University of Utah
Modeling and Tomography Development Project

1994 Midyear Report


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Preface

This midyear report summarizes the 1994 midyear research results of the Modeling and Tomography group at the University of Utah. Consortium members for 1994 include Advance Geophysical, Amerada Hess, Amoco, Chevron, Conoco, Exxon, Fujitsu, GRI, Noranda, Texaco, Unocal, and Phillips. Several other companies are considering membership.

Some research accomplishments for 1994 include:

- Theoretical understanding of random and quasi-random migration methods and why they provide better images than does standard migration. This establishes a theoretical framework for exploring applications of random or quasi-random integration methods to other processing algorithms such as DMO.

- Test of the quasi-random migration method on 3D field data. This test failed to show the effectiveness of the quasi-random migration relative to the standard migration procedure. We believe that the failure was caused by the irregular gridpoint distribution of the data. Phillips has agreed to provide us with a 3D data set obtained on a regular grid and we will soon begin our tests.

- Elimination of artifacts in elastic traveltime+waveform inversion (WTW) by using constraints. Crossing artifacts in the WTW tomograms are largely eliminated by inserting a penalty function in the misfit function. Also, simultaneous inversion of both P- and S-wave data shows much promise.

- Completion of 2D viscoelastic modeling code. The 2D viscoelastic modeling code will be used to understand the role of viscoelastic effects in CDP data, crosshole data and data processing.

- Conjugate gradient migration of synthetic VSP seismograms show good imaging capabilities.

- Improved processing procedure for the McElroy data result in improved migrated images.

- Improved procedure for 2D and 3D tomostatics with CPU time reduced by an order of magnitude.
Research for 1994 will aim to improve imaging methods for both crosshole, VSP and CDP data. For crosshole data, we will seek to improve the accuracy of the WTW method, and develop better ways to extract different wave modes from seismic data. We are investigating the problems associated with large well offsets, especially for media with low Q values. An important goal is to seek a robust and efficient method that updates both the velocity distribution and the migrated image.

For CDP data, we will test the quasi-random migration method on the Phillips 3D data set; the key feature about this data is that it is distributed on a regular grid. If successful then we will begin to explore the uses of quasi-random methods in velocity analysis. We will continue our theoretical analysis of quasi-random methods, including the application of it to data processing methods such as DMO. The 3D tomostatics method may be applied to a 3D CDP data set to compare its effectiveness to 2D tomostatics.

We will also explore the application of conjugate gradient migration to VSP data, and especially its use in single well imaging of salt domes.

Kim Olsen defended his PhD in May and in September he started a 2 year post-doc at UC Santa Barbara. Starting Nov. 23, Fuhao Qin and Wenying Cai will begin their new lives in Houston, Texas. On Dec 1 Fuhao will become a full-time employee of Amerada Hess, and Wenying is currently interviewing with various geophysical companies in Houston. Fuhao and Wenying recently bought a a new Honda Accord (with air bags and sidebeam protection) for their upcoming adventures on the Houston freeways. Lianxiang Ji and Dave Morey are new student members of our group, and I am expecting a new post-doc to join our group soon. I will soon buy a starter supercomputer, with the hopes that some of our member companies can financially help us to upgrade its memory and speed.

I would like to thank Gas Research Institute and, particularly, Tim Fasnacht for supporting our quasi-random migration work with a generous grant. This allows us to extend the work of Yonghe Sun and provides us with enough spare monies to buy a starter supercomputer. We also thank Fujitsu of America, and particularly Sia Hassanzadeh, for providing generous amounts of supercomputer time from 1993-1994; such CPU time enabled us to accomplish many important tasks in seismic imaging.

Jerry Schuster
Part I

Tomography Methods
Report 1

Turning Ray Tomography

Fuhao Qin, Wenyeng Cai, Gerard T. Schuster

1.1 Abstract

We present several case histories of applying a nonlinear inversion method to first arrival traveltimes from refraction seismic data. The key features of this inversion method are that: the model update is performed by back-projecting rays along thick ray paths, and the model is smoothed after each iteration to accommodate uneven ray coverage. Our numerical results suggest that this method is robust with respect to poor starting models and limited numerical tests suggest that the method is fast, stable and easy to use. It may find applications in both environmental engineering and statics corrections in seismic exploration.

1.2 Introduction

The refraction seismic technique is one of the most effective tools for investigating the shallow structure of the earth for both environmental studies and statics corrections in seismic exploration. Of the many available refraction methods (Marsden, 1993), tomographic methods are getting increased attention with the increasing power of portable computers. De Amorim et al. (1987) calculated statics correction from refraction traveltimes based on a one-layer model; and Olsen (1989) described a method that inverts for laterally varying velocities and shallow depths from the first arrival traveltimes. Both of these methods consider that waves are critically refracted from the top of the high velocity layer and can be used to invert for the layer thickness and velocity variations. They are ideally suited to areas where the near surface structure is restricted to two or three layers whose parameters vary over a predictable and limited range; this assumes that the interpreter correctly specifies the layer numbers and estimates their thickness and velocity.
Zhu et al. (1992) introduced a turning ray tomography method. They assumed that the first arrivals are not just arrivals critically refracted from an interface but are associated with rays that turn upward due to a velocity distribution that increases with depth. Using this concept, they inverted for the subsurface velocity distribution instead of the thickness and layer velocities and claimed that turning ray tomography can image near-surface velocities more accurately than refraction methods. However, in their method, rays are traced only once through a velocity model derived by interpreters; raypaths were kept unchanged even though the velocity models were updated iteration by iteration. The problems with this method are, first, that it requires a good initial model and, second, that some avoidable errors will arise from the fixed raypaths.

We present an improved method that does not always require a good starting model and still provides fast convergence. In this method, the velocity model and raypaths are both updated at each iteration. The traveltimes and raypath calculations are based on a finite-difference solution to the eikonal equation (Qin et al., 1992). This guarantees that the calculated traveltimes are first arrivals regardless of the wave types; thus it can deal with the irregular surface problem very easily. Similar to Zhu et al. (1992), the model is updated by a SIRT-like method.

Following this section, the refraction tomography method is briefly discussed and then three numerical examples are presented. Following this we discuss some details of implementation.

1.3 Refraction Traveltime Tomography

In refraction traveltime tomography the misfit function is defined as,

$$
\epsilon = \frac{1}{2} \sum_i (t_{i, obs} - t_{i, cal})^2,
$$

(1.1)

where $i$ represents the $i^{th}$ raypath, $t_{i, obs}$ is the recorded traveltimes and $t_{i, cal}$ is the calculated traveltimes of the first arrival. In refraction tomography we assume that the sources and receivers are distributed along the same surface, and that the subsurface slowness model is parametrized into a grid of equi-sized cells, with each cell having an unknown slowness.

To determine the slowness model that minimize the above traveltimes misfit function, a SIRT-like method (van der Sluis, A. and van der Vorst, H. A., 1987) is used. At each iteration the slowness $s_i$ in the $j^{th}$ cell is updated by the following formula:

$$
s_j = s_j - \alpha \frac{\sum_{i \in R_j} \Delta t_i}{N_j},
$$

(1.2)

where $N_j$ is the number of rays that visit the $j^{th}$ cell, $\Delta t_i = t_{i, obs} - t_{i, cal}$ is the traveltimes residual associated with the $i^{th}$ ray, and the summation $i$ is over the set of indices $R_j$ associated with raypaths (wavepaths) that visit the $j^{th}$ cell, and $\alpha$ is step length.
1.3. REFRACTION TRAVELTIME TOMOGRAPHY

To accommodate realistic field conditions and uneven ray coverage we have implemented the special procedures listed below.

1.3.1 Raytracing

Due to the extreme velocity gradients at the near-surface, ordinary raytracing methods might have difficulty in finding the correct raypath for the first arrival. Therefore, a finite-difference solution to the eikonal equation (Qin et al., 1992) is used to calculate all of the first arrival traveltimes. Rays are then traced from the receivers to the sources following the normal directions of the wavefronts (gradient directions of the traveltimes field). This guarantees that the traveltimes and raypaths are those for the first arrivals.

It is well known that band-limited waves do not simply travel along a geometrical ray path with zero thickness. A more physical description is that the band-limited energy primarily propagates along a thick ray or wavepath (Woodward, 1988, 1992; Luo, 1990). The thickness of the wavepath is proportional to the inverse of the source frequency and the length of the wavepath between the source and receiver (Woodward, 1992). For simplicity, we will approximate the wavepath by a thick ray with a constant width. The wavepath width is calculated as $A\sqrt{L}$, where $L$ is the raypath length and $A$ is a predetermined constant. Our experimental trials found that $A = 0.4$ works well for all of the models we tested. For simplicity, the raypath length is approximated by the direct distance from the source to the receiver. By doing this, not only does our thick ray approximate the actual wavepath but our experience suggests that it accelerates convergence for at least the first several iterations.

If a cell has no ray passing through it, we let it have the same gradient value as the cell just above it. The reason for doing this is to extend the gradient field downward from the deepest point where rays can reach. In other words, since we do not have information beneath the depth of maximum ray penetration, the best we can do is to assume that below this depth the velocity is the same. Figure 1.1 shows a layered earth model containing some refraction raypaths. The rays are concentrated near the interfaces and there is almost no information within each layer. However, the downward extrapolation of the gradient will provide each layer with a similar update in slowness. Our experience suggests that this strategy mitigates problems with convergence.

1.3.2 Irregular Surface Topography

In seismic exploration, many surveys are carried out in areas where elevation changes with offset. It is desirable to develop methods that can correctly deal with this problem. Our remedy is to give the region above the free surface a very low velocity value. The velocity is so low that no first arrival ray will go through it, but it is not too low so that it affects the stability of the finite-difference solution of the eikonal
Figure 1.1: Refraction ray paths and the downward extension of the gradient. Shading strength is proportional to gradient strength.

equation. We found that assigning the free-surface velocity to be one third to one half of the minimum subsurface velocity is sufficient for the models we tested.

1.3.3 Smoothing

When the inversion problem is underdetermined (more unknowns than equations), it is always desirable to apply a moving average smoothing filter to the gradient field. Even when the number of traveltime measurements exceeds the number of unknowns, some parts of the model may be overdetermined, and some other parts of the model may still be underdetermined. Thus, it is still helpful to apply a smoothing operator. For refraction tomography, we used rectangular smoothing filters with maximum and minimum widths that were equal to, respectively, three times and one-half the dominant source wavelength. The size of the smoothing operator should be gradually diminished with the number of iterations to achieve best results (Nemeth et al., 1993).

1.3.4 Large Models

A seismic line will extend to tens and even hundreds of kilometers, and can be too big to fit into the computer as a single model. We have found that it is convenient to subdivide the model into smaller segments. These segments should overlap one another and the overlap should be large enough to cover the region of edge artifacts in the tomogram. Edge artifacts appear near the ends of a model where the ray coverage is sparse relative to the middle of the model where there are typically many crossing rays. To invert for a certain model segment, only rays that start and end in this segment are used. After all model segments are reconstructed, they are combined into one large model by eliminating the overlapped parts that are considered to be affected by edge effects.
1.4. NUMERICAL EXAMPLES

This will conclude our methodology section. A typical refraction tomogram usually needs twenty to thirty iterations to converge, providing the homogeneous initial model is not very far away from the top layer velocity. Considering that the finite-difference eikonal equation solver is fast, a workstation will be able to handle most of the problems encountered in environmental engineering and seismic exploration.

1.4 Numerical Examples

In this section, one synthetic and two field seismic data sets are used to test the refraction inversion method. The synthetic data example shows how well refraction tomography can reconstruct complicated subsurface models. The next example, a field refraction survey over a shallow hazardous waste site, demonstrates the ability of refraction tomography in imaging a shallow stream channel. And the third example, a marine refraction field survey in the Gulf of Mexico, highlights the capability of refraction tomography for imaging a deeply buried salt dome.

1.4.1 Synthetic Seismic Survey Model

The refraction tomography method is applied to a synthetic seismic model provided by Kun Hua Chen from Chevron. It is the top part of a model based on a seismic survey in South America. The size of the model is 22 km by 12 km and the model for the refraction study is reduced to 21.61 km by 1.2 km (Figure 1.2). The maximum surface elevation change along this line is about 700 meters and the velocity varies from 2,000 m/s to 3,800 m/s. There are 261 sources evenly distributed within the offset range of 3,000 m and 18,600 m. The source interval is 60 m. For each source, 101 receivers are assumed to be located within a 6,000 m offset range which is centered at the source location. The receiver interval is also 60 m.

The 2-D acoustic wave equation was solved by a finite-difference method to calculate the synthetic seismograms. First arrival traveltimes were then picked from the seismograms, such as those from the common shot gather (CSG) in Figure 1.3. To amplify the first arrival signal for correct picking, an AGC gain with a window of 0.1 s was applied to the seismogram before an automatic picker picked the traveltimes. The picker picks the first arrival time at the point where 5% of the largest amplitude first occurs in that trace. A time shift is applied which is obtained from the zero offset trace. It is then considered to be the first arrival travelt ime.

Figure 1.4 shows the velocity tomogram from the picked refraction traveltimes. Comparing Figure 1.4 and Figure 1.2, we can see that the velocity of the top weathered zone is well reconstructed. The depth and shape of the refractor is also reconstructed except near the left and right edges. The velocity trend of the refractor is depicted in the tomogram and will be a good model for statics corrections. Note the vertical strips in this Figure, which are caused by the downward gradient extrapolation. The top of
Figure 1.2: Top part of the seismic survey model which is used in the following refraction study.
Figure 1.3: A finite-difference common shot gather displayed with an AGC window of 0.1 s.
these stripes is a good indicator of the maximum depth of refraction ray penetration. Any colors below this point do not represent reliable velocities.

