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Preface

This annual report summarizes the 1993 research results of the Modeling and Tomography group at the University of Utah. Consortium members for 1993 include Advance Geophysical, Amerada Hess, Amoco, Chevron, Conoco, Exxon, Fujitsu, GRI, Japon, Noranda, Oyo, Texaco, Marathon, Unocal, and Phillips.

Significant research accomplishments for 1993 include:

- SS, PP, SP, and PS waves were extracted from the McElroy data by FK-filtering, and were separately migrated to give PP-, SS-, SP- and PS- migrated images. There was fair to very good agreement with the synthetic seismograms obtained from the sonic logs. We find that a key step in successful migration or waveform inversion is to very carefully process the data, including separation of the wave modes by FK-filtering. A more precise mode separation method is a current topic of research. For successful migration, either an accurate velocity model should be used or depth corrections must be applied to each migrated shot gather.

- A correction method is developed that improves the resolution of stacked migrated image gathers; the individual traces in a common image gather are shifted or "shaken" until the migrated images from a common reflector point align with one another. Tests with field data show that the image resolution can be significantly improved, but a correct image of the earth's reflectivity is not guaranteed. We hope to extend this "shaking" method to updating the velocity model.

- Least squares migration of crosshole data shows a significant improvement in reflector resolution compared to standard migration if there are just a few source gathers (e.g., VSP data). However, the improvement is diminished when the migrated section is a composite of many prestack migrated sections. Application of this method to an RVSP data set shows that it has certain advantages over conventional VSP migration.

- Elastic traveltime+waveform inversion (WTW) of crosshole data. Elastic WTW is applied to synthetic and real crosswell data. In 1992, we successfully applied the acoustic WTW method to the Friendswood crosshole data, and this year
we applied the elastic crosshole WTW method to the Friendswood, Steepbank and McElroy data sets.

For the Friendswood data, there are no prominent S-wave reflections and so the WTW method could not reliably extract the S-velocity distribution. Apparently, the explosive source did not excite strong shear waves.

The McElroy data contained significant PP, SS, PS and SP arrivals, and so the elastic WTW method was able to reconstruct both the S-wave and P-wave velocity distributions. The S-wave and P-wave sonic logs showed fair to good agreement with the S-velocity and P-velocity tomograms. The high frequency details of the S-velocity tomogram do not compare well the S-sonic log, and we believe that this is due to artifacts from the FK filtering. These results also demonstrated that similar to the Friendswood data, the waveform tomograms provided almost an order of magnitude better resolution than the traveltime tomograms. These results also demonstrated that last year's successful waveform inversion of the Friendswood data was not a fluke, that WTW inversion could be successfully applied to field data in a relatively short amount of time (i.e., it required less than one month of effort to process the data and invert for the tomogram), and that the WTW method can be efficiently executed on a cluster of fast workstations. Remaining issues are to develop methods to eliminate tomographic artifacts, improve the tomogram's accuracy, and to assess the reliability of the waveform tomogram.

The Steepbank crosshole data contained prominent P-wave and S-wave reflections, but we were unsuccessful in obtaining a high resolution WTW tomogram; this failure is partly explained by the relatively low frequency content of the data. Attempts at inverting the Steepbank data are continuing.

- 3-D prestack migration of synthetic CDP data. We have developed a new Kirchhoff algorithm which, in principle, reduces the computational cost of standard 3-D prestack migration by almost an order of magnitude. We are in the process of applying this method to a 3-D CDP data set from Unocal.

- A Conjugate Gradient migration method is used to extract up- and down-going waves, P-, S-, P-P, S-S, S-P, and P-S wave modes in field crosshole data. Such filtered data might be used for both inversion and migration.

- Imaging shallow structures by inverting CDP data with a 2-D and a 3-D refraction tomography method. Application of the method to synthetic and field traveltime data show the viability of this approach.

Research for 1994 will aim to improve imaging methods for both crosshole and CDP data. For crosshole data, we will seek to improve the accuracy of the WTW method, and develop better ways to extract different wave modes from seismic data. We would like to investigate the application of the WTW and migration algorithms
to crosshole data with large well offsets. An important goal is to seek a hybrid migration+shaking method that updates both the velocity distribution and the migrated image.

For CDP data, we will test the effectiveness of the 3-D refraction inversion on field data, and compare its results to those from other methods. Current efforts are devoted to applying the Quasi-Monte Carlo 3-D migration methods to 3-D CDP field data. The hybrid migration+shaking method that updates both the velocity distribution and the migrated image will be developed for CDP data as well as for crosshole data.

Jerry Schuster
Part I

Tomography Methods
Report 1

Elastic Wave Equation Travel Time And Waveform Inversion Of Crosshole Seismic Data

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1.1 Abstract

An acoustic Wave Equation Travel Time And Wave Form (acoustic WTW) Inversion method was presented by Zhou et al. (1993) to invert the P-wave velocity distribution from crosshole seismic data. The acoustic WTW tomograms showed about 6 times greater spatial resolution than the corresponding travel time tomograms. In this paper we present the elastic wave equation travel time and waveform (elastic WTW) inversion method of crosshole seismic data which inverts for both the P-wave and S-wave velocity distributions. Comparison of the elastic WTW tomograms with the acoustic WTW tomograms shows that both methods can invert for the high resolution P-wave velocity structure when the S-wave energy is very weak in the recorded seismograms. Although the real data we used are dominated by unconverted P-wave arrivals, the elastic waveform inversion can still invert for the S-wave velocity structure. This comparison also shows that elastic WTW inversion is superior to acoustic WTW inversion when there are big energy converted S-waves in the recorded seismograms. The disadvantage of elastic WTW inversion is that it requires about 3 times more CPU time than the acoustic waveform inversion.

1.2 Introduction

The S-wave velocity distribution is important information for enhanced oil recovery (EOR) efforts because it can be used in conjunction with the P-velocity to estimate the porosity and lithology distributions. Detailed knowledge of lithology and fluid pathways can be crucial to the successful execution of steam, CO₂, or fire flood experiments. The problem with the current S-velocity inversion methods, such as S-traveltime tomography, is that they only provide a coarse estimate of the S-velocity field. The requirement is to find the S-velocity field within a resolution of about, say 5 feet, yet the reality is that S-traveltime crosshole tomography has a resolution many times coarser than that. The possible solution to this problem is to use elastic waveform tomography rather than traveltime tomography to invert for the S-velocity distribution. In this paper we present the elastic WTW method which, in principle, reconstructs both the P- and S-wave velocities by inverting both the traveltimes and waveforms in crosshole seismic data.

The elastic WTW inversion method is a straightforward extension of the acoustic WTW method presented by Zhou et al. (1993) and Luo and Schuster (1991). We only need to replace the acoustic wave propagation finite-difference modeling with elastic wave propagation finite-difference modeling and to calculate the gradients to update both the P-wave and S-wave velocity. For the real 3-D data, we use a filter in the frequency domain to transform the data to 2-D. The detailed analysis of this transform method shows that the 2-D approximation is justified (Xu, 1993).

