reflectivity model. Discrepancies between the migrated image and the model can
be used to judge the severity of the recording footprint noise. The problem with this approach is that it is computationally very expensive to generate 3-D data and migrated images, where it may take months on a supercomputer to both model and migrate the synthetic 3-D data.

In this paper, we present an analytical 3-D migration Greens function in the space-time domain that provides the 3-D prestack and poststack migrated images of a point scatterer. This Greens function is for a homogeneous medium, assumes a farfield approximation, and accounts for the discrete recording geometry of an orthogonal shooting pattern. Thus, a typical recording geometry can be specified and the point scatterer response can be computed by the numerical evaluation of this Greens function. Then, a trial and error procedure can be used to identify the recording geometry with the most acceptable point scatterer response. This work is an extension of Schuster’s (1996a) migration Greens function which was valid for a 2-D recording geometry with a continuous distribution of sources and receivers.

The next section will present the mathematical derivation, followed by the numerical results section where the Greens function is computed for a point scatterer and a meandering river channel. The final section presents a discussion and conclusion.

**3-D MIGRATION GREENS FUNCTION FOR CONTINUOUS RECORDING ARRAYS**

The 3-D migration Greens function for the prestack and poststack cases are derived for a continuous recording geometry. Section 4 will contain the derivation for the case of a discrete recording geometry.

**Poststack Greens function**

Suppose a point scatterer is imbeded in an homogeneous medium with velocity \( \frac{v}{2} \), and the line lengths of the midpoints in the \( x \) and \( y \) directions are \( L_{gx} \) and \( L_{gy} \), respectively. For monofrequency data, the traces can be migrated to give the zero-offset migration image:

\[
\tilde{m}(\mathbf{r} | \mathbf{r}_o, \omega) = \int_{L_{gx}}^{L_{gx}} \int_{L_{gy}}^{L_{gy}} e^{i2k(|\mathbf{r}_g - \mathbf{r}_o|)} h(x_g, y_g) dx_g dy_g, \quad (1.1)
\]

where \( h(x_g, y_g) \) is the 2-D sampling brush associated with the midpoint distribution. Here \( h(x_g, y_g) = 1 \) for a continuous sampling; \( k = \frac{\omega}{v} \) is the wave number; and \( \mathbf{r}_g \), \( \mathbf{r} \) and \( \mathbf{r}_o \) denote the receiver, interrogation and scatterer locations, respectively (see Figure 1.1).

By the farfield approximation, the farfield migrated section is given as (Chen 1996a):

\[
\tilde{m}(\mathbf{r} | \mathbf{r}_o, \omega) \approx \frac{2}{\pi r_o^2} \left[ \frac{\sin (\frac{L_{gx}}{r_o} k dx)}{\frac{L_{gx}}{r_o} dx} \right] \left[ \frac{\sin (\frac{L_{gy}}{r_o} k dy)}{\frac{L_{gy}}{r_o} dy} \right] e^{i2k\left((x_o - \frac{L_{gx}}{2})dx + (y_o - \frac{L_{gy}}{2})dy + z_o dz\right)}, \quad (1.2)
\]
where \( r_o = | \mathbf{r}_o | \), and \((dx, dy, dz) = (x - x_o, y - y_o, z - z_o)\). Note that equation 1.2 is a product of two SINC functions multiplied by a Fourier kernel. Thus, the integration of equation 1.2 with respect to frequency is a Fourier transform of two SINC function that is equivalent to the convolution of two distorted boxcar functions.

Integrating equation 1.2 over all frequencies yields the space-time migration Greens function:

\[
m(\mathbf{r} \mid \mathbf{r}_o) = \begin{cases} 
\frac{vL_{gx}L_{gy}}{2\pi r_o} & \text{if } dx = dy = dz = 0; \\
\frac{vL_{gx}}{2\pi r_o} & \text{if } dy = 0, \quad |x_o dx + z_o dz| < L_{gx} |dx|; \\
\frac{vL_{gy}}{2\pi r_o} & \text{if } dx = 0, \quad |y_o dy + z_o dz| < L_{gy} |dy|; \\
\frac{v}{4\pi r_o dx dy} \left[ x_o dx + y_o dy + z_o dz \right] & \text{if } x_o dx + y_o dy + z_o dz \left( x_o - L_{gx} \right) dx + \left( y_o - L_{gy} \right) dy + z_o dz | \\
& - \left( x_o - L_{gx} \right) dx + y_o dy + z_o dz | \\
& - x_o dx + \left( y_o - L_{gy} \right) dy + z_o dz | \\
& \left( x_o - L_{gx} \right) dx + y_o dy + z_o dz | < L_{gx} |dx| + L_{gy} |dy|; \\
0 & \text{otherwise.} \end{cases}
\]