1.4.2 Imaging A Shallow Stream Channel

A field experiment was conducted to detect the location of a shallow stream channel at a hazardous waste site in Ogden Utah. The hammer source and receiver spacings were both 5 feet, and at least 24 channels were active for each source. The entire length of the shooting line is 5x60=300 feet. This line location was selected because a nearby well indicated the presence of shallow (approximately 12-20 ft depth) stream channel overlain by silty fill material. The well log indicated a gravel channel underlain by competent sediments.

An example of a shot gather from the field experiment is shown in Figure 1.5, where 60 shot gathers were collected by shooting every 5 feet. More than 60x24=1440 first arrival traveltimes were picked and used for the inversion. The resulting velocity tomogram is shown in Figure 1.6 where the RMS traveltime error at the last iteration was less than 2 ms. The edges of the tomogram are characterized by dipping artifacts caused by the paucity of raypaths near the ends of the survey line. The middle part of the tomogram contains a trough structure which suggests the possibility of a river channel at a depth of approximately 15 feet. Just below this tomogram is the same tomogram except a common offset gather (offset=40 feet) is superimposed on it. Note that the first arrivals have time delays that are similar in character to the topography of the refractor. To verify that there is an anomaly at this location, common midpoint gathers were stacked to provide the stacked section shown in Figure 1.7; in addition, the common offset gather with an offset of 40 feet is shown just below the stacked section. The trough denoted by an arrow is roughly at the same location as the trough in the velocity tomogram. The moveout velocity that maximized the coherence of reflection stacking, not the tomogram's velocity, was used for the stacking velocity.

To further verify the presence of this trough, a Ground Penetrating Radar (GPR) survey with a 50 MHz antenna was carried out over the same site. The GPR common offset gather is shown in Figure 1.8 and shows a trough at about the same depth and location as the previous two figures. Note that the lateral resolution of the trough in the velocity tomogram appears to be better than that depicted by the GPR data.

1.4.3 Salt Intrusion Example

The method was also tested on a CDP field data set from the Gulf of Mexico. The CDP seismic data were collected from an area where there was an onshore salt intrusion. The length of the seismic line was approximately 65,340 ft and there were 192 sources with a source interval of 110 ft. Maximum fold of the line is 190 and for each source there were 250 receivers with a receiver interval of 110 ft.
Figure 1.4: Velocity tomogram inverted from the picked finite-difference seismogram traveltimes.
Figure 1.5: A shot gather from the Ogden, Utah refraction survey.
Figure 1.6: Velocity tomograms obtained by inverting first arrival traveltimes from the Ogden, Utah refraction data. The bottom tomogram is the same as the top tomogram except a common offset gather (40 feet offset) is superimposed.
Figure 1.7: A stacked section (top) and a common-offset gather (bottom) from the Ogden, Utah refraction survey. Here the common offset between the source and receiver is 40 feet.
Figure 1.8: A common-offset GPR gather taken from a GPR survey that was spatially coincident with the Ogden, Utah refraction survey. Here the spacing between the GPR source and receiver is 15 feet.
Approximately two thirds of the first arrival travel times were picked from the seismograms. For the inversion, the grid spacing used was 110 ft for a grid size of 595 by 21 grid points. The starting model is a homogeneous model with a velocity of 3000.0 ft/s. The RMS residual decreased from 1.547 s for the starting model to 0.155 s after 20 iterations. Figure 1.9 shows the inverted tomogram after 20 iterations. The salt dome was clearly imaged. This result agrees well with the geologist’s interpretation based on seismic and other information.

For this particular inversion, 30 iterations took about 3 hours running time on an IBM RISC 6000 computer.

1.5 Discussion

For the models tested, we found that the refraction tomographic inversion method is very stable and converges to a model that minimizes the traveltime residual. It does not always require a good initial model, although a good starting model may accelerate the rate of convergence. The ideal digitization of the model is to discretize the model into a very fine grid in order to get accurate finite-difference traveltimes and raypaths. A smoothing operator will mitigate the problems associated with the underdetermined system of equations (see Appendix for details on how to choose the size of the smoothing operator). To avoid excessive CPU time, the selection of the grid spacing should be based on the size of the model and the capacity of the computer. And a multigrid technique may be used to achieve both accuracy and efficiency (see Nemeth et al., 1994).

Like all nonlinear inversion processes, smaller step lengths seems to give a better result. In our inversion code, we constrained the step length so that, for each iteration, the velocity change did not exceed 10 percent of the largest velocity value in the model. Numerous tests always showed convergence, but this is no guarantee that the final result is the true model. It is recommended that different starting models be used to determine if they all converge to the same model.

This method has already been extended to three dimensions so that it can be applied to 3-D seismic surveys (Cai and Qin, 1994).

1.6 Appendix

This appendix establishes some relationships between the model resolution and the source-receiver configuration of the refraction experiment. We will assume a two-layer model that is parameterized as a stack of four wide rectangular cells of the same thickness (see Figure 1.10). The measured first arrival traveltimes are associated with the direct and refraction arrivals, and give rise to the following set of traveltine equations.
Figure 1.9: Velocity tomogram inverted from the refraction traveltime picked from the salt intrusion data set.
Figure 1.10: A two layer velocity model parametrized into 4 cells of unknown slowness.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
1 & 1 & 2l & 1 \\
1 & 1 & 3l & 1 \\
\end{bmatrix} \cdot \begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4 \\
\end{bmatrix} = \begin{bmatrix}
t_1 \\
t_2 \\
t_3 \\
t_4 \\
\end{bmatrix},
\]

(1.3)

where the slowness in the \(i\)th layer is denoted by \(s_i\), and \(t_i\) represents the first arrival traveltime of the \(i\)th ray. For simplicity we assume that the segment lengths of the rays are equal to integer multiples of the value \(l\). These traveltime equations can be compactly written in matrix-vector notation as \(L\vec{s} = \vec{t}\), where \(L\) is the raypath matrix.

Note that simple row operations will reduce the above equations to the following form

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 2l \\
\end{bmatrix} \cdot \begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4 \\
\end{bmatrix} = \begin{bmatrix}
t_1 \\
t_2 \\
t_3 - t_2 \\
t_4 - t_2 \\
\end{bmatrix},
\]

(1.4)

It is clear that the \(s_1\) and \(s_4\) slowness values can be explicitly computed, but the \(s_2\) and \(s_3\) slowness values are indeterminate. This is because the second and third columns in the original raypath matrix are linearly dependent. Thus the null space of the 4x4 \(L\) matrix has a dimension of 1. In fact, more refraction traveltime equations (i.e., more equations than unknowns) will not reduce this indeterminacy because we will still have 2 columns that are linearly dependent. We will now make some general statements about the model resolution associated with direct wave+refraction traveltime data from a single refractor model.

- If the model is parameterized into \(N\) equi-spaced layer cells between the surface and the first subsurface refractor then there will be a minimum of \(N - 1\) linearly
dependent columns in $L$. Thus, the null space dimension of $L$ (or $L^T L$) is at least equal to or greater than $N - 1$. The proof for this statement is a simple extension of the previous example of a 4 layer model to an N layer model. The minimum null space dimension of $L$ for refraction data is similar to that for CDP reflection tomography (Bube et al., 1985; Schuster, 1989). Similar to the refraction parameterization, we assume that the reflection model is parameterized into $N$ layer-like cells between the surface and the horizontal reflector.

- If the refraction velocity model between the surface and the subsurface refractor is parameterized into an N x M grid (N pixels deep and M pixels wide) of square pixels, the minimum null space dimension of $L^T L$ is equal to $N - 1$. The proof for this statement is given by a layer stripping argument similar to that used by Calnan and Schuster (1989); they proved that the minimum null space dimension of $L$ for crosswell tomography is $N - 1$.

- Applying null space concepts and layer stripping arguments to $L^T L$ in crosswell transmission tomography, Calnan and Schuster (1989) suggested that the pixel width should be at least twice the receiver spacing in order to reduce the null space dimension of $L$ in crosswell transmission traveltime tomography. We make the same recommendation for 2-D refraction models since the concepts for model resolution in refraction tomography are quite similar to those for crosswell transmission tomography.

- If the velocity increases in depth then the null space of $L^T L$ will be empty for a dense source-receiver coverage. The proof for this statement can be realized by approximating the gradient velocity model as a stack of very thin homogeneous layers. Since the rays refract off each of the layers then their velocity structure can be determined.

1.7 Acknowledgement

We thank the 1994 GG527 class of Department of Geology and Geophysics, University of Utah, for collecting the second data set. We also thank Kun Hua Chen of Chevron Overseas Co. for allowing us to use the synthetic model and an anonymous sponsor for providing the salt intrusion data set. We thank all the 1992 and 1993 sponsors of the University of Utah Modeling and Tomography Development Project for their financial support. They are: Amerada Hess, Amoco, Arco, Chevron, Conoco, Exxon, Fujitsu, GRI, Japon, Noranda, Oyo, Texaco, Marathon, Unocal, and Phillips.
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Report 2

Improvements to the 3D Turning Ray Tomography Method

Fuhao Qin, Wenying Cai

2.1 Abstract

The original 3D refraction tomography code (Cai and Qin, 1993) is modified to allow the user to use different grid spacings during the inversion process. The computational efficiency achieved by this modification can be as great as an order of magnitude depending on the desired accuracy of the inversion. A smoothing operator to the slowness field is also incorporated into the code to remove the horizontal artifacts seen in the 1993 annual report (Cai and Qin, 1993).

2.2 Instruction

The choice of grid spacing plays an important role in refraction tomography. Wider grid spacing usually speeds up the inversion process at the expense of resolution. On the other hand, smaller grid spacing usually increases the resolution of the inversion results (providing proper measures are taken to deal with the underdetermined nature of the fine grid inversion problem, Nemeth et al., 1993) at the expense of increased computation cost. This problem is much more serious in 3D than in 2D since the computational cost is proportional to $N^3$, where $N$ is the number of grid along one dimension of the 3D cube.

To achieve both efficiency and accuracy, it is desirable to use different grid spacings at different stages of the inversion process. For the first several iterations, a large grid spacing is used to speed up the inversion and to obtain a long wavelength approximation to the model. When the decrease in the data residual stalls, a finer grid spacing is then applied to reveal finer details of the velocity structure.
Besides the increased computational cost in 3D turning ray tomography, another problem is even more troublesome. The inversion is much more underdetermined due to the larger spacing between seismic lines and the three dimensional freedom of the raypaths. This problem is very difficult to solve completely, but greater damping or smoothing provides an easy way to alleviate it.

2.3 Code Modification

The original 3D turning ray tomography code (Cai and Qin, 1993) is modified so that it

1. accepts an initial model with any grid spacing size and interpolates the model to a finer grid spacing for the current iteration. Thus it is very easy for the user to change grid spacing from large to small according to the convergence of the inversion;

2. smooths the slowness fields every fifth iteration in addition to the smoothing applied to the slowness gradient in each iteration. This eliminates some of the inversion artifacts caused by sparse ray coverage.

Another trick we use is that the initial model is smoothed after interpolation to remove any possible high frequency artifacts caused by the interpolation.

2.4 Numerical Results

To test the new turning ray tomography code, the fault and dome model used in Cai and Qin (1993) is used here. Figure 2.1 shows the model. The size of the model is 240x240x80 m and the velocities for the top and bottom regions are 1000 m/s and 2000 m/s, respectively. For simplicity, the recording geometry does not resemble the usual field experiment. There are 3 source lines and 5 receiver lines evenly distributed along both the X and Y directions. There are 7 sources in each source line and 12 receivers in each receiver line.

A multigrid tomography method was first carried out in which the grid spacing decreased from 6 meters (9 iterations) to 4 meters (9 iterations) and, finally, to 2 meters (4 iterations). For comparison, a standard tomographic method was applied where the grid spacing was fixed at 2 meters for all iterations. Figure 2.2 depicts the cross sections of (a) the true model, (b) the multigrid tomogram and (c) the standard tomogram. The curve overlaid on (b) and (c) outlines the true model. It can be seen that there is not much of a difference between (b) and (c) except near the right edge where neither result is trustworthy due to edge effects. We conclude that the two results are basically of the same quality.
2.5 DISCUSSION

However the computation costs for the two results are very different. Figure 2.3 shows the variation of the RMS residual with respect to the CPU time used on a DEC 300 work station. It can be seen that to get the same residual level the multigrid method is much faster than the standard tomography method.

It can also be seen that the residual does not decrease monotonically. It is due to the smoothing after the interpolation of the initial model and at every fifth iteration. Compare Figure 2.2 (b) and (c) with the result of Cai and Qin (1993) (not shown here), and we can see that the new results eliminate most of the inversion artifacts due to uneven ray coverage.

2.5 Discussion

The standard 3D turning ray tomography method is computationally expensive, but the multigrid technique provides a good way to reduce the computational cost. Damping or smoothing can be used to alleviate the problems caused by sparse ray coverage. However, we have not found the complete solution to this problem. What if the seismic lines are too far apart? What is the criterion for selecting the smoothing size based on the line spacing? These are the problems that still need an answer.

2.6 Reference


Figure 2.1: The 3D fault and dome model.
Figure 2.2: Cross section of (a) true model; (b) multigrid tomogram; (c) standard tomogram.
Figure 2.3: CPU time (using a 300 DEC 300 work station) and the corresponding RMS residual for multigrid inversion and standard inversion.
Report 3

Elastic Waveform Inversion of Crosshole Pressure Field Data

Changxi Zhou

3.1 Summary

I perform simultaneous inversion of P- and S-velocity structure by applying the traveltime-wave equation inversion method to the McElroy crosshole seismic data. The method we use here requires less data processing and also honors the original amplitude information in the data compared to the "divide and conquer" strategy presented by Zhou et al. (1994). The resolution of the P- and S-tomograms is higher than that of the tomograms inverted by using a divide and conquer technique. Unlike the divide and conquer method, the converted waves are properly inverted properly and the correlations between the P- and S-tomograms are improved. The crossing artifacts have been reduced by applying a simple dipping constraint to the misfit gradient.