Seismic exploration data contain two distinct types of information: 1). information about the smooth velocity structure is primarily associated with the traveltime
1.3. Theory

In this section we present an outline of the elastic WTW algorithm. Our goal is to reconstruct the P- and S-wave velocity model which predicts the observed seismograms \( p(x_r,t|x_s)_{\text{obs}} \) that minimize the following misfit function:

\[
E = \frac{1}{2} \sum_x \sum_t (1 - w)[\delta \tau_{rs}]^2 + \frac{1}{2} \sum_x \sum_t \int dt \delta p_{rs}(t)w \delta p_{rs}(t). \tag{1.1}
\]

Here \( p(x_r,t|x_s)_{\text{cal}} \) is the calculated seismogram, \( \delta p_{rs}(t) = p(x_r,t|x_s)_{\text{obs}} - p(x_r,t|x_s)_{\text{cal}} \) is the seismogram residual, and \( \delta \tau_{rs} = \tau_{\text{obs}}(x_r,x_s) - \tau_{\text{cal}}(x_r,x_s) \) is the travel time residual, or the difference between the observed and calculated first arrival times for a source at \( x_s \) and a receiver at \( x_r \). The \( w \) (discussed in Luo and Schuster, 1990) is a weighting factor used to balance out the strength from these two residuals. We assume the density can be obtained from either well log data or by a simple empirical relation between P-wave velocity and density (Gardner et al., 1974). For two component data where the vertical \( v_z(x_r,t|x_s)_{\text{obs}} \) and horizontal \( v_x(x_r,t|x_s)_{\text{obs}} \) particle velocities are measured, the seismogram residual is given by

\[
\delta p(x_r,t) = v_x(x_r,t|x_s)_{\text{obs}} - v_x(x_r,t|x_s)_{\text{cal}} + v_z(x_r,t|x_s)_{\text{obs}} - v_z(x_r,t|x_s)_{\text{cal}} \tag{1.2}
\]

For an explosion source and two component geophones we perform the following operations for one iteration of the elastic waveform inversion.

1. Solve the elastic wave equation with zero initial conditions for each shot point by a 4th order finite-difference algorithm:

\[
\rho \frac{\partial}{\partial t} v_x^f - (\frac{\partial}{\partial x} \sigma_{xx}^f + \frac{\partial}{\partial z} \sigma_{xz}^f) = 0, \tag{1.3}
\]

\[
\rho \frac{\partial}{\partial t} v_z^f - (\frac{\partial}{\partial z} \sigma_{zz}^f + \frac{\partial}{\partial x} \sigma_{xz}^f) = 0, \tag{1.4}
\]

\[
\frac{\partial}{\partial t} \sigma_{xx}^f = (\lambda + 2\mu) \frac{\partial}{\partial x} v_x^f + \lambda \frac{\partial}{\partial z} v_z^f + \sum_j S_j, \tag{1.5}
\]

\[
\frac{\partial}{\partial t} \sigma_{zz}^f = (\lambda + 2\mu) \frac{\partial}{\partial z} v_z^f + \lambda \frac{\partial}{\partial x} v_x^f + \sum_j S_j, \tag{1.6}
\]
\[ \frac{\partial}{\partial t} \sigma_{zz}^I = \mu \left( \frac{\partial}{\partial x} v_z^I + \frac{\partial}{\partial z} v_z^I \right), \quad (1.7) \]

where \( S_j \) denotes the \( j \)-th source and \((v_x^I, v_z^I, \sigma_{xx}^I, \sigma_{zz}^I, \sigma_{xz}^I)\) denote particle velocities and stresses. During the computation, we sample the particle velocities at appropriate receiver locations to give \( p(x_r, t|x_s)_{\text{calc}} \) from equations (5).

2. Calculate the weighted residuals as defined in equation 1.1. The criteria we use for choosing the weighting factor is to set \( w = 0 \) for \( \delta \tau > T/4 \), and \( w = 1 \) for \( \Delta \tau \leq T/4 \), where \( T \) is the period corresponding to the peak frequency of the first arrival wavelet in a seismogram.

3. Compute the elastic wave equation in reverse time with zero final conditions by using the same 4th-order finite-difference algorithm for each shot point. Here the seismogram residual is treated as the source time history at the receiver location.

\[ \rho \frac{\partial}{\partial t} v_x^b = \left( \frac{\partial}{\partial x} \sigma_{xx}^b + \frac{\partial}{\partial z} \sigma_{xz}^b \right) = \sum_i \sum_h \hat{\gamma}_h^i, \quad (1.8) \]
\[ \rho \frac{\partial}{\partial t} v_z^b = \left( \frac{\partial}{\partial z} \sigma_{xz}^b + \frac{\partial}{\partial x} \sigma_{zz}^b \right) = \sum_i \sum_h \hat{\gamma}_h^i, \quad (1.9) \]
\[ \frac{\partial}{\partial t} \sigma_{xx}^b = (\lambda + 2\mu) \frac{\partial}{\partial x} v_z^b + \lambda \frac{\partial}{\partial z} v_x^b, \quad (1.10) \]
\[ \frac{\partial}{\partial t} \sigma_{zz}^b = (\lambda + 2\mu) \frac{\partial}{\partial z} v_z^b + \lambda \frac{\partial}{\partial x} v_z^b, \quad (1.11) \]
\[ \frac{\partial}{\partial t} \sigma_{xz}^b = \mu \left( \frac{\partial}{\partial x} v_z^b + \frac{\partial}{\partial z} v_x^b \right), \quad (1.12) \]

where \( \hat{\gamma}_h^i \) and \( \hat{\gamma}_h^i \) denote the appropriate source time histories, which are the residuals of the seismograms for the \( l \)-th receiver of the \( k \)-th shot array.

4. Compute the perturbation of Lame parameters (Mora, 1987).

\[ \delta \lambda = - \sum_j \int_0^T dt \left( \frac{\partial}{\partial x} v_x^I + \frac{\partial}{\partial z} v_z^I \right) \left( \frac{\partial}{\partial x} v_x^b + \frac{\partial}{\partial z} v_z^b \right), \quad (1.13) \]
\[ \delta \mu = - \sum_j \int_0^T dt \left( \frac{\partial}{\partial x} v_x^I \frac{\partial}{\partial x} v_x^b + \frac{\partial v_x^I}{\partial z} \frac{\partial v_z^b}{\partial z} \right) \]
\[ + \left( \frac{\partial v_x^I}{\partial x} + \frac{\partial v_z^I}{\partial z} \right) \left( \frac{\partial v_x^b}{\partial z} + \frac{\partial v_z^b}{\partial x} \right). \quad (1.14) \]

In this step, we use the boundary values which we saved in forward propagation modeling (step 1) to recover the forward propagation field in the reverse time array.

5. Calculate the perturbation of model parameters by using the weighted gradient of P- and S-wave velocities.
\[ \delta v_p = 2v_p \rho \delta \lambda, \quad (1.15) \]
\[ \delta v_s = -4v_s \rho \delta \lambda + 2v_s \rho \delta \mu. \quad (1.16) \]

At every iteration we perform these five steps and use the subspace method (Kennett et al., 1988) to calculate the step length to update the P and S wave velocities. We choose P and S wave velocity values as model parameters in our inversion because they are much better resolved than the Lame parameters, \( \lambda \) and \( \mu \) (Tarantola et al., 1985).

### 1.4 Numerical Examples

We apply the elastic waveform inversion method to both synthetic and real cross-hole seismic data. The real data is collected by Exxon (Chen et al., 1990) near Friendswood, Texas. In all cases we use a non-linear steepest descent method with pre-conditioning (Beydoun and Mendes, 1989). For each test we compare the elastic waveform inversion with acoustic waveform inversion to show the advantages of elastic waveform inversion.