Equation 3 says that the migration image is non-zero only in the region exterior to a distorted hourglass. This hourglass is formed by rotating two lines along the Z-axis at the scatterer position, where the dips of the two lines vary as they are rotated. The orientations of the two lines are determined by the survey configuration and the position of the scatterer as shown in Figure 1.2. If \( dx \) or \( dy \) equal zero, then the \( m(\mathbf{r} \mid \mathbf{r}_o) \) is the same as that in Schuster (1996a).
Figure 1.2: Point scatterer response of the zero-offset migration operator. The non-zero region is exterior to the distorted hourglass which is formed by rotating two lines along the z axis at the scatterer position. The orientation of these two lines is determined by the procedure outlined in the next figure.
Figure 1.3: Point scatterer response of the zero-offset migration operator. The hourglass is generated in the following ways: 1). Draw line $AB$ on the surface then a plane ABC is determined, where C is the scatterer's position. 2). The line $GCF$ is in the plane perpendicular to line $AC$. Similarly, $DCG$ is in the plane perpendicular to line $BC$. These two lines are in plane ABC. 3). Repeat steps 1) and 2), except choose a different orientation of line $AB$. The composite of lines $DCE$ and $FCG$ describe the distorted hourglass.
Prestack migration Greens function

In this case, the receiver is now offset from the source and the source samples a fixed patch of ground with area \( L_{sx} \times L_{sy} \). The farfield prestack migration equation for a single monofrequency source is written as (Chen, 1996a):

\[
\tilde{m}(r_s | r_o, \omega) = \int_{x_g}^{L_{gy}} \int_{y_g}^{L_{gx}} e^{ik(|x_g - x_o| - |x_g - x_s| + |x_s - x_o|)} |h(x_g, y_g)| dx_g dy_g.
\]

And integrating this equation over all frequencies gives:

\[
m(r_s | r_o) = \begin{cases}
\frac{L_{gy} L_{gx} v}{2\pi r_o^3} & \text{if } dx = dy = dz = 0; \\
\frac{v L_{gy}}{\pi r_o^2} dx & \text{if } dy = 0, |(2x_o - x_s)dx + 2z_o dz| < L_{gx} |dx|; \\
\frac{v L_{gx}}{\pi r_o^2} dy & \text{if } dx = 0, |(2y_o - y_s)dy + 2z_o dz| < L_{gy} |dy|; \\
\frac{v}{2\pi r_o^2} dy dx & \left| (2x_o - x_s)dx \\
+ (2y_o - y_s)dy + 2z_o dz \right| < L_{gx} |dx| + L_{gy} |dy|; \\
0 & \text{otherwise.}
\end{cases}
\]

Equation 5 says that the migration response is non-zero within the region intersected and surrounded by the rotation of several lines each of which is determined by the geophone and source configuration and the position of scatterer as shown in Figure 1.4. Also if \( dx \) or \( dy \) equals zero then the 3-D response reduces to the 2-D case, and the migration response is the same as in Schuster (1996a).

3-D MIGRATION GREENS FUNCTION FOR DISCRETE ARRAYS

The 3-D migration Greens function for the prestack and poststack cases are derived for a discrete recording geometry.
Figure 1.4: Point scatterer response of the prestack migration operator. The non-zero region is exterior to the distorted hourglass. The hourglass is generated in the following way: 1). Choose a shot point $A = (x_s, y_s, 0)$, then the point $B = ((L_{gX} + x_s)/2, (L_{gg} + y_s)/2, 0)$ is determined. Through points $A, B$ and $C$, a plane is determined where $C$ is the scatterer’s position. 2). The line $FCE$ is in the plane perpendicular to line $AC$. Similarly, $DCG$ is in the plane perpendicular to line $BC$. These two lines in the plane $ABC$. 3). Repeat steps 1) and 2), except choose a different source that determines another orientation of line $AB$. The composite of lines $FCE$ and $DCG$ describe the distorted hourglass.
**Poststack Greens function**