3.2 Introduction

Zhou et al. (1994) used a divide and conquer strategy in applying elastic waveform inversion to crosshole pressure field records; the strategy was to first invert the separated P-waves for the P-velocity distribution, and then invert the separated S-waves for the S-velocity field. Supposedly, this strategy aimed to de-complexify the field records so that the misfit function would not be so non-linear with respect to velocity perturbations. Unfortunately there are some drawbacks with this strategy:

1. Skillful data processing procedures are required to separate both the PP- and SS-reflections from the original data. This separation is done by FK filtering which usually contaminates the records with coherent noises. Figure 3.1 shows a field record, and Figures 3.2 and 3.3 show the record after PP- and SS-reflection
separation. The filtering eliminates many useful parts of the records, such as truncation of the PP reflections, and is unable to eliminate PS-converted waves in the SS sections. Consequently, the PS reflections will be treated as coherent noise in their inversion.

2. Scalar waveform inversion is used to separately invert for both the P- and S-tomograms. The field data must be amplitude corrected to match the amplitude distribution of scalar wave propagation. This rough correction can distort the amplitude information in the original data and might result in incorrect impedance reconstruction.

In this report we present an approach of elastic traveltime-waveform inversion which requires less data processing and simultaneously inverts for both the P- and S-velocity distribution.

## 3.3 Methodology

Generally, the elastic waveform inversion method is applied to two or three component particle velocity or displacement records (Tarantola et al., 1985; Gauthier et al., 1986; Mora, 1987; Crase et al. 1992; Zhou et al., 1994). It is not straight forward to apply this method to pressure field data because of the failure of the numerical modeling method in simulating wave propagation at the boundary between the solid and liquid interfaces in the well. Ignoring the thin water layer in the elastic inversion might prove harmful (Zhou et al., 1994), so we implement several modifications to the elastic inversion presented by Zhou et al., (1994):

1. In the step of forward modeling we put a column of water in the receiver well to simulate a hydrophone recording environment. Three cylindrical transition zone are created between the rock and water to simulate the water saturated mud and the wall of the receiver well.

2. In the step of residual back propagation, we assume that the receiver is very close to the wall of the the well. The thin water layer can be neglected. The pressure field residuals are directly added to \( \sigma_{xx} \) and \( \sigma_{zz} \) as the source time history.

3. In the step of calculating the perturbation of the model parameters, we apply the \( \partial V_p / \partial x \) and \( \partial V_s / \partial x \) constraints to eliminate the crossing artifacts in the tomograms.

4. The subspace method is used to determine the optimal step lengths for updating both the P- and S-velocities.
3.4 Real data examples

The modified inversion scheme is applied to the McElroy field data (Harris et al., 1992). There are 210 shots evenly distributed in the source well at the depth range of 500 feet. The offset of the wells is 184 feet. Each common shot gather has 186 traces. The P- and S-velocity range is from 8000 ft/s to 22000 ft/s. We use a grid size of 0.625x0.625 feet to discretize the survey area. Thus the model size is 295x801. The time interval used in elastic wave propagation modeling is 0.0125 ms. A total 4000 time steps will be calculated.

Instead of the complicated data processing procedures required by the divide and conquer technique, we only apply a 200 to 1400 Hz bandpass filter to the field data filtering out the high and low frequency noises. The field data are transformed from 3-D to 2-D by applying the filter $\sqrt{i/w}$ in the frequency domain.

First, the first arrival traveltimes are picked and used to invert for the P-tomogram. Figure 3.4 shows the traveltime tomogram after 10 iterations. Five major velocity structures can be seen in this tomogram. Then this tomogram is used as the initial guess of the P-velocity structure for the elastic waveform inversion. The S-velocity initial model (Figure 3.5) is assigned from the P-velocity tomogram by multiplying the ratio of the S- to P-direct arrival traveltimes. The density model is obtained by an empirical relationship between the P-velocity and density (Gardner et al. 1974). We suppress the direct wave energy in the record and balance the trace amplitude in each gather according to the geometrical spreading factor when doing elastic waveform inversion. After two iterations, we obtain the high resolution P- and S-tomograms shown in Figures 3.6 and 3.7. They show a thinly layered structure. The resolution is higher than that of 7th iteration P- and S-tomograms (Figure 3.8 and 3.9) inverted by the divide and conquer strategy. Furthermore, the P- and S-velocity structures in Figure 3.6 and 3.7 have more similarities than those in Figure 3.8 and 3.9. The first iteration gradients (Figure 3.10 and 3.11) show that the approximation at the rock and water boundary used in our inversion method distorts the gradient field near the receiver well.

3.5 Discussion

Because of the limited CPU time on the Fujitsu supercomputer, we only performed two iterations of our elastic waveform inversion. But these results improved the resolution of the tomograms over those by using the divide and conquer method. The squared RMS waveform residual was reduced by eleven percent. The distortion in the gradient fields near the receiver well shows that a better way to simulate the wave propagation through the rock and water boundary needs to be studied in the near future.
3.6 Acknowledgements

We are very grateful to Fujitsu Computer Company for the use of their VPX-240 supercomputer. Acknowledgement is made to the Donors of The Petroleum Research Fund, administrated by the American Chemical Society for partial support of this research (contract PRF# 22807-AC2, PID 8909029). We are also grateful for the financial support provided by the 1992 University of Utah seismic tomography consortium members; Amerada Hess, Amoco, Arco, Chevron, Conoco, Exxon, Gas Research Institute, Japon, Marathon, Noranda, Oyo, Phillips, Texaco, and Unocal.

3.7 Reference


Figure 3.1: A field record before data processing
Figure 3.2: The separated PP-reflections from the field record.
Figure 3.3: The separated SS-reflections from the field record.
Figure 3.4: The P tomogram of traveltime inversion.
Figure 3.5: The S-velocity initial model assigned from P traveltime tomogram.
Figure 3.6: The P-tomogram of elastic waveform inversion after two iterations.
Figure 3.7: The S-tomogram of elastic waveform inversion after two iterations.
Figure 3.8: The P-tomogram of divide and conquer strategy waveform inversion after seven iterations.
Figure 3.9: The S-tomogram of divide and conquer strategy waveform inversion after seven iterations.
3.7. REFERENCE

Figure 3.10: The P-gradient of the first iteration of elastic waveform inversion method for the pressure field data.
Figure 3.11: The S-gradient of the first iteration of elastic waveform inversion method for the pressure field data.
Report 4

Radar Velocity Inversion

Wenying Cai

4.1 Abstract

Crosshole radar data is inverted for the radar velocity distribution by using an electromagnetic wave equation traveltime (EM WT) method. For comparison, a ray tracing tomography scheme is performed on the same data set. The two methods give very similar results which correlate very well with the logging profiles.

Introduction

In the 1990 Annual Report of the University of Utah Tomography and Modeling Consortium, we adapted the wave equation traveltime (WT) inversion algorithm (Luo and Schuster, 1990) to crosswell radar velocity inversion. The EM WT method was tested on both synthetic and field radar data in 1991. The method has proved to be very robust in areas with low-moderate conductivities.

The field radar data used for the previous test (Cai, 1991) was acquired from an area having a very small velocity contrast. To convince ourselves that the EM WT method can handle other velocity structures, we applied this method to a data set collected from an area where the logging data indicated a layered medium. Again we compare the EM WT result to that of a ray tracing method to see how well they agree with each other.

4.2 Numerical Results

This data set was provided by Japex and belongs to the Public Works Research Institute, Ministry of Construction; the survey was conducted by Geophysical Surveying and Consulting Co., LTD. in 1992 using a RAMAC radar system. The central
frequency of the radar system is 20 MHz, and the survey depth is between 15 and 60 meters. Both the transmission and receiver intervals are 1.5m, and there are 31 transmission levels and 31 records for each transmission level.

The time zero is unknown in the RAMAC radar system. To estimate the zero time we assume that a constant time shift exists in all records and the first arrival traveltimes is a function of the distance between the transmitter and the receiver. The picked first arrival traveltimes are plotted against distance in Figure 4.1. Using a least square method to fit the traveltimes to a straight line, the zero time is estimated to be about $t_0 = -0.0946\mu s$, i.e., all records are recorded after the transmission is turned on at $0.0946\mu s$.

The EM WT inversion for the radar velocity distribution is depicted in Figure 4.2. There are two clear interfaces in the inverted tomogram, one of them at a depth of about 28m with a slight layer dip from the transmitter borehole to the receiver borehole, the other interface appears at a depth of about 55.5m in the transmitter borehole with a larger dip. Comparing the result to the neutron porosity logs we find that they correlate very well for the two interfaces as well as with another two smoother interfaces at depths of about 40m and 50m. These interfaces indicate lithology changes.

A SIRT like ray tracing tomography algorithm is also applied to the same data set. The ray tracing tomogram is depicted in Figure 4.3, and it is very similar to the EM WT result except the velocity range is wider than the EM WT tomogram.

### 4.3 Discussion

The radar velocity tomograms reconstructed by EM WT method and by ray tracing method agree very well. The tomograms match very well with the neutron porosity logging profiles, and the error of the zero time estimate is small so it does not severely affect the velocity structures.

### 4.4 Reference


Figure 4.1: A plot of the first arrival traveltimes against distance.
Figure 4.2: EM WT radar tomogram compared to the neutron porosity logging profiles.
Figure 4.3: Ray tracing radar tomogram compared to the neutron porosity logging profiles.
Part II

Migration
Report 5

Some Numerical Tests of Quasi-Monte Carlo Migration

Fuhao Qin

5.1 Abstract

Numerical tests on the synthetic French model data show that quasi-Monte Carlo migration still gives superb result over standard migration even when there is a 10 percent error in the migration velocity. Preliminary tests on a Unocal field data set do not give satisfactory results. Possible reasons are that the source and receiver distribution is not very regular, and source and the receiver intervals are too large.

5.2 Instruction

Sikorski and Schuster (1992) and Sun et al. (1994) proposed to perform Kirchhoff integral migration using quasi-Monte Carlo multidimensional integration methods. Tests on a 3-D synthetic model (French model) with ideal seismic data (no multiples and converted wave modes and etc.) using the correct migration velocity (Sun et al., 1994) showed that the quasi-Monte Carlo migration could be almost an order of magnitude more efficient than standard migration. Although the test results are very promising, they have not demonstrated the robustness of the method for migration velocity errors or multiples and most importantly, for real data. In this report, we use the synthetic French model data to suggest that the method works well with 10 percent migration velocity errors and we also apply this method to a real 3-D data set.
5.3 French Model Migration with Incorrect Velocity

We use the synthetic 3-D French model data to demonstrate how velocity inaccuracies affect the quasi-Monte Carlo migration. All parameters are the same as the test in Sun et al. (1994) except that the migration velocity used here is 10 percent less than the true velocity. Similar to Figure 9.5 and 9.6 in Sun et al. (1994), Figure 5.1 and 5.2 show the images from regular grid and Halton point migration, respectively. Each figure consists of 4 different images using 8100, 32400, 129600 and 518400 traces, respectively. It can be seen that the coherent noise is much less in the Halton migration image than in the regular grid migration using the same number of seismic traces. This suggests that although errors in the migration velocity move the reflectivity image to an incorrect depth and blur the migrated image, quasi-Monte Carlo migration can still enhance the image by decreasing the coherent noise.

5.4 Real Data Test

In this section, we try to apply the method to a real 3-D data set. This is a marine data set collected over an area with salt dome intrusions. To acquire the data, receiver cables were laid on the sea floor and airguns were shot along 12 shot lines on either side of the receiver cable. This formed a swath and there were 13 swaths in this survey. There are 6 long swaths with cable length approximately 50,000 ft (228 shots) and 7 short swaths with cable length of 32,000 ft (143 shots). The inline shot and receiver spacings were both 220 ft. The spacing between shot lines was also 220 ft.

From these data, we use about 500,000 traces that covers a small part of the survey area to perform the migration test. To minimize the computation time, we migrated only one section that cuts right through the center of the middle point distribution along the north/south direction (parallel to the lines, Figure 5.3). The migration velocity is a background velocity without the salt dome structure to avoid the head wave problem in the eikonal equation traveltime computation.

Figure 5.4 shows several CDP gathers of the data to be migrated. The gathers are corrupted by high frequency noise. To decrease the high frequency noise and to alleviate the spatial aliasing, a triangular smoothing filter is used for low pass filtering. Migration is performed first using all the seismic traces. Figure 5.5 shows the migration result. The image clearly depicts a stack of horizontal layers and outlines the salt dome. Since the migration velocity does not include the salt dome, the outline of the salt may not be precise.

Figure 5.6 and 5.7 are the migration results from 1/3 and 1/6, respectively, of the total traces selected by the Halton distribution. Both of the images contain the same structures as in the image from all traces except that the images are noiser.

Next, migration is performed by using every third trace (Figure 5.8) and every
5.5 DISCUSSION

sixth traces (Figure 5.9). If we consider this as migration by regular grid, we would expect that these images should be much worse than the Halton point migration images. However, unlike what we expected, these images are similar to those from the Halton point migration.

There must be something wrong either in the data or in the migration method. So we examined the shot location for a common receiver gather. Figure 5.10a shows all the shot locations of one CRG. Figure 5.10b,c show the shot locations after Halton selection and Figure 5.10d,e are the shot locations after the "regular" selection. It is very clear that there is not much of a difference between the Halton and "regular" grid migration results. The shot distribution of the regular selection looks as random as the Halton distribution. The reason is that the "regular" shot location is selected by choosing every third or sixth trace from the CDP gathers. They are not regularly distributed. Besides, the shot location for the whole CRG itself does not look very regular either. That explains the similarity between the Halton point migration and our pseudo-regular grid migration results.

5.5 Discussion

The 3-D migration tests on the synthetic French model data suggest that quasi-Monte Carlo migration is not too sensitive to migration velocity inaccuracies. It further suggests its potential utility for filed data applications.

The numerical tests on the Unocal 3-D data set is not very conclusive due to the irregularity of the source and receiver distribution, and inability to extract a regular distributed shot distribution. A fair comparison will be to first interpolate the data into a regular distributed source and receiver locations and then apply the migration. Future tests will carry this strategy out on both the Unocal data as well as other 3-D data sets.