#### 1.4.1 Synthetic Crosshole Data

Synthetic data will now be used to estimate the effectiveness of the WTW method in inverting both P and S velocities from (1). two-component seismic data and a good starting S velocity model, (2). two-component seismic data and a poor starting S-wave velocity model, and (3). single component pressure data. The synthetic data are generated by a 2-D elastic finite-difference method for the fault model shown in Figure 1.1a, where the P-wave velocity ranges from 2300 m/s to 3600 m/s. We assume that \( V_s \) equals 0.5\( V_p \) to get the initial S-wave velocity distribution (Figure 1.1b), and use an empirical formula \( \rho \approx \sqrt{v(z)_p} \) to assign the density distribution from the P-wave velocity distribution, and use the same empirical formula to update the density distribution at every iteration of the inversion. The "observed" seismograms in this case are generated by a 4th-order finite difference solution to the 2-D elastic wave propagation. The fault model is discretized onto a mesh with 162x242 grid points with 18 explosion line sources and 36 two component receivers along the left side and right side of the model respectively; a 50 gridpoint wide absorbing sponge zone is extant along each boundary. The source function is a Ricker wavelet function added onto the x and z stress components. The receivers record particle velocities in the receiver locations. The source wavelet has a peak frequency of 60 Hz and the starting P wave velocity model is homogeneous with a velocity value of 3000 m/s.

Figure 1.2a shows the first arrival traveltine inversion for the P-wave velocity after 10th iteration. We use this tomogram as the initial guess of the P-wave velocity.
distribution for elastic waveform inversion and use Poisson's relationship to assign the initial S-wave velocity model. After 6 iterations we get the P- and S-wave velocity tomograms shown in Figure 2b. The tomograms show very good interface definition compared to the true model.

For comparison, Figure 1.3 shows the acoustic WTW tomogram for the P-wave velocity distribution after 6 iterations. The recorded seismogram we used for the inversion is obtained by summing the z and x components of normal stresses at each receiver point; these are generated by the 4th-order elastic wave propagation finite-difference modeling at appropriate receiver locations. There is converted S-wave energy in the recorded data so that the acoustic WTW tomogram is not as accurate as the elastic WTW tomogram.

The elastic WTW is robust with respect to the initial guess of the S-wave velocity distribution. This is demonstrated by elastic WTW inversion of the previous fault model shot gathers except the homogeneous S-wave velocity (1500 m/s) distribution is assigned when we begin the waveform inversion. Figure 1.4 shows the P-wave tomogram after 6 iterations. The accuracy of the tomogram is quite acceptable.

In the previous tests, elastic WTW was applied to 2-component data to invert for both P- and S-velocities. The next test attempts to invert for both P- and S-velocities from single component pressure field data. The results are shown in Figure 1.5 and suggest that inverting single component for both P and S velocity distributions may not be successful.

1.4.2 Real Crosshole Data

The elastic waveform inversion is now applied to a real crosshole seismic data set collected by Exxon near their Friendswood, Texas test site (Chen et al., 1990, Zhou et al., 1993). The offset of the two wells is 600 feet, the depth of the wells is 1000 feet, and the source and receiver intervals are 10 feet. There are 98 sources and 96 receivers in the source and receiver wells. The source is an explosion source which consists of a small amount of dynamite and the seismic data has a usable bandwidth of 80 to 600 Hz. The receivers used here are the hydrophones which record the pressure field. A typical unprocessed shot gather at intermediate depth is shown Figure 1.6a.

The processing steps applied to the shot gathers include (Cai and Schuster, 1993) 1). eliminating the tube waves by median filtering; 2). free-surface reflections were muted out; 3). an 80-600 Hz bandpass filter was applied to the data; 4). each seismogram was normalized to its maximum value; and 5). direct arrivals were muted after the waveform inversion was turned on. Each forward modeled shot gather used a source wavelet extracted from the corresponding observed shot gather and the wavelet is used as the time history of the x and z components of the normal stress when doing elastic wave propagation modeling; e.g., Figure 1.6b shows the first arrival source wavelet associated with a trace at intermediate depth. To accommodate the 80-600
Hz bandwidth of the data, a 2-D finite difference mesh of 303x501 gridpoints was used for the forward modeling and back-projection, with the same well geometry as in the Friendswood experiment. Well deviations in the source and receiver wells were corrected by applying an appropriate time shift to the raw seismograms. The data were corrected to 2-D by multiplying the filter $\sqrt{1/\omega}$ by the spectrum of the observed seismograms. The final processed shot gather associated with Figure 1.6a is shown in Figure 1.6c.

Because the recorded data are dominated by unconverted P-waves, we apply the acoustic wave equation traveltime and waveform inversion to the 98 shot gathers of the processed Friendswood data. After 46 iterations we get the P-wave tomogram in Figure 1.7a. Then we use this tomogram as the initial P-wave velocity model for the elastic WTW inversion and use the relationship $V_s = 0.5V_p$ to assign the S-wave velocity starting model from the P-wave velocity. After 6 iterations of elastic waveform inversion we get the P- and S-wave tomograms in Figure 1.7b and Figure 1.7c. The final tomograms provide a fine resolution of both the layer interfaces and velocity distribution. Because the recorded data are dominated by unconverted P-waves, the elastic WTW tomogram (Figure 1.7b) is close to the acoustic WTW tomogram (Figure 1.7a). This structure and resolution are verified in Figure 1.8 which compares the smoothed sonic log (solid line) in the source hole to a vertical slice of the final elastic waveform inversion P-wave velocity tomogram (6b). The slices were taken from the tomogram along a vertical line 40 feet from the sonic log.

Although we have no S-velocity well logs, the S-wave tomogram shows a good correlation with the P-wave velocity structure. However, the absence of noticeable S reflections in the data suggests that the S-velocity tomogram alone is not a reliable representation of the S-velocity distribution. Our synthetic results from the previous section also suggest that the inversion of two-component data may be more accurate.

Finally we compare the synthetic shot gathers computed from the velocity field in the, respectively, acoustic WTW 46th iteration tomogram and the final elastic WTW tomogram with the recorded shot gather (Figure 1.9). In the elastic waveform inversion synthetic shot gathers, the seismic events match the observed shot gathers better than those from acoustic synthetic shot gathers.

1.5 Ongoing Research

We are currently inverting a 2-component crosshole seismic data set from Canada and a 1-component crosshole data set from Texas. These two data sets were collected by mechanical sources and so are dominated by significant P- to S-wave converted waves. Our experience thus far suggests that these different wave types must be first separated from the data in order to effectively apply the elastic WTW method. Such a separation was not necessary in the Friendswood data because the source was explosive which apparently did not generate large amplitude P- to S-converted waves at 600 foot offsets.
1.6 Conclusion

We presented the elastic WTW method for inverting 2-component crosshole seismic data. Inversion of both synthetic and real crosshole seismic data shows that this method can provide a significantly better model resolution than that given by traveltine tomography. The synthetic test shows this method also can provide high resolution S-wave velocity tomograms for the data which contain significant S-wave information. Compared to acoustic WTW inversion, the elastic WTW inversion method provides more information about the model from the same recorded seismic data. Results also show that inverting both P- and S-velocities from data that does not contain significant P- to S-wave energy is unlikely to be successful. Our experience with crosshole data that has significant converted P to S energy suggests that converted modes must be first separated from the data prior to WTW inversion.