The representation of the poststack migration Greens function for a monofrequency source is the same as in equation 1.1. However the sampling function is given by a convolution of two 1-D comb functions (Schuster, 1996b), i.e.

\[ h(x_g, y_g) = n(x_g, y_g) * e(x_g, y_g), \]  

where \( e(x_g, y_g) = \delta(y_g) \sum_{n=0}^{N_gx-1} \delta(x_g - n\Delta x_g); \) and \( n(x_g, y_g) = \delta(x_g) \sum_{m=0}^{N_gy-1} \delta(y_g - m\Delta y_g). \) Here \( \Delta x_g \) and \( \Delta y_g \) denote the geophone intervals in the \( x \) and \( y \) directions, respectively; and \( N_{gx} \) and \( N_{gy} \) denote the number of geophones in the \( x \) and \( y \) directions, respectively. Plugging equation 1.6 into equation 1.1 and integrating over all frequencies gives the discrete poststack Greens function for migration:

\[ m(x \mid x_0, \omega) = \frac{v}{2\pi r_o^2} \sum_{n,m} \delta\left(\frac{2}{r_o}(x_o - n\Delta x_g)dx + (y_o - m\Delta y_g)dy + z_odz\right), \]  

where the integers \( n \) and \( m \) satisfy \( 0 \leq n \leq N_{gx} - 1; 0 \leq m \leq N_{gy} - 1. \)

Comparing equation 1.7 with equation 3, the Greens function for a discrete source-receiver geometry is represented by rotated fans, not the single distorted hourglass defined by the rotation of two planes. The dip angle of each discrete plane in a fan is determined by the index \((n, m)\), as shown in Figures 1.5 and 1.6. The angle between the two neighboring planes in a fan is determined by the values of \( \Delta x_g \).

Figure 1.7 displays the poststack Greens function evaluated at the depth of the scatterer. The radial lines that emanate from the central spike are aliasing artifacts caused by sampling wide band data with a finite midpoint interval. These radial lines represent the directions that are perpendicular to the resonant directions of a propagating aliased wavefront. For example, Figure 1.8 shows that waves propagating along the \( x \)-axis, diagonal axis or \( y \)-axis are densely sampled at regular intervals, while any other direction will be irregularly and/or coarsely sampled. Thus, sampling high frequency waves along the former set of directions resonate with aliased energy. For example, high frequency waves propagating parallel to the line \( AB \) in Figure 1.5 map to the line \( AB \) in Figure 1.7.

**Prestack Greens function**

The representation of the Greens function for 3-D prestack migration with a discrete source-receiver geometry can be represented as

\[ \tilde{m}(x \mid x_0, \omega) = \int_{o}^{L_{sx}} \int_{o}^{L_{sy}} \int_{o}^{L_{gz}} e^{ik(|x - x_0| + |x - x_s| + |x - x_o|)} \left| \frac{h(x_s, y_s, x_g, y_g)dx_sdy_sdx_gdy_g}{\frac{F_g - x}{F_g - x_0} \frac{F_g - x_s - F_s - x_o}} \right|, \]

where

\[ \frac{F_g - x}{F_g - x_0} \frac{F_g - x_s - F_s - x_o}, \]  

(1.8)
Figure 1.5: Poststack migration Greens function for a discrete source-receiver geometry is represented by rotated fans, where each fan appears as above. Rotating line $\overline{AB}$ in the horizontal plane will introduce a new migration fan in the subsurface, rotated by the same amount.
Figure 1.6: a). The energy of the trace at A is smeared over the plane O’A’A”, where AO’ is perpendicular to this plane and O’ is the scatterer location. Note that the dashed line denotes the intersection of the O’A’A” plane and the horizontal plane at the scatterer depth. b). For the poststack Greens function, the dip angle of each plane is associated with the scatterer’s position and the midpoint location, while the angle between each plane is determined by the midpoint interval.
Figure 1.7: Plot of 3-D poststack Greens function at the depth of the scatterer, where there is a 101 \times 101 grid of uniformly-spaced receivers, the image point is at the center of the square and the image plane is at a depth of 10,000 feet. The radial spokes that emanate from the central spike are aliasing artifacts due to the wide band source and the finite midpoint interval.