5.6 Acknowledgment

We thank Dr. Wook Lee and Unocal for providing the 3-D field data set.

5.7 Reference

Figure 5.1: 3-D migration French model images using various number of traces for a regular distribution of sources and receivers.
Figure 5.2: Same as Figure 5.1 except a Halton distribution of sources and receivers are used.
Figure 5.3: Source-receiver mid-point distribution and location of migration section.
Figure 5.4: Several CDP gathers of the unmigrated data.
Figure 5.5: Migration image using all seismic traces.
Figure 5.6: Migration image using 1/3 of the total traces by Halton point selection.
Figure 5.7: Migration image using 1/6 of the total traces by Halton point selection.
Figure 5.8: Migration image using 1/3 of the total traces by "regular" selection.
Figure 5.9: Migration image using 1/6 of the total traces by "regular" selection.
Figure 5.10: Shot locations of a CRG; (a) all shots; (b) 1/3 of the shots by Halton selection; (c) 1/6 of the shots by Halton selection; (d) 1/3 of the shots by "regular" distribution; (e) 1/6 of the shots by "regular" selection.
6.1 Abstract

I present both theoretical and numerical results that explain why random or quasi-random migration provides a better migrated image than standard migration for coarsely sampled data. Results show that migrating seismic data on a random or quasi-random grid tends to destroy the nasty grating lobes of an image. In contrast, the images obtained from a coarse regular array of data always contain nasty grating lobes. These grating lobes are equivalent to aliasing artifacts, and so quasi-random or random migration has a built-in anti-aliasing filter. The penalty for this anti-aliasing feature is a relatively higher sidelobe level, which decreases the dynamic range of the migrated image. However, the limited numerical tests of Sun et al. (1993) suggest that the benefits of anti-aliasing outweigh the acceptable loss of dynamic range.

Theoretical analysis shows that images from 2D random prestack migration can have twice the horizontal resolution and fewer aliasing problems than images from 2D poststack migration. 3D poststack migration will have roughly the same resolution as 2D prestack migration provided the source and receiver apertures are the same. And images from 3D prestack migration can have twice the resolution as images from 3D poststack or 2D prestack data.

This new understanding provides the framework for exploring new applications of quasi-random or random integration in seismic exploration.
Figure 6.1: Migration images of the French model for various numbers of synthetic seismograms on a regular grid. The imaging technique is a 3-D prestack Kirchhoff migration method applied to a regular grid of prestack seismic data (from Sun et al., 1993).

6.2 Introduction

Sun et al. (1993) discovered that migrating seismic data from a coarse quasi-random grid led to migrated images with fewer aliasing artifacts than those obtained from a coarse regular grid of data. Figures 6.1 and 6.2 illustrate, respectively, their migrated images obtained from regular grids of seismograms and from quasi-random grids of seismograms. It is clear that the ghosts due to a sparse distribution of data are much more pronounced for the standard Kirchhoff migration (Figure 6.1) compared to the quasi-random Kirchhoff migration (Figure 6.2).

Why did the quasi-random Kirchhoff migration work better than the standard Kirchhoff migration? Up until now I did not know the precise answer to this question. A suitable answer might lead to the design of optimal source-receiver geometries for sparse data migration and economical 3D field experiments.

This paper presents both theoretical and numerical results that explain why quasi-random migration provides a better migrated image than standard migration for coarsely sampled data. In a nutshell, migrating seismic data on a random or quasi-random grid tends to destroy the nasty grating lobes of an image; this compares favorably to images obtained from a coarse regular array of data where grating lobes
Figure 6.2: Same as previous figure except a quasi-random distribution of seismic data is migrated.

are always present. The grating lobes are equivalent to aliasing artifacts, and so quasi-random or random migration has a built in anti-aliasing filter. This new understanding provides the framework for exploring new applications of quasi-random integration in seismic exploration.

### 6.3 Resolution of Migrated Images

A form of 2D post-stack Kirchhoff migration can be represented by

\[
m(\vec{r}, \omega) = \int_{-L}^{L} e^{i2\omega|\vec{r} - \vec{r}'|/c} \frac{f(\vec{r}', \omega)}{|\vec{r}' - \vec{r}|} h(\vec{r}') dx',
\]

where \( f(\vec{r}, \omega) \) represents the surface seismic data measured on the recording line of length \( 2L \), \( c \) is the subsurface homogeneous velocity, \( h(\vec{r}') \) is the sampling function, \( m(\vec{r}, \omega) \) is the migrated image at some subsurface location \( \vec{r} \), and \( \omega \) is frequency.

Since seismic wavefields are always sampled by a discrete array of geophone groups, the sampling function can be described by

\[
h(\vec{r}) = \sum_{i=-N}^{N} \delta(\vec{r} - \vec{r}_i),
\]
where the number of geophone groups (i.e., gridpoints) is $2N + 1$, the gridpoint distribution is described by the set of vectors $(r_{-N}, r_{-N+1}, ..., r_N)$, and $r_i$ is the vector pointing to the $i^{th}$ gridpoint on the recording array. A regular sampling function is defined such that the grid points are evenly distributed along the recording line, and a quasi-random sampling function is one where the gridpoints are distributed in a quasi-random manner.

### 6.3.1 Vertical Spatial Resolution.

The vertical and spatial resolution of the migrated image will be controlled by two factors: the source bandwidth and the geometry of the recording array. The bandwidth of the source will largely dictate the vertical resolution $\Delta z$ of the migrated image by the following formula:

$$\Delta z = 0.5c\delta t,$$

where $\delta t$ is the pulse width that is inversely proportional to source bandwidth. This criterion implies that if two point scatterers are vertically separated by a distance greater than $\Delta z$ then they can be seismically distinguished from one another if the pulse width honors the above equation.

### 6.3.2 Horizontal Spatial Resolution.

In contrast to vertical resolution, the horizontal resolution is largely determined by the aperture size. This can be seen with the example of a buried point scatterer shown in Figure 6.3. For a point scatterer buried at the location $r_0$ the zero-offset impulse response is represented in the frequency domain by

$$f(r', \omega) = \frac{e^{-i2\omega|\vec{r}' - \vec{r}_0|/c}}{|\vec{r}' - \vec{r}_0|},$$
where \( \vec{r}' \) denotes the receiver location on the earth's surface. Plugging the above into equation 6.1 we get

\[
m(\vec{r}, \omega) = \int_{-L}^{L} e^{i k (|\vec{r}' - \vec{r}| - |\vec{r}' - \vec{r}_0|)} \frac{h(\vec{r})}{|\vec{r}' - \vec{r}|} dx',
\]

where \( k = 2\omega/c \) and \( \vec{r} = (x, z) \). If assume that 1. the aperture width of \( 2L \) is much less than both the depth of the point scatterer \( (2L/|z_0| << 1) \) and the imaging depth \( (2L/|z| << 1) \), and 2. \( r >> L^2k/2\pi \), then we can approximate the argument in the first exponential by

\[
|\vec{r}' - \vec{r}| = \sqrt{z^2 + (x'-x)^2} = r\sqrt{1 + \frac{x'^2 - 2xx'}{r^2}} = r - \frac{x'}{r} + \frac{x'^2(1 - \sin^2 \theta)}{2r} + \text{higher-order terms}
\approx r - x'\sin \theta,
\]

(6.6)

where \( r = \sqrt{x^2 + z^2} \) and \( \sin \theta = x/r \). This characterization of distance as \( r - x'\sin \theta \) is known as the far-field approximation (Steinberg and Subbaram, 1991). In a similar fashion the other exponential argument becomes

\[
|\vec{r}' - \vec{r}_0| = \sqrt{z'^2 + x'^2}
\approx r_0.
\]

(6.7)

Substituting equations 6.6 and 6.7 into equation 6.1 we get

\[
m(\vec{r}, \omega) \approx A \int_{-L}^{L} e^{-ikx'sin \theta} h(\vec{r}') dx',
\]

(6.8)

where \( A = e^{ik(r-r_0)/r_0^2} \). Setting \( h(\vec{r}) = 1 \) yields

\[
= \frac{2A \sin(kL \sin \theta)}{k \sin \theta}.
\]

(6.9)

Equation 6.9 says that the migrated image of the point diffractor is a sinc function, not a point diffractor. The halfwidth of the sinc function's main lobe is determined by finding its first zero crossing, i.e.,

\[
\frac{\pi}{2} = kL \sin \theta,
\]

\[
= \frac{kLx}{r},
\]

\[
\approx \frac{kLx}{r_0},
\]

(6.10)
which can be solved for $x$ to give the width $\Delta x$ of the main lobe

$$\Delta x = \frac{\pi r_0}{kL}. \quad (6.11)$$

Thus, the beam width is proportional to the depth of the scatterer $r_0$ and inversely proportional to the wavenumber and aperture size. Not surprisingly, this says that wider apertures and wider source bandwidths improve the lateral resolution of the imaged point scatterer. These resolution characteristics are consistent with the enlargement of the first Fresnel zone as a function of reflector depth and source bandwidth.

A rough criterion for horizontal resolution can now be defined: if two point scatterers are horizontally separated by a distance greater than $\Delta x$ then they can be seismically distinguished from one another if the aperture width, frequency and reflector depth honors equation 6.11.

### 6.3.3 Regular Grid Migration and Grating Lobes.

Until now we assumed that the sampling function was unity over the entire aperture, i.e., $h(-L < x' < L) = 1$. I will now substitute the discrete equi-spaced sampling function

$$h(x) = \sum_{j=-N}^{N} \delta(x - j\delta x) \quad (6.12)$$

(where $\delta x = 2L/(2N + 1)$) into equation 6.8 to give

$$m(\vec{r}, \omega) = \frac{2AL}{2N + 1} \sum_{j=-N}^{N} e^{-ik(\delta x \sin \theta)j}. \quad (6.13)$$

Using the sinc function identity on page 302 in Oppenheim and Wilsky (1983) the above expression is reduced to

$$m(\vec{r}, \omega) = \frac{2AL\sin[(2N + 1)\Omega/2]}{(2N + 1)\sin(\Omega/2)}, \quad (6.14)$$

where $\Omega = 2kL\sin \theta/(2N + 1)$. Equation 6.14 describes the periodic curve shown in Figure 6.4, which is similar to that of a sinc function except the main lobe is repeated at integer multiples of $\Omega = 2\pi$. These false images or repeated main lobes are called grating lobes by the radar imaging community (Steinberg and Subbaram, 1991), and give rise to the spatial aliasing artifacts in a migrated image. The grating lobes move closer to the main lobe as the geophone spacing becomes coarser. Thus, the effect of a sparse regular grid of geophones is to introduce spatial aliasing artifacts into the migrated image.

It can also be seen from equation 6.14 that the ratio of the mainlobe amplitude to sidelobe amplitude is proportional to $2N + 1$. Hence, the dynamic range of the
Figure 6.4: Plot of $\frac{\sin(5f/2)}{\sin(f/2)}$ against $f$. The repeated main lobes at integer multiples of $2\pi$ represent grating lobes, or false images due to spatial aliasing.

Figure 6.5: Plot of $e^{-k^2\sigma^2\sin^2\theta}\frac{\sin(5f/2)}{\sin(f/2)}$ against $f$. The grating lobes are suppressed compared to the previous figure.

migrated section becomes greater as the number of geophones increase. Here, the dynamic range of the migrated section is proportional to the difference in reflection strengths between the strongest and weakest diffractors that can be simultaneously imaged. In order to image a weak diffractor, its main lobe amplitude should be reasonably larger than the sidelobe amplitude of the strongest diffractor.

### 6.3.4 Random Grid Migration and Attenuated Grating Lobes

Now I assume a random distribution of gridpoint locations so that $j_0x$ in equation 6.13 is replaced by $j_0x + \epsilon_j$, where $\epsilon_j$ is an independent random variable with a Gaussian probability distribution; also, $\epsilon_j$ is assumed to have a mean of zero and a standard deviation denoted by $\sigma$. Plugging this random variable into equation 6.13 we get the
random array image \( m(\vec{r}, \omega)_{\text{rand}} \), i.e.,

\[
m(\vec{r}, \omega)_{\text{rand}} = \frac{2AL}{2N+1} \sum_{j=-N}^{N} e^{-ik(\delta x \sin \theta)j} e^{-ik(\sin \theta)\epsilon_j}.
\] (6.15)

Defining the weighted expectation operator \( E \) as

\[
E(g(\epsilon)) = \frac{1}{2\pi \sqrt{\sigma}} \int_{-\infty}^{\infty} e^{-\frac{25\epsilon^2}{\sigma^2}} g(\epsilon) d\epsilon,
\] (6.16)

recalling the identity

\[
e^{-k^2\sigma^2} = \frac{1}{2\pi \sqrt{\sigma}} \int_{-\infty}^{\infty} e^{-ik\epsilon} e^{-\frac{25\epsilon^2}{\sigma^2}} d\epsilon,
\] (6.17)

and applying the expectation operator \( E \) to equation 6.15 results in

\[
E(m(\vec{r}, \omega)_{\text{rand}}) = \frac{2AL}{2N+1} \sum_{j=-N}^{N} e^{-ik(\delta x \sin \theta)j} E(e^{-ik(\sin \theta)\epsilon_j})
\]

\[
= \frac{2AL}{2N+1} \sum_{j=-N}^{N} e^{-ik(\delta x \sin \theta)j} e^{-k^2\sigma^2\sin^2(\theta)}
\]

\[
= e^{-k^2\sigma^2\sin^2(\theta)} \frac{2AL}{2N+1} \sum_{j=-N}^{N} e^{-ik(\delta x \sin \theta)j}
\]

\[
= e^{-k^2\sigma^2\sin^2(\theta)} m(\vec{r}, \omega).
\] (6.18)

Thus the random migrated image is the same, on average, as the regular migrated image except for the multiplicative Gaussian scaling function \( e^{-k^2\sigma^2\sin^2(\theta)} \). This Gaussian function suppresses the grating lobes in Figure 6.4 to give the attenuated spectrum shown in Figure 6.5, and so (on average) attenuates the aliasing artifacts in the migrated image. The penalty for suppressing the grating lobes is that the ratio of the main lobe to maximum sidelobe amplitude is decreased (p. 37, Steinberg, 1983). Steinberg and Subbaray (1992) explain "Randomizing the element positions in a thinned array destroys the grating lobes but not the energies in these lobes. The energy in the grating lobes of the periodically thinned array must equal the energy in the sidelobes of the randomly thinned array with the same average thinning factor."