1.7 References


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Figure 1.1: The P- and S-wave velocity structure of the Fault model
Figure 1.2: (a) The acoustic WTW tomogram after 10 iterations. (b) The elastic WTW tomograms after 16 iterations (inverting P- and S-wave velocity structure in the same time).
Figure 1.3: Acoustic WTW tomogram after 16 iterations.

Figure 1.4: Elastic WTW tomogram after 16 iterations for a homogeneous S-wave velocity model as the starting model.
Figure 1.5: Elastic WTW tomogram after 16 iterations by using single component pressure field data.
Figure 1.6: A typical shot gather of Friendswood crosshole data collected by Exxon near their Friendswood, Texas test site. The source depth is 520 feet. (a) Raw shot gather. (b) First arrival wavelet extracted from the Figure 5a shot gather. (c) Figure 5a shot gather data after signal processing.
Figure 1.7: (a) Acoustic WTW tomogram after 46 iterations. (b) and (c) Elastic WTW P- and S-wave velocity tomograms after 6 iterations by using the Figure 1.7a tomogram as the initial model.
Figure 1.8: Elastic WTW P-wave tomogram velocity profiles (dashed lines) compared to the sonic log (solid line) in the source well. The profiles are extracted from the tomograms 40 feet from the source well.
Figure 1.9: Synthetic acoustic and elastic common shot gathers associated with the (a) acoustic waveform inversion tomogram in Figure 1.7a, and the (b) elastic waveform inversion tomogram in Figures 1.7b and 1.7c. Figure 1.9c depicts the corresponding observed shot gather. The source location for these common shot gathers is at the depth 520 feet.
Report 2

Crosshole Elastic WTW Applied to the 2-component Steepbank Data

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2.1 Abstract

We apply the elastic wave equation traveltime and waveform inversion (elastic WTW) method to the 2-component Steepbank crosshole data. These data are characterized by a well offset of 103 meters and a somewhat low frequency source (180 Hz). Numerical tests indicate that the following processing steps are necessary, but not sufficient, for successful waveform inversion: FK filtering of the up- and down-going waves, FK-fan filter extraction of the PP- and SS-reflections, and balancing the amplitudes of the up- and down-going reflections. Preliminary results show that the elastic WTW tomogram is not significantly more resolved than the traveltime tomogram. This is somewhat surprising since the seismogram calculated from the elastic WTW tomogram matches the observed seismogram much better than the seismogram calculated from the traveltime tomogram. The somewhat low spatial resolution of the WTW tomogram is partly explained by the low frequency character of the data, an insufficient number of WTW iterations, and interfaces with large impedance contrasts that dominated the waveform residual and misfit gradient.
2.2 Introduction

Zhou et al. (1993) successfully applied the acoustic WTW method to the Friendswood crosshole data set (Chen et al., 1990) to obtain a high resolution image of the interwell velocity distribution. The primary reasons for their success were that the Friendswood data were of high quality and were generated by an explosive source. The explosive source generated a simple radiation pattern and primarily excited P waves so that a simple acoustic WTW method could be used for inversion.

Unfortunately, many crosshole data sets are generated with mechanical sources that excite complicated radiation patterns along with high energy direct and converted S waves. The associated elastic records not only contain direct P-waves and reflected PP-waves, but also contain direct S-waves, SS-, PS-, SP-reflections, and transmitted PS- and SP-waves, which do not exist in the acoustic wave propagation record. Hence, an elastic WTW method must be used to invert such data.

It is much more difficult to successfully invert elastic data compared to inverting acoustic data. The many different elastic wave modes introduce a much greater complexity to the records, and so it becomes more important to start out with a very good initial guess model. Otherwise the elastic WTW method will get hopelessly lost in local minima. For example, the P- and S-, PS- and SP-transmitted waves contribute to the long wavelength components of the gradient field and, typically, will dominate the gradient field because of their large amplitudes relative to reflected waves (Schuster, 1994). Thus, an imperfect ability to model these transmitted arrivals will lead to large transmission waveform residuals. These residuals will dominate the gradient with long wavelength artifacts and prevent the short wavelength velocity components from being updated. In the acoustic WTW method, this was not a problem because the only large amplitude transmitted wave was the direct P wave, which could easily be muted out.

2.2.1 Divide and Conquer Strategy

The key ingredient for success in elastic WTW inversion is to reduce the complexity of the elastic records. To achieve this goal we use a "divide and conquer" strategy. The data are processed (via median and FK filtering) so that all but the PP reflections are eliminated. We then apply the elastic WTW method (setting the shear modulus to zero in the forward modeling code) to reconstruct the P-velocity model. We repeat this step again, except we now process out all arrivals but the SS reflections, and then invert for the S-velocity model. Now more complex arrivals can be allowed into the inversion process to reconstruct even more details about the velocity model.

The problems associated with a complex radiation pattern are ameliorated by normalizing the maximum amplitude of any seismogram to one. Emphasizing just PP or SS reflections and normalizing amplitudes transforms the elastic WTW method into something akin to a glorified reflection traveltime tomography method.
2.3. **DATA PROCESSING**

It this paper, we apply the elastic WTW method to Chevron's Steepbank (Canada) crosshole data. Many of the problems discussed above are encountered, and partial remedies are described. We first describe the WTW method, show the processing steps and finally present the results of applying the elastic WTW method to the Steepbank data.

### 2.3 Data Processing

The crosshole data we use in this project are from the Steepbank survey in Canadas (courtesy of Chevron). The crosshole experimental geometry is shown in Figure 2.1, where the offset and depth of the wells are 103 meters and 158 meters, respectively. There are 77 sources and 80 receivers in the source and receiver wells, with source and receiver intervals of 2 meters. Figure 2.2 shows a common shot gather (gather-15) for a shot at a depth of 28 meters. Figure 2.2a shows a vertical component record and Figure 2.2b shows the in-line horizontal component record. The center frequency of the data is about 180 Hz and we estimate that the average wavelength associated with the seismic arrivals is about 14 meters, compared to the approximately 5 meter variations in the sonic log. The are several strong reflections in this recorded data (arrows A and B in Figure 2.2), some of which come from above and below the wells (arrows C and D in Figure 2.2). A dominant mode of energy in this record is the direct S-wave.

Prior to applying WTW inversion, we apply the following data processing procedures to emphasis the PP reflection information (Cai and Schuster, 1993):

1. Apply a 50 to 300 Hz bandpass filter to the field data to filter out the high and low frequency noise.

2. Pick the first arrival traveltimes and invert for the smooth P-velocity tomogram by travelt ime tomography. Mute out the direct arrivals.

3. Approximately mute the signal below the direct S-wave.

4. Apply FK-filtering to separate up- and down-going waves and suppress the reflected PS- and transmitted PS-waves.

5. Mute the reflections that come from above and below the wells.

6. Mute the strong SP-reflected waves. By now the processed records should mainly contain PP-reflections. In the final step we add the FK-filtered up- and down-going shot gathers together to get one common shot gather.

7. The processed data are transformed from 3-D to 2-D by applying the filter $\sqrt{i/\omega}$ in the frequency domain.
Figure 2.3 shows gather-15 after data processing. The major arrivals in this processed gather are PP-reflected waves. Figure 2.4 shows the source wavelet extracted from gather-15 by use of Promax software.