where \( h(x_s, y_s, x_g, y_g) = e(x_s, y_s) * n(x_s, y_s) * e(x_g, y_g) * n(x_g, y_g) \), and the sampling comb functions are given as:

\[
e(x_s, y_s) = \delta(y_s) \sum_{m=0}^{N_{sx}-1} \delta(x_s - m\Delta x_s); \quad n(x_s, y_s) = \delta(x_s) \sum_{n=0}^{N_{sy}-1} \delta(y_s - n\Delta y_s); \quad \text{(1.9)}
\]

\[
e(x_g, y_g) = \delta(y_g) \sum_{j=0}^{N_{gx}-1} \delta(x_g - j\Delta x_g); \quad n(x_g, y_g) = \delta(x_g) \sum_{l=0}^{N_{gy}-1} \delta(y_g - l\Delta y_g); \quad \text{(1.10)}
\]

where \( L_{sx} = (N_{sx} - 1)\Delta x_s, L_{sy} = (N_{sy} - 1)\Delta y_s, L_{gx} = (N_{gx} - 1)\Delta x_g, L_{gy} = (N_{gy} - 1)\Delta y_g \); \( N_{sx}, N_{sy}, N_{gx} \) and \( N_{gy} \) is the number of shot points and receivers in the \( x \) and \( y \) directions, respectively; \( (\Delta x_s, \Delta y_s) \) and \( (\Delta x_g, \Delta y_g) \) are the source and receiver intervals respectively.

Assuming the farfield approximation and integrating over all frequencies, we get the Greens function for the migration image:

\[
m(\mathbf{r} | \mathbf{r}_o) = \frac{v}{2\pi r_o^2} \sum_{k,l,m,n} \delta \left( \frac{1}{r_o} ((2x_o - k\Delta x_g - m\Delta x_s)dx + (2y_o - l\Delta y_g - n\Delta y_s)dy + 2z_0dz) \right), \quad \text{(1.11)}
\]
Figure 1.8: The sampling of high frequency waves propagating along the x-, y- or diagonal axes will be strongly aliased (i.e. resonating) because they are regularly and densely sampled. Other directions of propagation will be coarsely and/or irregularly sampled and so will not produce strong "resonances" in the sampled wavefield.
Figure 1.9: Aliased energy associated with the Greens function for post stack migration. This figure is obtained by cutting off the main pulse in the Greens function for migration. It is clear that the artifacts are strongest along the x-, y- and diagonal axes.

where $0 \leq k \leq N_{gx} - 1; 0 \leq l \leq N_{gy} - 1; 0 \leq m \leq N_{sx} - 1; 0 \leq n \leq N_{sy} - 1; 0 \leq m \leq N_{sx} - 1; 0 \leq n \leq N_{sy} - 1$.

For prestack migration, the Greens function is constituted by a set of fans, where each fan in a set consists of planes with orientations determined by the source-receiver pairs in a shot gather; and the orientation of each fan is determined by the scatterer position and source-receiver geometry as shown in Figures 1.10- 1.11. The angle between adjacent fans is determined by the source interval.

A plot of the prestack 3-D Greens function evaluated at the scatterer’s depth is shown in Figure 1.12, where the scatterer is buried at a depth of 10,000 ft, the receiver configuration is the same as in Figure 1.1 and the source configuration is a $21 \times 21$ distribution of shot points. From Figures 1.7 and 1.12, it is obvious that the Greens function for migration consists of the centered main pulse and the aliased energy is caused by the non-zero source and receiver intervals and infinite bandwidth of the source function. These aliasing artifacts, known as the recording footprint noise, will contaminate the migration result at each imaging depth.

From Figure 1.13, it can be seen that the artifacts along the x-axis, y-axis and diagonal-axis directions are stronger than in any other direction. As discussed previously, these directions are perpendicular to the resonant directions of propagation for a sampled propagating wave.
Figure 1.10: The prestack migration Greens function for a discrete source-receiver geometry is represented by a set of fans, where each fan consists of planes determined by the source-receiver pairs in a shot gather and the orientation of each fan is determined by the scatterer position and source-receiver geometry. For example, for the source-receiver pair shown above, the plane A’A”O is perpendicular to line AO and plane B’B”O is perpendicular to line BO.
Figure 1.11: The angle between adjacent planes within a fan is determined by the receiver interval.
Figure 1.12: Greens function for 3-D prestack migration, where there is a $101 \times 101$ grid of uniformly-spaced receivers and a $21 \times 21$ grid of uniformly-spaced sources; the image point is at the center of the square and the image plane is at a depth of 10,000 feet.