A decrease in the ratio of the main lobe to side lobe amplitude means that the dynamic range of the migrated image is decreased compared to an image from a regular array of geophones. Apparently the migration tests of Sun et al. (1993) suggest that this loss in dynamic range is acceptable, and so justifies the anti-aliasing benefits of migrating a quasi-random array of data.

### 6.3.5 Quasi-random Grid and Attenuated Grating Lobes.

The previous section showed that the chief benefit of random array imaging is that the grating lobe amplitudes are suppressed, and so aliasing artifacts are mitigated. An
6.3. **RESOLUTION OF MIGRATED IMAGES**

intuitive explanation is that on a regular grid distribution the quadrature errors of the Kirchhoff integral at each of the gridpoints are in phase and so upon summation lead to coherent amplification of local quadrature errors. In contrast, the local quadrature errors on a random grid are out of phase with one another and so do not, on average, greatly amplify the quadrature error. The problem with a sparse random gridpoint distribution is that it can have a patchy appearance on a large scale; some parts might have a denser sampling of points than other parts. This can lead to severe quadrature errors. To avoid this problem, quasi-random sampling schemes were developed (see Chapter 1 in Niederreiter, 1992) so that "the samples are spread in a uniform manner over the integration domain". Thus, quasi-random sampling avoids the patchiness problem and (I believe) still retains the the anti-aliasing benefits of an irregular spatial sampling on a local scale. I will now show some numerical tests which support this statement.

Figures 6.6 and 6.7 depicts, respectively, the Halton and Hammersley point distributions on a 2-dimensional grid. Note that the quasi-random Halton and Hammersley points are irregularly spaced on a local scale, but they have a uniform density on a larger scale.

Applying a 2048x2048 point FFT to the quasi-random sampling brushes in Figures 6.6 and 6.7 gives the magnitude spectrums shown, respectively, in Figure 6.8 and 6.9. For comparison, the uniform spectrum of a regular sampling brush is shown in Figure 6.10. Note that the uniform spectrums in Figure 6.10 have teeth of uniform height located at integer multiples of the Nyquist frequency, and these teeth in the \((k_x, k_y)\) domain get further apart as the sampling teeth get closer together in the \((x, y)\) domain. If the teeth spacing is smaller than the spatial bandwidth of the data then the high frequencies will masquerade as low frequencies, i.e., the image will be aliased. In contrast, the Figure 6.8 and 6.9 spectrums associated with the quasi-random sampling brushes have a unit spike at the origin surrounded by a sea of shorter spikes with irregular height. The amplitudes of these irregular spikes in the \((k_x, k_y)\) domain become smaller as the sampling teeth get closer together in the \((x, y)\) domain. The important observation is that, unlike the large spikes at integer multiples of the Nyquist frequencies in, e.g., Figure 6.10d, there are no grating lobe spikes in the, e.g., Figure 6.8d quasi-random spectrum. In fact, the only large amplitude tooth in Figure 6.8d is located at the origin. This suggests that the aliasing artifacts inherent in a quasi-random migrated image might not be as pronounced compared to the migration artifacts obtained from a regular grid of data. Indeed, the better performance of quasi-random migration compared to standard migration for the French model (Sun et al., 1993) seems to verify this suggestion.

Comparing Figure 6.10d with Figure 6.8d shows that the quasi-random spectrum has shorter teeth, but many more teeth, than does the uniform spectrum. This is consistent with theoretical analysis that shows that the ratio of the main lobe to sidelobe amplitudes is less than that for a regular array (p. 37 in Steinberg, 1983). Turning grating lobes into a sea of small amplitude sidelobes is certainly desirable, but
Figure 6.6: Halton point distributions on a 2-D grid. The associated sampling brush $h(x, y)$ is equal to one at the gridpoints and is equal to zero elsewhere.

Figure 6.7: Same as previous figure except these are Hammersley point distributions.
Figure 6.8: Magnitude spectrums of the Halton distributions shown in a previous figure. A 2048x2048 regular grid was superimposed on the Halton point distributions, the unit spikes at the Halton points were moved to their nearest point on the regular fine grid, and a 2048x2048 FFT was applied to the resulting sampling brush.
Figure 6.9: Same as previous figure except the magnitude spectrums are for the Hammersley sampling brushes shown in a previous figure.
Figure 6.10: Same as previous figure except the magnitude spectrums are for the regular sampling brushes.
is it worth it if the penalty is the appearance of short teeth at other frequencies? The limited migration tests shown in Figures 6.1 and 6.2 suggest that this is a beneficial tradeoff.

### 6.3.6 Post-stack vs Pre-stack Migration

We all know by both intuition and numerical tests that prestack migrated images are better resolved than poststack migrated images. Let us quantify this statement with the analysis from the previous sections.

The composite image obtained by summing the migrated common shot gathers is represented by the 2-D prestack Kirchhoff integral:

\[
m(\vec{r}, \omega) = \int_{-L}^{L} \int_{x_s-\Delta}^{x_s+\Delta} e^{ik_s(x_s+\Delta)} \frac{e^{ik_s(x_s-\Delta)}}{|r^g - \vec{r}|} \frac{f(r^g, r^g_s, \omega)}{r^g_s - \vec{r}} h(r^g) h(r^s) dx_s dx_s,
\]

where each shot is surrounded by a symmetric aperture of receivers, and the aperture width of the receiver array is \(2\Delta\). The superscripts \(g\) and \(s\) denote, respectively, the geophone and source coordinates, \(f\) represents the prestack data, and \(2L\) is equal to the width of the shot aperture. As before, I apply a farfield approximation to the exponential arguments in equation 6.19 to get the following equation:

\[
m(\vec{r}, \omega)_{\text{prestack}} = \alpha \int_{-L}^{L} e^{-ik_s x_s} \int_{x_s-\Delta}^{x_s+\Delta} e^{-ik_s x_s} dx_s dx_s,
\]

where \(\alpha\) is an expression similar to \(A\) in equation 6.8, and \(k_s\) and \(k_g\) are the source and geophone wavenumbers (similar to \(\omega \sin \theta / c\) in equation 6.8). Carrying out the above integral I get

\[
m(\vec{r}, \omega)_{\text{prestack}} = 2\alpha \int_{-L}^{L} \frac{e^{-ik_s x_s} \sin(k_s \Delta)}{k_s} dx_s
\]

\[
= 4\alpha \frac{\sin([k_g + k_s]L) \sin(k_s \Delta)}{(k_g + k_s)k_s}.
\]

Equation 6.21 says that the scatterer's image is approximated by a concatenation of two sinc functions. In fact, if \(k_g \approx k_s\) then the main lobe of this image has half the width of that for the post-stack image (see equation 6.9). Thus, the horizontal resolution of prestack migration can be twice as good as poststack migration.

If a random, instead of a regular, array of sources and receivers were used for prestack migration then an expression similar to equation 6.18 would be obtained, except the random prestack image would be represented by a concatenation of two sinc-like functions and two scaled Gaussian functions, i.e.,

\[
m(\vec{r}, \omega)_{\text{random prestack}} = e^{-(k_s^2 + k_g^2)\sigma^2} m(\vec{r}, \omega)_{\text{prestack}}.
\]
For $k_g = k_s$ this means that the grating lobes are better attenuated than in the poststack image. A prestack image would be less prone to aliasing artifacts than a poststack image.

It is easy to extend this analysis to 3D poststack and 3D prestack migration. Roughly speaking, 3D poststack migration should have the same image resolution as 2D prestack migration for 2D structures. Also, 3D prestack migration can have about twice the image resolution as 3D poststack or 2D prestack migration.

Some untested practical applications of the previous analysis include:

- The spatial resolution formulas, e.g. equation 11, can be used to determine the variable gridpoint spacing as a function of depth in a discrete migrated image. Fewer imaging points in a grid mean less computation costs.

- Better design of CDP surveys. Formula 21 contains $L$, the width of the source aperture, and $\Delta_s$, the width of the receiver aperture. These variables can be adjusted in the field experiment so as to match the resolution requirements of the target zone. For a 3D prestack experiment, the equivalent imaging formula will contain both the width and length of the receiver and aperture arrays. An array that is wider than it is long will lead to better subsurface resolution along the wide axis; how much better can be determined by a formula.

- We might borrow other methods from the radar imaging community. For example, amplitude weighting of the array or diversity stacking (Steinberg and Subbaram, 1991).

5 Summary

Resolution analysis for migration of regular, random and quasi-random distributions of seismic data reveal the following principles:

1. An image migrated from a coarse random array of seismic data is less likely, on average, to be aliased than an image migrated from a regular array of data. This is because the grating lobes of the image are suppressed due to the incoherent arrangement of the geophones. In migrating these data, the geophones act as an incoherent arrangement of loudspeakers that broadcast the reflected wavefield back into the earth; the broadcast will produce reinforcement (cancellation) of seismic energy at the diffractor (grating lobe) location. Numerical tests suggest that these principles for random migration also hold true for quasi-random migration.

2. A decrease in the dynamic range of the random migrated image is the penalty for the anti-aliasing benefits of random or quasi-random migration. The ratio of the mainlobe to sidelobe image amplitudes is decreased
for a random array compared to a regular array. Sun’s numerical tests suggest that this might be an acceptable cost.

3. The spectrums for 2-D quasi-random sampling brushes also show reduced grating lobe amplitudes, and thereby support the idea that quasi-random migration has the same anti-aliasing benefit as random migration.

4. Theoretical analysis shows that images from 2D prestack migration can have twice the horizontal resolution and fewer aliasing problems than images from 2D poststack migration. 3D poststack migration will have roughly the same resolution as 2D prestack migration provided the source and receiver apertures are the same. And images from 3D prestack migration can have twice the resolution as 3D poststack data.

This new understanding of random and quasi-random migration provides the framework for exploring new applications of quasi-random integration in seismic processing. Future work should study the benefits of applying velocity analysis and DMO to coarse quasi-random grids of seismic data.

I was able to analyze the random migration problem by mostly borrowing the quantitative tools used by the radar imaging community. Further study of their literature might prove to be productive.

References


Report 7

SS Reflection Extraction and Migration of McElroy Crosswell Data

Wenyeng Cai and Fuhao Qin

7.1 Abstract

Further processing of the McElroy crosswell data has been conducted with the aim of improving the SS migration image. A median filter other than an f-k filter is used to remove tube waves. Results show that median filtered data results in improved SS migrated images.

7.2 Introduction

In the 1993 Annual Report of the Modeling and Tomography Project of the University of Utah (Cai and Qin, 1994), we used a sequence of processing steps to extract PP and SS reflections from the McElroy data set. The processing includes picking first P or S arrivals, preprocessing, removing direct P or S arrivals, and f-k filtering to separate wavefields. As we can see from the PP- and SS- reflection migration sections presented in the 1993 annual report, this processing flow is sufficient for extracting the PP reflections while it is not very suitable for extracting the SS reflections. In particular, the f-k filter was not effective in removing the tube waves. To effectively suppress tube waves, we now use a median filter followed by a high cut filter to remove the tube waves after the preprocessing step.

In this report we present SS reflection migration images using a median filter that suppresses the tube waves in the data processing procedure. These migrated sections will be compared with the corresponding migration sections obtained without a median filter to suppress the tube waves (presented in 1993 Annual Report of the Modeling and Tomography Project, University of Utah).
7.3 SS Reflection Migration

The S-velocity used for migration of the McElroy crosswell data is taken from the P-velocity tomogram scaled by a factor of 0.55. This scaling factor was estimated from the P- and S-sonic logs in the source well.

Figure 7.1 depicts a processed shot gather that contains mostly downgoing SS reflection arrivals. Compared to the same gather that was processed without a median filter for tube wave suppression (Figure 7.2), it is obvious that using a median filter for suppressing tube waves is more effective than using an f-k filter. The migrated SS reflection section obtained from median filtered data also shows significant improvement. Figure 7.3a depicts an upgoing SS reflection migration section which is compared to the corresponding section (Figure 7.3b) computed without the use of a median filter. Obviously, the migrated section in Figure 7.3a has fewer fault-like artifacts and the interfaces are more continuous than those in Figure 7.3b. The downgoing migrated section depicted in Figure 7.4a shows similar features, compared to the corresponding section in Figure 7.4b. The resulting composite migrated section and the sonic log synthetics are shown in Figure 7.5. For comparison, the corresponding section and the sonic log synthetics are depicted in Figure 7.6. We can see that the final migrated SS reflection image in Figure 7.5 has more continuous reflectors and better correlation with the sonic logs compared to the image in Figure 7.6 for which no median filters were used to remove the tube waves.

7.4 Discussion

The McElroy data are dominated by complex wave modes, which include PP and SS reflections, direct P and S arrivals, P-S transmitted and reflected conversions, and tube waves. The tube waves act as coherent noise that severely affect the SS reflection migration images. Using a median filter to suppress tube waves prior to the use of an f-k filter (to separate upgoing and downgoing SS reflections) can improve the migration image quality.

Our conclusion is that high resolution SS-reflectivity images can be obtained from crosswell data if careful data processing procedures are designed to extract the SS reflections.