2.4 Elastic WTW Inversion of Steepbank Data

We pick the first arrivals of the Steepbank data and do crosshole travelt ime inversion to get the P-velocity tomogram. Because the first arrival energy is very weak in this data set (as shown in Figure 2.2), the travelt ime picks have large errors which show up in the travelt ime tomogram as several large artifacts in Figure 2.5. We use this tomogram as the initial velocity model for waveform inversion. The density distribution we use here is taken from an empirical relationship between P-wave velocity and density (Gardner et al., 1974).

After a total of 12 elastic WTW iterations (setting the shear modulus to zero) we get the P-velocity tomogram shown in Figure 2.6. The WTW tomogram image is not significantly better resolved than the travelt ime tomogram in Figure 2.6. Comparing the shot gathers calculated from the travelt ime and waveform tomograms shows that the waveform synthetics (Figure 2.7b) contain many more reflections than the travelt ime synthetics (Figure 2.7a). The WTW synthetic seismograms in Figure 2.7b show several reflection events (arrows A, B, C and etc) that resemble the events in the observed gather (Figure 2.7c). The source location of these gathers is at 28 meters. Figure 2.8 shows that the waveform residual decreases with each WTW iteration, and the total waveform RMS residual is decreased by 30 percent after 12 iterations. Because of the difficulties in picking the first arrival travel times for this data set, we cannot provide the plots of traveltime residuals vs. iteration number.

We believe that the low resolution of the WTW tomogram might be caused by:

1. the strong reflection energy generated from the large velocity discontinuity at the 100 meter depth. If the modeling method cannot perfectly simulate these reflections, then this will lead to large amplitude waveform residuals that will dominate the elastic WTW gradient field. Such a large residual can prevent updating other parts of the velocity model. We are currently looking for remedies to this problem.

2. not enough WTW iterations. Computer limitations prevented computing many WTW iterations, each of which required about 30 CPU minutes on a Fujitsu VPX/240 supercomputer.

3. improper FK-filtering that introduced artifacts into the processed records.

4. low frequency character of the data. The dominant wavelength associated with the data is about 14 meters, which may be too long for reconstruction of fine-detailed features.
2.5 Conclusion

This is not a successful example of elastic waveform inversion, but valuable lessons were learned. We believe that the divide-and-conquer strategy, FK-filtering of the data and balancing the up- and down-going wave amplitudes are essential for implementing the WTW method. Nevertheless, the elastic WTW method did not generate a tomogram that was significantly more resolved than the traveltime tomogram. Some reasons for this low resolution include artifacts from the FK filter, low frequency character of the data, insufficient number of WTW iterations, and dominance of the misfit gradient by waveform residuals from large amplitude reflections (or other modes). Future work will attempt to correct these problems and obtain a more highly resolved tomogram.

2.6 References


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Zhou, C., Schuster, G., T. and Hasanzadeh, S., 1994, Wave Equation Traveltime and Waveform Inversion of the McElroy data for P- and S-velocities, 1994 University of
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2.7 Acknowledgements

Acknowledgement is made to the Donors of The Petroleum Research Fund, administered by the American Chemical Society for partial support of this research (contract PRF\$ 22807-AC2, PID 8909029). We are also grateful for the financial support provided by the 1992 University of Utah seismic tomography consortium members; Amerada Hess, Amoco, Arco, Chevron, Conoco, Exxon, Fujitsu Computer Company, Gas Research Institute, Japon, Marathon, Noranda, Oyo, Phillips, Texaco, and Unocal.
Figure 2.1: The geometry of the Steepbank crosshole experiment.
Figure 2.2: Field CSGs (gather-15) containing P- and S-arrivals. (a) vertical component. (b) horizontal component.
Figure 2.3: Vertical component CSG gather-15 after data processing. The PP-reflections are enhanced.

Figure 2.4: The source wavelet extracted from gather-15 by using Promax software.
Figure 2.5: P-velocity first arrival travelttime tomogram.

Figure 2.6: P-velocity elastic waveform tomogram after 12 iterations. The tomogram in Figure 2.5 was used as the initial model.
Figure 2.7: Z-component synthetic CSGs computed from the (a) elastic WTW tomogram in Figure 2.6, and the (b) traveltime tomogram in Figure 2.5. The field CSG is shown in (c). Each of these CSGs have the same source location.
Figure 2.8: RMS waveform residual vs iteration number for the WTW iterations.

Figure 2.9: Reflectivity image after applying constrained Kirchhoff migration to the Steepbank data (courtesy of Fuhao Qin).
Report 3

Crosshole Elastic WTW Inversion of the McElroy Data

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3.1 Abstract

We apply the elastic wave equation traveltime and waveform inversion (elastic WTW) method to the McElroy crosshole data. These data are characterized by a well offset of 184 ft and a wide-band source wavelet (250-2000 Hz). Numerical tests indicate that the following processing steps are necessary, but not sufficient, for successful waveform inversion: FK filtering of the upgoing and downgoing waves, FK-fan filter extraction of the PP and SS reflections, and balancing the amplitudes of the upgoing and downgoing reflections. To reduce the complexity of the data we follow the divide and conquer strategy, i.e., extract the PP reflections and SS reflections, then invert each wave mode separately. Numerical results show that the vertical spatial resolution of the WTW P-wave and S-wave tomograms are approximately 7-10 feet and 4 feet, respectively. This compares favorably to the 40-50 feet vertical resolution of the P-velocity tomogram obtained from the first arrival traveltime data. There is good to very good agreement between the sonic logs and the velocity from the WTW tomogram. These preliminary results demonstrate that high resolution Poisson ratio, S-velocity, and P-velocity tomograms can be extracted from crosshole data, and therefore can be used for lithological interpretation.
3.2 Introduction

Zhou et al. (1994a) presented the elastic WTW method for crosshole inversion. They showed that the elastic WTW method was not suited for the Friendswood data set because it was obtained with an explosive source, which generated very weak shear arrivals. They later applied the elastic WTW method to the Steepbank crosshole data (Zhou et al., 1994b) and found that the convergence rate was slowed down by large amplitude reflections from a single interface. Since elastic wave records are very complicated and contain many types of pure and converted modes, Zhou et al. (1994b) adopted a "divide and conquer" processing strategy. That is, they used muting, median filtering and FK filtering to eliminate all but the PP and SS reflections in the data. They then attempted to separately invert these two types of reflections. These simplified records should, in principle, allow for a faster and more robust convergence to the actual P- and S-velocity models.

In this paper we apply the elastic WTW method to the crosshole McElroy data. We use the same "divide and conquer" strategy as used for the Steepbank data. The resulting P-velocity and S-velocity WTW tomograms show good to very good agreement with the P- and S-sonic logs. In fact, the vertical resolution of the P-wave tomogram is almost an order of magnitude better than that of the traveltime tomogram. We now describe the strategy and show the results of applying the elastic WTW method to the McElroy data.

3.3 McElroy Data Processing

The crosshole data we use are from the McElroy survey conducted by Jerry Harris in West Texas. The experimental parameters are given in Table 1 where the well offset is 184 feet. There are 201 sources and 186 receivers in the, respectively, source and receiver wells, with source and receiver intervals of 2.5 feet. The source is a piezoelectric mechanism with a bandwidth between 250 Hz and 2000 Hz, and the receivers are hydrophones. Figure 3.1 shows a common receiver gather (gather-90) for a receiver at a depth of 2920 feet. Figure 3.2 shows the processing flow, and the processed shot gathers that contain the PP reflections and the SS reflections are shown, respectively, in Figures 3.3 and Figure 3.4. The WTW method is applied to these processed data, first to invert for the P-velocity tomogram (setting shear modulus to zero) and, then to invert for the S-velocity tomogram (setting bulk modulus to zero).