Figure 1.13: Aliased energy associated with the Greens function for prestack migration. This figure is obtained by cutting off the main pulse in the Greens function for migration. It is clear that the artifacts are strongest along the x- y- axes.
BANDLIMITED GREENS FUNCTION

The above discussion is based on a wide band frequency assumption; however in practice the signal is bandlimited. Here we examine the Greens function for migration in the band limited case. The wide band Greens functions for poststack and prestack migration are given in equations 1.7 and 1.11, respectively. Here the source wavelet for the Greens function is a \( \delta \) function. To obtain the bandlimited Greens function, we use the bandlimited wavelet to filter the wide band Greens function. Since equation 1.7 and 1.11 are in the space domain and, according to the convolution theorem, the Greens function for migration in the bandlimited case can be represented as a convolution in the space domain:

\[
m(\mathbf{r} | \mathbf{r}_o) = m(\mathbf{r} | \mathbf{r}_o) |_\delta \ast s(\mathbf{r}),
\]

where \( m(\mathbf{r} | \mathbf{r}_o) |_\delta \) is the migrated image for the wide band signal, and \( s(\mathbf{r}) \) is the band limited seismic wavelet. Here we choose the source wavelet as a gate function in the wave number domain. For zero offset migration, the Greens function for a SINC source wavelet is given by (see appendix):

\[
m(\mathbf{r} | \mathbf{r}_o) = \frac{a v}{\pi r_o^2} \sum_{j,l,m,n} \sin \left( \frac{4k_o}{r_o} \right) \frac{1}{r_o} \left( (x_o - j \Delta x_g - m \Delta x_s) dx + (y_o - l \Delta y_g - n \Delta y_s) dy + z_o dz \right),
\]

where \( 0 \leq j \leq N_{gx} - 1; 0 \leq l \leq N_{gy} - 1 \) and \(-k_o \leq k \leq k_o\). \( k_o \) is the wave number corresponding to the maximum frequency, \( a \) is a constant.

For non-zero offset migration, the Greens function for a SINC source wavelet is (see appendix)

\[
m(\mathbf{r} | \mathbf{r}_o) = \frac{a v}{\pi r_o^4} \sum_{j,l,m,n} \sin \left( \frac{k_o}{r_o} \right) \frac{1}{r_o} \left( (2x_o - j \Delta x_g - m \Delta x_s) dx + (2y_o - l \Delta y_g - n \Delta y_s) dy + 2z_o dz \right),
\]

where \( 0 \leq j \leq N_{gx} - 1; 0 \leq l \leq N_{gy} - 1; 0 \leq m \leq N_{sx} - 1; 0 \leq n \leq N_{sy} - 1, a \) is a constant and \( (j,l,m,n) \) are integers.

In the frequency band limited case, the delta function is replaced by a bandlimited signal. Because of the sidelobes in the bandlimited signal, the migration image is smeared. This smearing can be explained by recalling that, under the farfield approximation, the diffraction isochrone is locally replaced by a plane. If the receiver interval is too large, then the angles between the adjacent planes within one fan of planes will not be small. Consequently, there will be incomplete cancellation of migration energy outside the scatterer’s location and result in much migration noise. From the above formula, one way to overcome the problem of spatial aliasing is to choose the shot and receiver points at a random distribution of locations. In this way, the migration noise will be suppressed more efficiently than by regular signal alignment (Chen, 1996b).
Figure 1.14: The Greens function for poststack migration evaluated at a depth of 10,000 ft, where the source wavelet is represented by a SINC function with a peak frequency of 50 Hz; and there is an $101 \times 101$ orthogonal grid of receivers. The image point is centered below the recording plane at a depth of 10,000 feet.

Figure 1.15: The Greens function for prestack migration. The configuration is the same as that in Figure 1.14 except there is a $21 \times 21$ orthogonal grid of evenly distributed sources.
The Greens function for migration can be used to design, by trial and error, an optimal survey configuration. That is, a reflectivity model can be represented as a composite of point scatterers so that the migration response of primary reflection data is given as follows (Schuster, 1996a):

\[
m(\mathbf{r}) = \int_{\Omega} G(\mathbf{r} \mid \mathbf{r}_o) R(\mathbf{r}_o) d\mathbf{r}_o,
\]

where \(G(\mathbf{r} \mid \mathbf{r}_o)\) is the migration Green function, \(R(\mathbf{r}_o)\) is the reflectivity strength at \(\mathbf{r}_o\) and \(\Omega\) is the half infinite space.