7.5 Reference

Figure 7.1: A processed shot gather that primarily contains downgoing SS reflections. A median filter is used to suppress the tube waves.
Figure 7.2: A processed shot gather that primarily contains downgoing SS reflections. The processing steps are the same as those in Figure 7.1 except no median filter is used.
Figure 7.3: Upgoing SS migrated sections. (a). A median filter is used to suppress the tube waves in the processing procedure; (b). No median filter is used.
Figure 7.4: Downgoing SS migrated sections. (a). A median filter is used to suppress the tube waves in the processing procedure; (b). No median filter is used.
Figure 7.5: Composite SS migrated section compared to the sonic log synthetics. A median filter is used to suppress the tube waves in data processing.
Figure 7.6: Composite SS migrated section compared to the sonic log synthetics. No median filter is used to suppress the tube waves in data processing.
Report 8

Imaging fault structures with VSP migration

Tamas Nemeth

8.1 Abstract

In this paper an iterative VSP migration is applied to two synthetic data sets to image complex fault structures. The first model contains a steeply dipping fault in the middle of the model and the second model has several dislocation faults. The obtained migrated sections indicate that the iterative VSP migration is capable of imaging complex structures.

8.2 Introduction

VSP migration has become a useful tool in recent years in imaging the subsurface reflectivities. Several papers (Amundsen, 1993; Payne, 1994) have addressed the problem of imaging complex structures with VSP migration.

VSP migration usually provides a better resolution than the corresponding surface migration. However, it needs an accurate background velocity distribution to accomplish this task. In many cases an accurate velocity model is not available, especially for regions far away from the receiver well. In this case a VSP migration velocity analysis (Nemeth, 1994b) is needed. Another issue for VSP migration is to extract amplitude information from the seismograms and to eliminate the migration artifacts. An iterative migration (Nemeth, 1994a) helps to solve these problems.

In this paper I process two synthetic data sets to image complex structures. The model for the first data set has a steeply dipping fault, and the model for the second data set has several dislocation faults. The results indicate that the VSP migration is capable of imaging these structures.
8.3 Numerical results

Test 1.
Test 1 was a blind test performed on 7 common source gathers. The goal of the test was to study the imaging capabilities of the iterative Kirchhoff migration on a (supposedly) complex model. No velocity distribution was provided for the test.

The processing included the following procedures: seismogram filtering, seismic tomography, and migration. The seismogram filtering procedure started with direct arrival time picking for the subsequent seismic tomography. Then the direct waves were removed from the seismogram by a median filter (Cai, 1992). No frequency filtering was necessary, since there was no high or low frequency noise in the seismograms. I mention that the signal was a relatively low frequency signal, consequently the migrated images have ample reflector width.

The next procedure was to use seismic tomography applied to the first arrival travel times to provide the background velocity for migration. The model size was defined as 3000 ft by 6000 ft. The receiver well was located at the right side of the model and the seven sources were distributed evenly along the 3000 ft offset. There were receivers every 40 ft in the well. The resulting tomogram (not shown) showed a 5-layer model near the receiver well and a dipping structure away from the well. But due to the lack of the ray coverage away from the well I did not consider those velocities locally reliable and modified the the obtained velocity model by extrapolating the more reliable near-well velocities in the lateral direction.

The next procedure was the prestack iterative Kirchhoff VSP migration (Nemeth, 1994). The output of this routine was 7 prestack migrated sections. Before stacking them I arranged the migrated data into common image gathers to perform migration velocity analysis. To my surprise no adjustments were necessary on common image gathers, since the events lined up horizontally on common image gathers (CIG's). Since the input velocity model was (probably) not correct, I expected some curvature on the CIG's. I assume that the reason for the lack of the curvature is that the migrated images contain low frequency events due to the low frequency seismograms. In this case the difference between the original and the applied background velocity is not enough to cause the migrated images to be shifted at least a quarter-half a wavelength. The composite migrated section was obtained by stacking the seven prestack migrated sections. It is depicted in Figure 8.1 and the reversed polarity pair is shown in Figure 8.2. Using low frequency events to study both composite migrated sections together is instructive. The composite migrated sections reveal a general dipping layer structure and a steeply dipping fault in the middle part of the section. Compare the migrated sections with the original model (Figure 8.3), provided later.

Test 2.
This test was performed to study the imaging capabilities of VSP migration for complex geologic structures. The velocity distributions provided for these models are shown in Figures 8.4, 8.6 and 8.8. The recording geometry was the same for these
models. The receiver well was located at an offset of 3500 ft and there were 4 equi-
spaced sources along the surface between the source and receiver well. The receivers
were located every 15 ft from the surface to the 2800 ft well bottom.

Since the background velocity model was provided, only an iterative VSP migra-
tion was applied to the four common source gathers. Ten iterations were used and a
near horizontal constraining matrix was applied to eliminate the diffraction artifacts,
which originated from the fault edges. The resulting composite migrated sections are
depicted in Figures 8.5, 8.7 and 8.9, respectively. The migrated sections indicate
that either one of the faults can be imaged.

8.4 Discussion

In this paper several case studies were performed to test the imaging capabilities of
VSP migration. The results indicate that the VSP migration is a useful tool to image
subsurface reflectivities. It has been found that VSP migration can image steep faults
and discontinuous fault dislocations.

8.5 Acknowledgement

I thank Dr. Kun Hua Chen from Chevron for providing me with the synthetic data
sets and challenging my VSP migration code.

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Figure 8.1: The composite migrated section of Test 1.
Figure 8.2: The reversed polarity composite migrated section of Test 1.
Figure 8.3: The original model of Test 1.
Figure 8.4: The first model of Test 2.

Figure 8.5: The composite migrated section of the first model of Test 2.
Figure 8.6: The second model of Test 2.

Figure 8.7: The composite migrated section of the second model of Test 2.
Figure 8.8: The third model of Test 2.

Figure 8.9: The composite migrated section of the third model of Test 2.
Report 9

Iterative Kirchhoff Migration of Radar Data. Some Numerical Results.

Tamas Nemeth

9.1 Abstract

An iterative Kirchhoff migration method is applied to synthetic radar data to correct the amplitudes and to eliminate migration artifacts. In the theoretical part of the paper a description of the conjugate gradient radar migration is given. In the subsequent numerical results section the asymptotic forward modeling and the imaging and amplitude aspects of the migration are studied. The results of the numerical tests show that iterative migration can extract amplitude information from the observed radargram and eliminate artifacts in the migrated section for the simple numerical models.

9.2 Introduction

Radar data acquisition has become widespread in the last few years with the rapid development of digital ground penetrating radar equipment. The radar technique was tested on many geotechnical problems with success. As the sophistication of the radar data acquisition increased, so did the related data processing methods (Fisher et al., 1992; Portniaguine, 1994).

Migration is one of the most important data processing methods which translates the observed radargrams into migrated reflectivity sections. Iterative migration helps to eliminate artifacts and provide a migrated section which is consistent with the observed radargrams. Some effects to be taken into account in shallow depth exploration are the large offset-vs-depth ratios or the interpretation of both reflections and
diffractions. In this paper I study these effects using simple but realistic numerical modeling cases.

9.3 Theory

In this section we present an outline of the iterative radar migration algorithm. We define our migration as finding the electromagnetic velocity perturbation (reflectivity) distribution which predicts the observed radargrams $E(x, t)^{obs}$ and minimizes the following objective function:

$$\varepsilon = \sum_i \left\| E^{cal} - E^{obs}_i \right\|^2 + \alpha \sum_i \left\| C\Delta c - C\Delta c^{apr}_i \right\|^2. \quad (9.1)$$

Here $E(x, t)^{obs}$ denotes the observed radargram, $E(x, t)^{cal}$ is the calculated radargram, $\Delta c$ is the electromagnetic velocity perturbation, $\Delta c^{apr}$ is an a priori electromagnetic velocity perturbation, $C$ is a damping matrix, specified for each application and $\alpha$ is a Lagrange multiplier. The subscript $i$ stands for the $i$-th radargram.

The calculated radargram is computed in the following way (Cai, 1992):

$$E(x, t)^{cal} = - \int_0^t dt_1 \int dx_1 \frac{2\Delta c(x_1)}{c^3(x_1)} \frac{\partial^2 E_0(x_1, t_1)}{\partial t_1^2} G(x, t; x_1, t_1). \quad (9.2)$$

Here $E_0(x, t)$ denotes the radargram due to the background electromagnetic velocity distribution $c(x)$, $G(x, t; x_1, t_1)$ is the Green’s function satisfying the second-order Maxwell’s equation

$$\left( \nabla^2 - \mu \sigma \frac{\partial}{\partial t} - \frac{1}{c^2(x)} \frac{\partial^2}{\partial t^2} \right) G(x, t; x_0, t_0) = \delta(x - x_0)\delta(t - t_0). \quad (9.3)$$

Here $\mu$ denotes the magnetic permeability and $\sigma$ denotes the conductivity. For a high frequency approximation (Portniaguine, 1994) the Green’s function is:

$$G(x, t; x_0, t_0) = e^{-\beta \tau_{x_0} x} \frac{\delta(t - \tau_{x_0} x)}{|x_0 - x|}, \quad (9.4)$$

where $\beta = \frac{\sigma}{2\epsilon_0}$ is the damping coefficient and $\tau_{x_0} x = \frac{|x_0 - x|}{c}$ is the propagation time. Here we assume homogeneous velocity and resistivity distributions.

For the minimization of equation (9.1) we use a conjugate gradient algorithm (Nemeth, 1993):
9.4. NUMERICAL RESULTS

\[ \Delta c_0 = 0; \quad d_o = \begin{pmatrix} E_{obs}^c \\ C\Delta c^{apr} \end{pmatrix}; \quad I_0^\alpha = \begin{pmatrix} L \\ \alpha C \end{pmatrix}^T \begin{pmatrix} E_{obs}^c \\ C\Delta c^{apr} \end{pmatrix}; \quad \hat{I}_0^\alpha = I_0^\alpha \]

and for \( n = 0, 1, 2, \ldots \)

\[ f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} L \\ C \end{pmatrix} \hat{i}_n^\alpha \]

\[ \hat{k}_n^\alpha = \frac{\|f_2\|^2}{\|f_1\|^2 + \alpha \|f_2\|^2} \]

\[ \Delta c_{n+1} = \Delta c_n + \hat{k}_n^\alpha \hat{i}_n^\alpha; \quad d_{n+1} = d_n - \hat{k}_n^\alpha f \]

\[ I_{n+1}^\alpha = \begin{pmatrix} L \\ \alpha C \end{pmatrix}^T d_{n+1} \]

if \( I_{n+1}^\alpha \leq \varepsilon_0 \) then quit

\[ \beta_n^\alpha = \frac{\|I_{n+1}^\alpha\|^2}{\|I_n^\alpha\|^2} \]

\[ \hat{I}_{n+1}^\alpha = I_{n+1}^\alpha + \beta_n^\alpha \hat{i}_n^\alpha \]

In this scheme \( L \) denotes the linear operator which connects \( \Delta c(x) \) with \( E(x, t)^{cal} \) in equation 9.2: \( L\Delta c = E^{cal} \) and consequently \( L^T \) is its transpose.

9.4 Numerical Results

Forward modeling simulation.

First, study the ability of the forward modeling operator (equation 9.2) to include the attenuation effect. In the first experiment there are two reflectors, 8 and 20 m deep. Compute the common source gathers with different attenuation parameters. For the source wavelet a realistic radar impulse source wavelet was used. Figure 9.1- 9.6 depicts the common source gathers with resistivities \( \rho = 100000; 10000; 1000; 500; 300; 100 \) Ohm-m, respectively. As we can see, as the resistivity decreases, the attenuation becomes significant. For the very low resistivities (100-300 Ohm-m) the attenuation is so large (-100; -140 dB) that the lower reflection cannot be registered by the radar equipment. Also for low resistivity, phase changes occur which cannot be taken into account with asymptotic forward modeling. These two effects set the exploration limit for the lower bound of resistivities to a range of about 300-500 Ohm-m.

The second experiment simulates an engineering case. There are three layers and a scatterer (pipeline) in the model. The bottom of the first layer is at 2 m, the second layer is at 9.5 m and the pipeline is at 3.5 m deep in the offset location 10.5 m. The velocity of the first layer is half the speed of light \( c_0 \) in a vacuum, the second layer velocity is \( \frac{1}{2} c_0 \), the first layer's resistivity is 300 Ohm-m and the second layer's resistivity is 500 Ohm-m. The source wavelet is a realistic radar impulse wavelet. The
calculated gather was a common-offset gather, with 2 m offset between the source and the receiver. The true amplitude common-offset gather is depicted in Figure 9.7. The attenuation is large enough to decrease the amplitude of the second reflector so much, that it can be seen only on the AGC gained common-offset gather (Figure 9.8).

In summary, for the given range of parameters (high frequency, medium-high resistivity) the asymptotic forward modeling is able to compute realistic data.

Imaging effects in migration.
Most radar measurements are applied to shallow depths. There is a number of specific circumstances, related to shallow depth measurements, such as large offset-vs-depth ratios, the presence of strong point scatterers and large source wavelength-vs-depth ratio. In the following I address these problems.

Large offset-vs-depth ratio for shallow reflection data causes wavelet stretching during migration. For example, in Figure 9.9 such a CSP gather is depicted. (The first reflector is at an 8 m depth and the second reflector is at 20 m depth.) The corresponding migrated section is shown in Figure 9.10. There is a strong stretching effect for the first layer and a milder stretching for the second layer. Usually the longer the wavelet, the more severe the stretching effect is. A destretching operator can be designed which eliminates the effect of stretching. The input parameters for this destretching operator are the length of the wavelet and the zero-offset two-way traveltime. The resulting destretched migrated section is depicted in Figure 9.11.

Kirchhoff migration is a diffraction migration. If both reflectors (layers) and diffractors (buried pipelines) are present in the common-offset gather, as in Figure 9.7, there might be a problem to distinguish between the point scatterers and the migration artifacts. To solve this problem, each point scatterer must be identified and to the remaining part of the model a constraining operator can be applied. For example, Figure 9.12 shows the migrated section of the common-offset data in Figure 9.7 by applying constraining operator to it.