Detailed steps to the processing are shown below.

1. Apply a 200 to 1400 Hz bandpass filter to the field data to filter out the high and low frequency noise.

2. Pick the first arrival traveltimes (provided by J. Harris and Mark Van Schack) and invert for the smooth P-velocity tomogram by traveltime tomography. Mute out the direct arrivals.
3. Mute the arrivals that arrive during and after the onset of the direct S-wave for the extraction of PP waves; and mute out arrivals that arrive before and during the onset of the direct S-wave for the extraction of SS waves.

4. Apply FK-filtering to separate upgoing and downgoing waves and suppress the reflected PS- and transmitted PS-waves. Different FK fans are used for the SS reflections and PP reflections.

5. Mute the reflections that come from above and below the wells.

6. Mute the strong SP-reflected waves. By now the processed records should mainly contain either PP reflections or SS reflections. In the final step we add the FK-filtered upgoing and downgoing shot gathers together to get one common shot gather. The amplitudes of the upgoing and downgoing waves are balanced with one another.

7. The processed data are transformed from 3-D to 2-D by applying the filter $\sqrt{i/\omega}$ in the frequency domain.

### 3.4 Elastic WTW Inversion of McElroy Data

We use the first arrival McElroy traveltimes picked by Stanford University researchers and do crosshole traveltime inversion to get the P-velocity tomogram shown in Figure 3.5. We use this tomogram as the initial velocity model for waveform inversion. The density distribution we use here is taken from an empirical relationship between P-wave velocity and density (Gardner et al., 1974).

#### 3.4.1 WTW Inversion of PP Reflections

The elastic WTW method (setting the shear modulus to zero) is applied to the McElroy data processed for PP reflections. In all cases, only the shot gather data were inverted, but a better result would have been obtained if both the shot gathers and receiver gathers had been simultaneously inverted.

After seven WTW iterations we get the P-velocity tomogram shown in Figure 3.6; for comparison we also show the P-wave migrated section obtained by Cai (1994, this volume). The WTW tomogram and the migrated section both show the same dipping interface near the bottom of the model. The reflector interfaces are very prominent in the migrated section compared to the interface images in the WTW tomogram. This is because the migrated section emphasizes the discontinuous features of the velocity model (i.e., the reflectivity distribution) while the tomogram is an integrated reflectivity distribution, i.e., the velocity distribution. It is easy to transform the tomogram into a reflectivity distribution, which will then be comparable to the migrated section. However, the WTW reflectivity distribution should be, in principle,
more reliable because it was generated from a, presumably, more accurate velocity function. Note the long wavelength diagonal-like artifacts in the WTW tomogram can be removed by FK filtering or regularization constraints.

Figure 3.7 compares the WTW tomogram's velocity profile with the sonic log at the source and receiver wells. There is a very good correlation between the variations of the WTW velocity and the sonic log velocity at the receiver well (between depths of 2850 to 3050 feet). This contrasts with the poor WTW and sonic log velocity correlation in the source well. A better correlation could have been achieved if the WTW method had been applied to the receiver gathers as well as the source gathers. Unfortunately, time limitations prevented the inversion of the receiver gathers. Note that the velocity variations in the sonic log are larger than those in the WTW method. We believe that this is because the forward modeling code is designed not to generate converted waves, while the field data was not subject to such a restriction. Finally, we notice that the vertical spatial resolution of the WTW tomogram near the receiver well is about 7-10 feet, compared to the 40-50 feet resolution provided by the traveltime tomogram.

Figure 3.8 shows the squared RMS waveform residual with respect to the WTW iteration number. After seven iterations the waveform residual has decreased by approximately 10 percent. Time limitations prevented further iterations, but we believe that using constraints with more iterations will yield a more accurate velocity profile.

3.4.2 WTW Inversion of SS Reflections

The elastic WTW method (after setting the bulk modulus to zero) is applied to the McElroy data processed for SS reflections. In all cases, only the shot gather data were inverted, but a better result would have been obtained if both the shot gathers and receiver gathers had been simultaneously inverted. The starting S-velocity model is taken from the P-velocity tomogram scaled by a factor of 0.55.

After five WTW iterations we get the S-velocity tomogram shown in Figure 3.9; for comparison we also show the S-wave migrated section obtained by Cai (1994, this volume). The WTW tomogram and the migrated section both show the same dipping interface near the bottom of the model. For the same reasons mentioned above, the S-reflector interfaces are more prominent in the migrated section compared to the interface images in the WTW tomogram. There are also noticeable artifacts in the S-wave tomogram, which can be mitigated by the use of regularization constraints, more iterations and data weighting. Unfortunately time limitations prevented the use of such procedures.

Figures 3.10 and 3.11 show the, respectively, sonic log vs WTW velocity comparison and the squared RMS waveform residual with respect to the WTW iteration number. After five iterations the waveform residual has decreased by approximately 8 percent. Time limitations prevented further iterations and artifact reduction.
3.5 Discussion

This is a successful example of elastic waveform inversion applied to crosshole data, although much work remains to be completed. We are especially interested in exploring ways to reduce artifacts in the waveform tomogram. Some possible remedies include the additional use of constrained linear conjugate gradient iterations rather than just non-linear iterations, regularization constraints, data weighting schemes, more iterations and better data processing schemes. Finally, we will use the SS- and PP-waveform tomograms as the starting models for fully elastic waveform inversion of the data. The above are to be considered preliminary results because our work on inverting the McElroy data did not begin until the end of January, 1994. All results were calculated on a Fujitsu VPX/240 supercomputer and required about 40 CPU minutes per iteration. The grid size was 248x501 and there were 1400 time steps per iteration.

3.6 References


Jannane, M., and Equipe de Tomographie Geophysique, 1989, The wavelengths of Earth structures that we can resolve from seismic reflection data, Geophysics, 54, 906-910.


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3.7 Acknowledgements

Acknowledgement is made to the Donors of The Petroleum Research Fund, administrated by the American Chemical Society for partial support of this research (contract PRF\# 22807-AC2, PID 8909029). We are also grateful to Jerry Harris and Mark Van Schack for donating the data to us.
Figure 3.1: Fields CRGs (gather-90) at depth depth 2920 feet.
Figure 3.2: Processing flow for McElroy data.
Figure 3.3: CSG (gather-51) after data processing for the PP reflections.
Figure 3.4: CSG (gather-51) after data processing for the SS reflections.
Figure 3.5: P-velocity first arrival traveltime tomogram.
Figure 3.6: P-velocity elastic waveform tomogram after 7 iterations compared to the P-wave migrated section. The tomogram in Figure 3.5 was used as the initial model for the waveform inversion.
Figure 3.7: Comparison between the P-velocity WTW profile with the sonic log.
Figure 3.8: Squared RMS waveform residual vs iteration number for the P-velocity WTW iterations.
Figure 3.9: S-velocity elastic waveform tomogram after 5 iterations compared to the S-wave migrated section.
Figure 3.10: Comparison between S-velocity WTW profile with the sonic log.
Figure 3.11: Squared RMS waveform residual vs iteration number for the S-velocity WTW iterations.
Report 4

3-D Traveltime Refraction Imaging

Wenyeng Cai and Fuhao Qin

4.1 Abstract

I present a nonlinear 3-D refraction traveltime inversion method that reconstructs the earth’s 3-D shallow velocity structure from first arrival refraction traveltimes. The inversion method employs finite-difference solutions of the eikonal equation and does not always require a good starting model. Numerical tests show that 3-D refraction imaging can correctly reconstruct 3-D structures and is more accurate than 2-D imaging in some cases. The possible applications are for 3-D statics corrections and 3-D migration.