The quality of the migration image depends on the impulsiveness of the Greens function, and so the survey should be designed to maximize the impulsiveness of the Greens function and minimize aliasing artifacts.

Figure 1.16: Prestack Kirchhoff migration image of a scatterer, where there is a 51 \(\times\) 51 grid of uniformly-spaced receivers and a 21 \(\times\) 21 grid of uniformly-spaced sources; the image point is at the center of the above square and the image plane is at a depth of 10,000 feet. The scatterer point is located at (1000', 1000', 10,000').

Figure 1.16 - 1.18 compares the prestack migrated image obtained from Kirchhoff migration with the image computed by the Greens function in equation 1.14. As shown in Figures 1.16 and 1.17, the image computed by the farfield Greens function is a good approximation to the Kirchhoff image in a small region around the scatterer’s position. When the image point distance increases from the scatterer’s position and decreases from the surface, the approximation becomes worse as partly shown in Figure 1.18.
Figure 1.17: Same as previous figure, except the image is computed by evaluation of the Greens function in equation 1.14.

Figure 1.18: Difference between Greens function and Kirchhoff migration images.
Based on the Greens function and equation 1.15, the 3-D prestack migrated image associated with a meandering river channel is given in Figure 1.19. This image was generated by representing the river channel as a sinusoidal string of point scatterers, and evaluating equation 1.15 at the depth of the river channel. As seen in the Figure 1.19 image, there are artifacts aligned along the x-axis and diagonal directions. The explanation is that several scatterers at the apices of the channel are aligned along the x-axis direction, which reinforce the aliased energy along the x-direction. Similarly, the diagonal artifacts arise from the scatterers aligned along the resonant diagonal direction. This example suggests appears that geological trends parallel to the resonant sampling directions will amplify the aliasing artifacts in those directions.

Figure 1.19: Prestack migrated image of a meandering river channel computed by evaluating equation 1.15 for 1,147,041 traces recorded by an orthogonal recording array. Here the imaging depth is the same as the actual depth of the river at 10,000 ft, and the sinusoidal feature represents the location of the river channel. The computation of this image required about 60 CPU hours on a DEC Alpha Station 250 4/266.

CONCLUSIONS

Under the farfield approximation, the poststack and prestack Greens functions for migration in a homogeneous medium are derived. These formulae provide a fundamental understanding of how 3-D migration artifacts are influenced by different recording geometries. As an example, we see that the artifacts in the migration image are mainly perpendicular to the resonant sampling directions of the recording
array. For example, geological structures with trends parallel to the resonant sampling direction will amplify the migration artifacts. It therefore might be desirable to orient a recording geometry so that its resonant sampling directions are not parallel to the geologic trend.

Using these formulae, migration images can be generated without a large amount of forward calculations. For a continuous configuration and a wide band source, the Greens function for seismic migration is defined by a distorted hourglass cone with the scatterer at the apex. In a discrete configuration, the Greens function is represented by a fan of discrete planes, where the migration amplitude on each plane is a constant. The angle between each plane depends on the source and receiver intervals in the shooting geometry. For a discrete configuration and a bandlimited source, the Greens function for migration is the same as that of the wide band source but now the waveform is a band limited signal. Using the analytic Greens function for migration, we can be guided to an optimal recording geometry by evaluating the migration image for different recording geometries. Thus the migration Greens function can be used as a criterion to evaluate how the survey design affects the migration image, which is an important problem in 3D seismic exploration.

REFERENCES


Appendix

The bandlimited Greens function can be obtained by filtering the wide band Greens function, where the filter response is the bandlimited source wavelet. A SINC wavelet with a central frequency of $\omega_o$, has a corresponding wave number $k_o$. The migration Greens function of a SINC wavelet can then be represented as:

$$m(\mathbf{r} | \mathbf{r}_o) = \int_{-k_o}^{k_o} \tilde{m}(\mathbf{r} | \mathbf{r}_o, k) |_\delta e^{ik\mathbf{r}} dk$$  \hspace{1cm} (1.16)

Using equations 1.7 and 1.11, and integrating according to equation 1.16 yields the poststack and prestack formulae given in equation 1.13 and 1.14.