The large source wavelength-vs-depth ratio causes the imaged reflector to be very thick, so that different reflectors will overlap with each other or make the determination of the correct depth difficult. Figure 9.13 shows the migrated section of a two-layer model with a realistic source wavelet. This migrated section is obtained by a conventional Kirchhoff migration. If the migrated operator is the transpose of the forward modeling operator of equation 9.2 it includes an additional source wavelet convolution, resulting in the autocorrelation of the wavelet. Figure 9.14 shows the migrated section obtained by this more precise migration operator. As we see in Figure 9.14 we did not get exact autocorrelation functions for either of the reflectors. For the deeper reflector the side maximums are not equal. This effect is the result of applying different geometrical correction factors at different depths, although the original data came from the same depth by a source wavelet convolution. The shallower reflector also has this effect and additionally its autocorrelation is not complete, since the reflector is too close to the surface in terms of the wavelength-vs-depth. Fixing for
9.5. DISCUSSION

Each reflector depth range the geometrical spreading factor yields the migrated section in Figure 9.15. As we can see, the autocorrelation functions are much better here.

**Amplitude effects in migration.**

To determine the reflectivity strength or the resistivity contrast of the medium, we have to study the ability to obtain correct amplitudes by the Kirchhoff migration. The next experiment was designed to study the recovery of amplitudes due to both geometrical spreading and attenuation. The model had two reflectors at depths 2 m and 9.5 m. Both reflectors have a strength of 1. Common-offset data without the geometrical spreading and the attenuation was computed. Figure 9.16 shows the migrated section of this gather. Both migrated images have the same strength and shape.

Then we created a common-offset gather with geometrical spreading but without attenuation. For this simple geometry we decided to apply the pseudo-inverse Hessian operator \( \left( \frac{1}{\text{diag}(L^T L)} \right) \). The resulting migrated section is shown in Figure 9.17. The amplitudes of the reflectors are recovered but the shape of the migrated events is not exactly the same as the original shape. The reason of this effect is again in the different geometrical corrections for different parts of the wavelet.

The next step was to create a common-offset gather with both the geometrical spreading and the attenuation (\( \rho=1000 \) Ohm-m) effects. Again, a pseudo-inverse Hessian operator was applied to the migration operator. The resulting migrated section is depicted in Figure 9.18. The amplitude level was recovered but the signal shape suffered a slight distortion due to reasons explained above. The success of this amplitude recovering experiment shows that it is possible to extract amplitude information from radargrams.

### 9.5 Discussion

The main benefits of iterative radar migration are to get the correct amplitudes to eliminate artifacts. It was shown in this paper that both purposes can be achieved with iterative migration for relatively simple source-receiver geometries. The ability of this migration to handle amplitudes enables it in principle to estimate both the reflector strength and the resistivity separately. However, in practice we can encounter difficulties in achieving this purpose. The difficulty is related to the problem of separating the geometrical spreading from the attenuation effects. In principle, knowing the migration velocities we can calculate the reflectivity, but in practice the long wavelength and the short wavelength parts of the velocity field are decoupled. In this case the one amplitude effect is caused by two phenomena, making the separate determination impossible. To solve this problem, some a priori information must be used.
9.6 Reference


Figure 9.1: A common source gather with resistivity $\rho=100000$ Ohm-m.

Figure 9.2: A common source gather with resistivity $\rho=10000$ Ohm-m.
Figure 9.3: A common source gather with resistivity $\rho=1000$ Ohm-m.

Figure 9.4: A common source gather with resistivity $\rho=500$ Ohm-m.
Figure 9.5: A common source gather with resistivity $\rho=300$ Ohm-m.

Figure 9.6: A common source gather with resistivity $\rho=100$ Ohm-m.
Figure 9.7: A common offset gather. The distance between the source and the receiver is 2 m.

Figure 9.8: The same as the previous figure, except that an AGC is applied.
Figure 9.9: A common source gather with large source wavelength.

Figure 9.10: A conventionally migrated section of the previous radargram.
Figure 9.11: The migrated section with destretching operator of the previous radargram.

Figure 9.12: The migrated section of the radargram in Figure 7.
Figure 9.13: A conventionally migrated section preserving the source wavelet.

Figure 9.14: A migrated section producing the autocorrelation of the source wavelet.
Figure 9.15: A migrated section producing the autocorrelation of the source wavelet with fixing the geometrical spreading term for the range of the reflectors.

Figure 9.16: A migrated section of a radargram containing no geometrical spreading and attenuation effects. On the right hand side the migrated images are seen.
Figure 9.17: A migrated section of a radargram containing geometrical spreading effect but no attenuation. On the right hand side the migrated images are seen.

Figure 9.18: A migrated section of a radargram containing both the geometrical spreading effect and attenuation. On the right hand side the migrated images are seen.
Part III

Depth Focusing
Report 10

Preliminary Test on Depth Focusing Technique

Fuhao Qin

10.1 Abstract

A Kirchhoff integral migration method is used to carry out depth focusing analysis (DFA). Preliminary results suggest that it can provide a good correction to the migration image for shallow events. Future study is needed to explore its potential in 3D velocity analysis.

10.2 Introduction

The quasi-Monte Carlo migration method proposed by Sikorski and Schuster (1992) and tested by Sun et al. (1993) provides an efficient way to perform 3D Kirchhoff integral migration. It may achieve an efficiency of 6 to 10 times over the ordinary 3D Kirchhoff migration according to Sun et al. (1993). This makes the velocity analysis through 3D migration more realistic.

Here we test one of the migration based velocity analysis methods, namely, depth focusing analysis (DFA). We first test its performance for 2D cases and, we hope to implement it in 3D cases using the quasi-Monte Carlo migration technique.

Many authors (Yilmaz and Chambers, 1984; Jeannot et al. 1986; MacKay and Abma, 1989) discussed DFA method or related issues. A more recent paper (MacKay and Abma, 1992) analyzes the limitations of velocity estimation using DFA and describes the usefulness of focal-surface imaging.

In this report, we first describe shortly the method we use and then give the results of a very preliminary test.
10.3 Methodology

When the migration velocity is accurate the migration will generate images focused at the exact locations of the reflectors. If we migrate a common shot gather, this means the back-propagated wavefield will focus at the reflector at time zero (actually it is the time that takes the wave to travel from the source to the reflector). The migrated images are those wavefields back-propagated at their zero times (imaging condition). When the migration velocity is inaccurate, the migrated reflectors will be at different locations compared to the actual reflectors and will not be well focused. This means that the back-propagated wavefield does not focus at the reflector at time zero. Thus applying the imaging condition does not give us crisp reflector images at the correct locations. The back-propagated wave field may still focus, but it focuses at incorrect locations and nonzero times.

Figure 10.1 shows the relation between the migration velocity and migration image, where \( I \) is the half offset between the source and receiver of the none zero offset raypath and \( h_t \) is the true depth of the reflector \( M \). Assume that the true velocity is \( v_t \) and the migration velocity is \( v_m \). In migration, for the zero offset ray:

\[
\frac{2h_m}{v_m} = \frac{2h_t}{v_t} \tag{10.1}
\]

where \( h_m \) and \( h_t \) are the migrated depth and true depth of the reflector, respectively. Thus,

\[
h_m = \frac{v_m}{v_t} h_t \tag{10.2}
\]

For the nonzero offset ray:

\[
\frac{2\sqrt{l^2 + h_m^2}}{v_m} = \frac{2\sqrt{l^2 + h_t^2}}{v_t} \tag{10.3}
\]

Assume the offset \( (l) \) is small. To zero-order approximation of the square root operation, i.e., completely omit the offset term, we get the same result as equation 10.2:

\[
h_m \approx \frac{v_m}{v_t} h_t \tag{10.4}
\]

Therefore, the overall migration depth is approximately \( \frac{v_m}{v_t} h_t \). Since there is a zero-order approximation in the derivation, the image is not well focused.

Now let's see if we can let the two rays focus somewhere. Since there is no solution that fits both equations 10.1 and 10.3, we add to them another unknown \( \delta t \) which is equivalent to adding a DC shift to the time zero of the imaging condition, i.e.,

\[
\frac{2h_t}{v_m} = \frac{2h_t}{v_t} + \delta t, \tag{10.5}
\]
10.4. NUMERICAL RESULTS

\[
\frac{2\sqrt{l^2 + h_f^2}}{v_m} = \frac{2\sqrt{l^2 + h_t^2}}{v_t} + \delta t,
\]

where \( h_f \) is the focusing depth.

Solving the above two equation and using the first order approximation of the square root operation \( \sqrt{1 + \alpha} \approx 1 + \frac{\alpha}{2} \), we get,

\[
h_f \approx \frac{v_t}{v_m} h_t.
\]

(10.7)

Comparing equation 10.4 to 10.7, we can see that

\[
h_t^2 \approx h_m h_f.
\]

(10.8)

Although the above equation comes from just two rays, it holds true for all near offset rays since we only used the near offset assumption. We also implicitly assumed that the reflector is flat by assuming that the reflection rays are symmetrical. This formula is actually equation (4) of MacKay and Abma (1992).

So, the depth focusing idea is to first find both the migration depth \( h_m \) and the focusing depth \( h_f \), and then to figure out the true reflector depth \( h_t \) by using equation 10.8.

In the next section, a very simple synthetic model is used to test the depth focusing scheme.

10.4 Numerical Results

Figure 10.2 shows the model geometry. There are 5 layers with velocity ranging from 2000 m/s to 3000 m/s. The width of the model is 2500 m and depth is 1750 m. The acquisition geometry consisted of 101 shots collected at a 25 m shot interval by a symmetric split-spread cable with 101 receivers spaced at 25 m. The half length of the cable is 1250 m. Synthetic seismic data is generated by a finite difference acoustic wave equation solver (PP4). The source is a Ricker wavelet with a peak frequency of 20 Hz.

The prestack depth migration section using the true velocity is shown in Figure 10.3. The images of the structure are very clear. There are migration artifacts near the edges of the structures and there are also multiples due to the lack of angle coverage. The images are weaker near the edge of the model since fewer rays visit those areas.

Figure 10.4 shows the migration image using a homogeneous velocity of 2500 m/s. These images are badly focused and at the wrong positions except for the top layer since the migration velocity above it agrees with the true velocity.

To improve the migration image, DFA is carried out at 10 offset locations. Figure 10.5 shows the one at the offset location of 600 m. It can be seen that the focusing depth error can be easily picked for the first three layers. It is impossible to precisely
pick the images in depth. A possible reason is that the reflections are weak and the angle coverage is narrow.

The focusing depth error of the top three layers are picked for 10 different locations and the information is used to update the migration images using equation 10.7. Depth errors are linearly interpolated throughout the model from the 10 DFA locations. Figure 10.6 is the updated migration image. Comparing it with Figure 10.4, it is clear that Figure 10.6 image is much better focused and the reflector images are more or less moved toward their correct locations for the top three layers. The error is at the edges of the image at 1000 m, and is due to the DFA error caused by sparse ray coverage near the edges of the model.

To date, updating the velocity model has not been very successful. The possible reason is the dip of the second layer which violates the flat layer assumption.

10.5 Discussion

This is just a very preliminary test. More work is needed to further explore and understand the DFA method.

The simple test seems to suggest that,

- DFA gives the correct information about the depth of the images even when there is moderate dip in the layers;
- DFA is sensitive for shallower structures;
- A formula to update the velocity when dipping layers are present needs to be addressed;
- DFA for deeper events also needs attention.

10.6 Reference


Figure 10.1: Geometrical relation between migration velocity and migration image
Figure 10.2: Model geometry for the numerical test.
Figure 10.3: Prestack Kirchhoff migration image using true velocity.
Figure 10.4: Prestack Kirchhoff migration image using a homogeneous velocity of 2500 \textit{m/s}
Figure 10.5: DFA depth error image at the offset of 600 m.
Figure 10.6: Migration image after depth correction using the DFA depth error information.
Report 11

Relating Depth-Focusing Analysis to Migration Velocity Analysis

Tamas Nemeth

11.1 Abstract

Migration velocity analysis and depth-focusing analysis are one of the most important velocity estimation methods for seismic migration. The migration velocity analysis is based on the kinematic imaging principle while the depth-focusing analysis is based on the dynamic imaging principle. In this paper I relate the depth-focusing analysis to migration velocity analysis by displaying it in time-shifted common-image gathers. Using both imaging principles for time-shifted common-image gathers, a more robust velocity and depth estimation is expected.

11.2 Introduction

In recent years three main optimization criteria were developed and applied to seismic migration/inversion. The first criterion is the waveform inversion (Tarantola, 1987), i.e. to find a model which gives the best match between the observed and the calculated data in some sense (for example, the L-2 norm). The second criterion is the migration velocity analysis technique. The basis for this method is that if the correct velocity distribution is used for migration, reflectors from the individual migrated images will be located at the same depth and stacked coherently. If the migration velocity is incorrect, the individual migrated images from a common reflector will be imaged at different depths. Knowing this depth mismatch, the correct velocity can be estimated. This criterion makes use of the kinematic imaging principle. The third criterion is the depth-focusing analysis (DFA). DFA (McKay and Abma, 1992) is based on the dynamic imaging principle. Its goal is to find the depth at which the individual migrated images are most focused, i.e. the individual migrated images,
are maximally coherent. If the correct velocity is used during migration, the focusing depth will be the same as the real depth. However, if incorrect velocities are used for migration, the focusing depth will be different from the real depth. By measuring the focusing depth, the correct velocity can be estimated.

In a recently published paper Versteeg (1994) describes the Marmousi experiment. Several different companies processed the Marmousi data set using the three above mentioned optimization criteria for the velocity distribution estimation. The most successful among these applied techniques was the migration velocity analysis method, followed by the depth-focusing analysis and no results were given for waveform inversion. The reason for the success and failure lies probably in the robustness of the technique with respect to real conditions.

Several papers (Doherty and Claerbout, 1974; Faye and Jeannot, 1986; Yilmaz and Chambers, 1984; McKay and Abma, 1992; McKay and Abma, 1993) describe in detail the depth-focusing analysis. For the technical details refer to McKay and Abma (1992). The main idea behind the DFA method is that during the downward continuation (migration) of the observed seismogram the reflected energy focuses at a depth, called the focusing depth. This depth can be measured and related to the correct depth, and also a velocity update can be made based on this information.