4.2 Introduction

A turning ray tomography method was proposed by Zhu et al. (1992) to image 2-D near-surface velocities. This method uses a linear inversion algorithm and so requires a good initial velocity model; the final result will therefore be very sensitive to the initial model. To overcome this sensitivity, Qin et al. (1992) proposed a nonlinear inversion method that does not always require a good initial model. Their 2-D inversion algorithm proved to be both effective and efficient (Qin et al., 1993). We now extend this inversion algorithm to 3-D problems. Hopefully, the 3-D inversion can provide a good velocity structure for 3-D migration and statics correction.

This paper will briefly present the 3-D inversion algorithm and show two synthetic model tests. A fault model is designed to compare the effectiveness of 2-D versus 3-D traveltime inversion. Then a 3-D model with a fault and a bump structure is used to test the effectiveness of the 3-D inversion program.
4.3 Methodology

The details of the 2-D inversion method were presented in Qin et al. (1993). We will only illustrate different parts of the 3-D inversion algorithm from the 2-D method.

The 3-D refraction traveltime inversion scheme calculates traveltimes by finite-difference solutions to the 3-D eikonal equation (Qin et al., 1992). Topographic changes in the 3-D model are more complicated than those of the 2-D models but can be handled by interpolation between neighboring source and receiver points. Similar to the 2-D tomography algorithm in Qin et al. (1993), a horizontal smoothing operator (in the x- and y-directions) is applied to the gradient field to alleviate the dependencies nature inherent in this underdetermined and ill-posed problem.

4.4 Numerical Tests

This section will test the 3-D imaging method on two models: a fault model and a bump+fault model. The 2-layer fault model is used to demonstrate that the 2-D inversion method cannot give the correct depth of the interface if the 2-D survey lines are above and parallel to the fault surface. This compares to the 3-D inversion method which can reconstruct the correct depth of the interface. A 2-layer 3-D structure model (consisting of a bump and a fault) is designed to test the effectiveness of the 3-D inversion code. The velocity contrast between the two layers for both models is 1:2. The model size is $121 \times 121 \times 41$ grid points and the initial models are all homogeneous. The "observed" traveltime data are computed by finite-difference solutions to the 3-D eikonal equation (Qin et al., 1992).

4.4.1 Fault Model

Figure 4.1 shows a vertical slice (perpendicular to the fault striking direction along the y-axis) of the fault model. The 3-D survey lines (6 source lines and 10 receiver lines) are along the x- and y-directions. Five receiver lines are evenly distributed along the x-direction and the other five receiver lines along the y-direction. In each horizontal direction, there are three source lines distributed along the three receiver lines near the center of the model. There are 7 sources in each source line and 12 receivers in each receiver line. A source line along the y-direction and the same line of receivers will be used in the 2-D inversion. The 2-D tomogram will be compared to a vertical slice of the 3-D tomogram along the same profile.

Figures 4.2 and 4.3 depict, respectively, the 2-D inversion and 3-D inversion results. Obviously the 3-D inversion method correctly maps the velocity interface while the 2-D inversion method does not give the correct depth to the interface beneath the survey line. If a 2-D survey line along the x-direction is used in 2-D inversion, the 2-D inversion result should be as accurate as the 3-D result. Unfortunately, the real situation is not so simple and we do not always have a priori information about the
near surface structures. A three-dimensional reconstruction method is much more flexible and accurate.

4.4.2 Three Dimensional Model

The 3-D model geometry is shown as a fault and a bump structure in Figure 4.4. The same number of sources and receivers as that in the fault model survey is used and the source and receiver lines are distributed exactly the same way as the fault model survey. The velocity contrast across the interface is 1:2. A vertical slice along the x-direction of the 3-D tomogram is shown in Figure 4.6 and compared with the same slice of the true model displayed in Figure 4.5. Another vertical slice along the y-direction of the 3-D tomogram is shown in Figure 4.8 and compared with the same slice of the true model displayed in Figure 4.7. These figures show that the 3-D inversion method accurately reconstructed the true model.

4.5 Discussion

Preliminary numerical tests show that nonlinear 3-D refraction travelt ime inversion is effective. For complicated 3-D subsurface models it may provide a more accurate tomogram than provided by the 2-D inversion method. The 3-D method is very stable and is not sensitive to the initial model though a good starting model may speed up the convergence. The computing time for the 3-D tomogram with a grid size of $121 \times 121 \times 41$ grid points (with 42 shot gathers and 120 traces per gather) is about 3 hours per iteration on a RSIC 6000 530 workstation compared to about 1 minute per profile for the 2-D tomogram.

4.6 References


Figure 4.1: A vertical slice of the fault model. It is perpendicular to the x-axis.
Figure 4.2: 2-D profile of the 2-D tomogram for the fault model.

Figure 4.3: 2-D profile of the 3-D tomogram for the fault model.
Figure 4.3: 2-D profile of the 3-D tomogram for the fault model.
Figure 4.4: 3-D model geometry.
Figure 4.5: A vertical slice along the x-axis of the 3-D model.

Figure 4.6: 3-D reconstruction result: the vertical slice is along the same profile as in Figure 4.5.
Figure 4.6: 3-D reconstruction result: the vertical slice is along the same profile as in Figure 4.5.
Figure 4.7: A vertical slice along the y-axis of the 3-D model.

Figure 4.8: 3-D reconstruction result: the vertical slice is along the same profile shown in Figure 4.7.
Figure 4.8: 3-D reconstruction result: the vertical slice is along the same profile shown in Figure 4.7.
Report 5

First Arrival Traveltime Inversion of Surface Seismic Data

Fuhao Qin and Wenyong Cai

5.1 Abstract

Here we present a nonlinear inversion method that inverts the shallow velocity distribution from first arrival traveltimes. This method is robust with respect to poor starting models and limited numerical tests suggest that the method is fast, stable and easy to use. It may find applications in both environmental engineering and statics corrections in seismic exploration.

5.2 Introduction

The refraction seismic technique is one of the principal tools used for investigating the shallow structure of the earth for both environmental studies and statics corrections in seismic exploration. Of the many available refraction methods (Marsden, 1993), tomographic methods are getting increased attention with the increasing power of portable computers. De Amorim et al. (1987) calculated statics correction from the traveltimes of the refractions based on a one-layer model; Olsen (1989) described a method that inverts for laterally varying velocities and shallow depths from the first arrival traveltimes. Both of these methods consider that waves are critically refracted from the top of the high velocity layer and can be used to invert for the layer thickness and velocity variations. They are ideally suited to areas where the near surface structure is restricted to two or three layers whose parameters vary over a predictable and limited range; this assumes that the interpreter correctly specifies the layer numbers and estimates their thickness and velocity.