The theory described in the above mentioned papers is based on the zero-offset approximation of the CDP-gathers. I extend the focusing depth criterion to nonzero offsets in this paper in addition. The DFA focusing criterion is a dynamic criterion, since it measures the depth at which the maximum coherency of the prestack migrated images occur. I relate this method to kinematic imaging criterion and show the relationship between the DFA method and the migration velocity analysis method.

11.3 Theory

The DFA analysis in this paper will be carried out by Kirchhoff migration. In this case the DFA volume for a given offset coordinate can be computed as a migration of the observed data $P_s(t)$ with a range of imaging conditions specified by $\Delta \tau$:

$$DFA(z, \Delta \tau) = \sum_S \sum_R \int P_s(t) \frac{\delta(t - \tau^{sp} - \tau^{pr} - \Delta \tau)}{A_{sp} A_{pr}} \, dt$$  \hspace{1cm} (11.1)

Here $\Delta \tau = 0$ corresponds to the "normal" migration; $A_{sp}$ and $A_{pr}$ are the amplitude terms of the Green's functions from the source to the scattering point and from the scattering point to the receiver, respectively. The summations indicate that we stack the prestack migrated sections to obtain a composite depth trace $z$ at offset $x$. $P_s(t)$ is a trace associated with source $S$ and receiver $R$.

Calculation of the nonzero offset focusing depths.

Now let us calculate the focusing depths as a function of offsets for a 2-layer medium with a horizontal interface. In this case the traveltime from the true depth $h$ with
the true velocity \( V \) (left-hand side of the following equation) must be the same as the traveltime from the focusing depth \( h_f \) with the migration velocity \( V_m \) plus a time shift (right-hand side of the following equation):

\[
\frac{2}{V} \sqrt{h^2 + (l + \Delta l)^2} = \frac{2}{V_m} \sqrt{h_f(l)^2 + (l + \Delta l)^2} + \Delta \tau
\]

\[
\frac{2}{V} \sqrt{h^2 + l^2} = \frac{2}{V_m} \sqrt{h_f(l)^2 + l^2} + \Delta \tau
\] (11.2)

Equation 11.2 is true for any offset. Choose \( \Delta l \) to be small. In this case solving for \( h_f(l) \) gives:

\[
h_f(l) = \frac{1}{\gamma} \sqrt{h^2 + (1 - \gamma^2) l^2},
\] (11.3)

where \( \gamma = \frac{V_m}{V} \). Choosing \( l = 0 \) (zero-offset approximation) gives the well-known depth-focusing relationship (Faye and Jeannot, 1986): \( h_f V_m = h V \). As we can see, the focusing for larger offsets occurs at a focusing depth further away from the true depth or the focusing depth for zero-offset.

**Calculation of the migrated depths for different offsets at retarded times.** During prestack imaging we obtain images from the imaged scattering point as a function of offset. To study this dependence we form common-image gathers (CIG) by collecting the migrated traces from different prestack migrated sections with the same offset location \( x \). Analyzing the curvature of these images on CIG's forms the basis of the migration velocity analysis method. The accumulated experience shows (see, for example, Versteeg, 1994) that analyzing the CIG's is a robust way to adjust the background velocities for migration.

It is these CIG's which can be used to demonstrate the focusing effect of DFA. To do this, we migrate the mutigather data with different retarded (advanced) time shifts and display them on CIG's. For the horizontal layer model the migration imaging equation with time shifts can be written as:

\[
\frac{2}{V} \sqrt{h^2 + l^2} = \frac{2}{V_m} \sqrt{h_m(l)^2 + l^2} + \Delta \tau
\] (11.4)

Both sides of equation 11.4 are equal to the measured arrival time \( t \). The left-hand side contains the true velocity \( V \) and true depth \( h \) and the right-hand side expresses it with the migration velocity \( V_m \), migrated image depth \( h_m \) and the retarded time shift \( \Delta \tau \). Rearranging this equation shows that the migrated image depth at offset \( l \) can be expressed as

\[
h_m(l, \Delta \tau) = \sqrt{\gamma^2 h^2 + (\gamma^2 - 1) l^2 + \frac{\Delta \tau^2 V_m^2}{4}} - \gamma \sqrt{h^2 + l^2} \Delta \tau V_m.
\] (11.5)

Setting \( \Delta \tau = 0 \), equation 11.5 corresponds to the one obtained by Al-Yahya (1989), which forms the basis of migration velocity analysis. Equation 11.5 extends Al-Yahya's results for the case of retarded or advanced imaging conditions. Specific choices of \( \Delta \tau \) give the focusing depths as we shall see later.
Figure 11.1: Common-image gather for migration velocity 25% higher than the correct velocity. The different curves depict different migrated images with different time shift $\Delta \tau$. $Hm$ - migrated image curve for $\Delta \tau = 0$, corresponding to the conventional migration. $F0$ - migrated image curve with a $\Delta \tau$ so that the images at zero offset are stacked coherently. The depth of the zero offset image is at the conventional focusing depth. $F$ - curve at which a given offset image is stacked coherently.

Figures 11.1 and 11.2 illustrate the family of curves $h_m(l, \Delta \tau)$ for both high and low migration velocities. The true depth is 1 in these figures. To analyze Figure 11.1, start with the curve $Hm$ which corresponds to the images of migration with $\Delta \tau = 0$. As $\Delta \tau$ decreases, we get different curves at shallower depths. From these curves we mention curve $F0$; This curve corresponds to the focusing depths for zero-offset. As we can see, the horizontal derivative of this curve at zero-offset is zero. This allows the zero and near-zero offset images points to be stacked coherently and thereby gives a maximum amplitude on the DFA panels. As $\Delta \tau$ further decreases, larger offset images are stacked coherently. Curve $F$ depicts the depth at which a given offset image is stacked coherently and it corresponds to equation 11.3. Figure 11.2 shows a very similar picture. The difference is that migrating with $\Delta \tau = 0$ yields images shallower than the real depth. Then one has to increase the time shift $\Delta \tau$ to obtain positions where the images are stacked coherently to give large amplitudes on the DFA panels.
Figure 11.2: Common-image gather for migration velocity 20% lower than the correct velocity. The different curves depict different migrated images with different time shift $\Delta r$. $H_m$ - migrated image curve for $\Delta r = 0$, corresponding to the conventional migration. $F0$ - migrated image curve with a $\Delta r$ so that the images at zero offset are stacked coherently. The depth of the zero offset image is at the conventional focusing depth. $F$ - curve at which a given offset image is stacked coherently.
11.4 Numerical results

To check the validity of the above described equations, I performed some numerical experiments. A horizontal two-layer model was used, with homogeneous velocities in each layer. The thickness of the first layer was 5 m and the propagation velocity is 2000 \( \frac{m}{s} \). Then a common midpoint gather was formed with an offset-vs-depth ratio of up to almost 4. A 900 Hz peak frequency Ricker-wavelet was used for the source wavelet. Then I migrated this CMP gather with different velocities and different time shifts and formed common image gathers. Of course, this whole procedure "knew" nothing about the equations described above.

Figure 11.3 depicts the CIG gather with the correct velocity and with no time shift. As expected, the migrated images are placed at the correct depth. We notice a signal stretching at large offsets, a phenomenon similar to the NMO-stretching effect.

Then I migrated the CDP gather with a velocity 20% lower than the correct velocity. Figure 11.4 depicts the common image gather with no time shift. As expected, imaging occurs at depths shallower than the real depth. The far-offset images are increasingly pushed away from the real depth and they are stretched, too. The following sequence of figures (Figures 11.5, 11.6, 11.7, 11.8, 11.9) shows the images at time shifts \( \Delta \tau = 1.2; 3; 4; 5; 6 \) msec, respectively. Figure 11.6 corresponds to the time shift when the zero offset images are stacked coherently, determining the conventional focusing depth for DFA. But we notice that starting with this time shift, there is a whole range of time shifts \( \Delta \tau \) when the images are stacked more or less coherently. This effect makes it difficult to determine the focusing depth using conventional DFA analysis. But we can measure the curvature and knowing the time shift, we can backpropagate the depth differences into the model.

Figures 11.10-11.15 show a similar sequence of CIG’s for velocities 25% higher than the real velocity. We can see that with \( \Delta \tau = 0 \), the migrated images are deeper than the real depth. Applying negative time shifts the migrated images gradually move upward and get focused (see Figures 11.12-s 11.14).

In summary we mention that the migrated images behave quite the way they were predicted by the preceding equations. The retarded or advanced migrated images on the CIG’s give the opportunity to study the focusing properties during migration.

11.5 Discussion

Displaying time-shifted migrated images allows us to determine both the focusing depth and the curvature of the migrated images at the same time. This double characteristic might be more robust than the DFA analysis, since it includes both the dynamic imaging condition (focusing) and the kinematic curvature estimating methods (migration velocity analysis).

Future work will study the images from dipping layers and the effect of inhomogeneous velocities and crosswell geometries.
11.6 Reference


Tarantola, A., Inverse problem theory: Elsevier


Figure 11.3: The common image gather with the correct velocity.

Figure 11.4: The common image gather with velocity 20% lower than the correct velocity. The applied time shift is $\Delta \tau = 0$ msec.
Figure 11.5: The common image gather with velocity 20 % lower than the correct velocity. The applied time shift is $\Delta \tau = 1.2$ msec.

Figure 11.6: The common image gather with velocity 20 % lower than the correct velocity. The applied time shift is $\Delta \tau = 3$ msec.
Figure 11.7: The common image gather with velocity 20 % lower than the correct velocity. The applied time shift is $\Delta \tau = 4$ msec.

Figure 11.8: The common image gather with velocity 20 % lower than the correct velocity. The applied time shift is $\Delta \tau = 5$ msec.
Figure 11.9: The common image gather with velocity 20% lower than the correct velocity. The applied time shift is $\Delta \tau = 6$ msec.

Figure 11.10: The common image gather with velocity 25% higher than the correct velocity. The applied time shift is $\Delta \tau = 0$ msec.
Figure 11.11: The common image gather with velocity 25% higher than the correct velocity. The applied time shift is $\Delta \tau = -1$ msec.

Figure 11.12: The common image gather with velocity 25% higher than the correct velocity. The applied time shift is $\Delta \tau = -1.8$ msec.
Figure 11.13: The common image gather with velocity 25 % higher than the correct velocity. The applied time shift is $\Delta \tau = -2.3$ msec.

Figure 11.14: The common image gather with velocity 25 % higher than the correct velocity. The applied time shift is $\Delta \tau = -2.6$ msec.
Figure 11.15: The common image gather with velocity 25 % higher than the correct velocity. The applied time shift is $\Delta \tau = -3.0$ msec.
Part IV

Forward Modelling
Report 12

Some Preliminary Results in Viscoelastic Modeling

Xu Ji

12.1 Abstract

I present some preliminary results for modeling viscoelastic wavefields in the earth. For realistic source-receiver offsets and Q values, the viscoelastic seismograms show noticeable differences in traveltimes and waveforms compared to the equivalent elastic seismograms. This suggests that migration, full waveform inversion or traveltime tomography algorithms should take into account the viscoelastic effects of the medium.

12.2 Introduction

Robertsson et al. (1994) developed a finite-difference formulation to solve the viscoelastic equations of motion. Bill Symes was kind enough to provide us with the interior differencing portion of their code, and I have embedded it within our modeling format at the University of Utah. I now present some preliminary numerical results for modeling the viscoelastic response of the Friendswood model.

12.3 Preliminary Numerical Results

Figure 12.1 shows the Friendswood P-velocity model obtained by a hybrid traveltime+waveform inversion of Exxon’s Friendswood crosshole data (Zhou et al., 1993). Assuming a Q distribution that increases from 80 to 160 linearly with velocity, I computed both the elastic and viscoelastic responses to a line source located on the left at a depth of 520 feet. Figure 12.2 and Figure 12.3 show the wavefields and common shot gathers, respectively. The most noticeable difference between the elastic and
viscoelastic data is the decrease in source bandwidth in the viscoelastic wavefields, which appear to be of lower frequency content than the elastic wavefields. Figure 12.4 plots the first arrival traveltimes from these gathers and shows that there is an average delay of 0.83 ms due to the attenuation in the medium. This discrepancy can affect both the waveform and traveltime tomograms unless viscoelastic effects are taken into account.

Figures 12.5 and 12.6 are the same as Figures 12.3 and 12.4, respectively, except the Q distribution is taken to increase linearly with velocity from 10 to 100. The waveform and traveltimes difference are even more pronounced compared to the previous figures.

12.4 Discussion

As well offsets go out to 2000 feet or more it will become mandatory to take into account viscoelastic effects in high frequency crosshole data. My future work will explore the viscoelastic effects in such data and I will seek to adapt standard migration and tomography algorithms to viscoelastic data. Pending permission from Bill Symes, I plan to distribute the 2D viscoelastic code at our annual meeting.

12.5 Reference


Figure 12.1: Friendswood P-velocity model.

Figure 12.2: (a). Elastic and (b). viscoelastic snapshots of the line source response for the Friendswood model. The $Q$ distribution in the viscoelastic Friendswood model linearly increased from the top ($Q=10$) to the bottom ($Q=100$).
Figure 12.3: Shot gathers for the a) elastic and (b) viscoelastic responses in the Friendswood model.

Figure 12.4: a) Elastic and viscoelastic first arrival traveltimes picked from the shot gathers in the previous figure.
Figure 12.5: Same as previous figure with shot gathers except that the data were generated from the Friendswood model with a $Q$ distribution that linearly increased from the top ($Q=10$) to the bottom ($Q=100$).
Figure 12.6: Same as previous figure with traveltimes except that the data were generated from the Friemdswood model with a Q distribution that linearly increased from the top (Q=10) to the bottom (Q=100).