Zhu et al. (1992) introduced a turning ray tomography method. They assumes that the first arrivals are not pure refractions critically refracted from an interface
but rays gradually turned upward. Using this concept, they inverted for the sub-
surface velocity distribution instead of the thickness and layer velocities and claimed 
that turning ray tomography can image near-surface velocities more accurately than 
refraction statics methods. However, in their method, rays are traced only once 
through a velocity model derived by interpreters; raypaths were kept unchanged even 
though the velocity models were updated iteration by iteration. The problems with 
this method are, first, that it requires a good initial model and, second, that some 
avoidable errors will arise from the fixed raypaths.

We present an alternative method that does not require a good starting model 
and still converges fast. In this method, the velocity model and raypaths are both 
updated in the tomographic iterations. The traveltime and raypath calculation are 
based on the finite-difference solution to the eikonal equation (Qin et al., 1992). This 
guarantees that the calculated traveltimes are first arrivals regardless of the wave 
types; thus it can deal with the irregular surface problem very easily. Similar to Zhu 
et al. (1992), the model is updated by a SIRT-like method.

Following this section, the method is briefly discussed and then two numerical 
examples are presented. After that, some remaining details of the implementation of 
the method are discussed.

5.3 Methodology

Refraction traveltome tomography is actually a special case of crosswell traveltome 
tomography. Most optimization techniques applicable to crosswell tomography can 
be used here. Refraction traveltome tomography differs from crosswell tomography in 
that: (1) the sources and receivers are all located at a similar elevation level. If the 
rays are straight lines, the ray coverage beneath the earth’s surface will be zero; (2) 
the topography changes tremendously in some areas. This makes raytracing difficult 
to find the rays with minimum traveltome.

To illustrate the inversion procedure, we divide this section into several subsections.

5.3.1 Irregular Surface

In seismic exploration, many surveys are carried out in areas where elevation changes 
with offset. It is desirable to develop methods that can correctly deal with this 
problem. The criterion of correctness is that the region above the free surface should 
not affect the first arrival traveltome calculation by any means. Our procedure is to 
give the region above the free surface a very low velocity value. The velocity is so low 
that no first arrival ray will go through it, but it is not too low to affect the stability 
of the finite difference eikonal equation solver. We found that one third to one half 
of the minimum velocity of the model is sufficient for the models we tested.
5.3. METHODOLOGY

5.3.2 Raytracing

Due to the topographic changes along the seismic survey line, ordinary raytracing methods might have difficulty in finding the correct ray path even though the area above the free surface has a lower velocity. Therefore, a finite-difference solution to the eikonal equation (Qin et al., 1992) is used to calculate all of the first arrival traveltimes. Rays are then traced from the receivers to the sources following the normal directions of the wavefronts (gradient directions of the traveltime field). This guarantees that the traveltimes are first arrivals and the raypaths are correct although they may not be very precise.

It is well known that waves do not travel along a geometrical ray path with no thickness at all. A more physical description is the concept called wavepath (Woodward, 1988, 1992; Luo, 1990). The thickness of the wavepath is proportional to the inverse of frequency and the wavepath length. In traveltime inversion, there is no frequency involved so we only consider the raypath length that affects the thickness of the wavepath. For simplicity, we will also consider that the wavepath thickness is a constant throughout the raypath. The wavepath width is calculated as $A\sqrt{L}$, where $L$ is the raypath length and $A$ is a predetermined constant. By doing this, not only does it give us some physical meaning to the raypaths, but also makes the code converge faster for at least the first several iterations.

5.3.3 Gradient Downward Extrapolation

As mentioned in the beginning of this section, most optimization methods used in crosswell tomography can be used for refraction traveltime tomography. We define the misfit function as,

$$
\epsilon = \frac{1}{2} \sum_i (t_{i, obs} - t_{i, cal})^2,
$$

where, $i$ represent the $i_{th}$ raypath, $t_{i, obs}$ is the recorded traveltime and $t_{i, cal}$ is the calculated traveltime.

To minimize the above travelt ime misfit function, a SIRT like method (van der Sluis, A. and van der Vorst, H. A., 1987) is used. The negative gradient of the traveltime misfit function (the model updating direction) is then

$$
g_j = -\frac{\sum_{i=1} \Delta t_i}{N_j},
$$

where $g_j$ is the negative gradient in the $j_{th}$ cell, $N_j$ is the number of rays that visit the $j_{th}$ cell, $\Delta t_i = t_{i, obs} - t_{i, cal}$ is the travelt ime residual and the summation $i$ is over the indices associated with raypaths (wavepaths) that visit the $j_{th}$ cell.

If a cell has no ray passing through it, let it have the same gradient value as the cell just above it. The reason for doing this is to extend the gradient field downward from the deepest point where rays can reach. In other words, since we do not have
information beneath the depth of maximum ray penetration, the best we can do is to assume that below this depth the velocity is the same. Figure 5.1 shows a layered earth model with some refraction raypaths in it. The rays are concentrated near the interfaces and there is almost no information within each layer. However, the downward extrapolation of the gradient will provide each layer with a similar updating direction. Our experience suggests that this strategy mitigates to avoid convergence problems.

5.3.4 Smoothing

When the inversion problem is a under-determined (more unknowns than equations), it is always desirable to apply a moving average smoothing filter to the gradient field. Even when the data number exceeds the number of unknowns, some parts of the model may be over-determined, and some other parts of the model may still be under-determined. Thus, it is still helpful to apply a smoothing operator. For refraction tomography, we used a rectangular smoothing filter. The size of the smoothing operator should be large in the first several iterations and become smaller with the number of iterations to achieve best results (Nemeth et al., 1993).

5.3.5 Large Models

A seismic line will run tens and even hundreds of kilometers. It will be too big to fit in a computer as a single model. However, we found that it is convenient to subdivide the model into smaller segments. These segments should overlap one another and the overlap should be large enough to cover the region of edge effects in the inversion result. To invert a certain segment, only rays that start and end in this segment are used. After all segments are inverted, they are combined into one large model by throwing away the overlapped parts that are considered to be affected by edge effects.

This will conclude our methodology section. A typical refraction tomogram usually needs twenty to sixty iterations to converge, providing the homogeneous initial model is not very far away from the top layer velocity. Considering that the finite-difference eikonal equation solver is fast, a workstation will be able to handle most of the problems encountered in environmental engineering and seismic exploration.

5.4 Numerical Examples

In this section, CDP refraction traveltime data from a synthetic model and a field experiment are used to test the refraction inversion method. The synthetic model test is demonstrates the effectiveness of this method in inverting for a shallow velocity distribution. The field data example shows the ability of this method to invert the near surface structure which, in this case, is an onshore salt dome intrusion.
5.4. NUMERICAL EXAMPLES

5.4.1 Synthetic Model

The model used is the top part of a model based on a seismic survey in South America. The size of the model is 22 km by 12 km (Figure 5.2) and the model for the refraction study is reduced to 21.61 km by 1.2 km (Figure 5.3). The maximum surface elevation change along this line is about 700 meters and the velocity varies from 2,000 m/s to 3,800 m/s. There are 261 sources evenly distributed within the offset range of 3,000 m and 18,600 m. The source interval is 60 m. For each source, 101 receivers are assumed to be located within a 6,000 m offset range which is centered at the source location. The receiver interval is also 60 m. The finite-difference eikonal equation solver was used to calculate the first-arrival traveltimes. In the forward modeling process, the region above the irregular free-surface was given a velocity of 1,000 m/s.

In the inversion process the grid spacing was set to 10 m. We can not use a very large grid spacing since: (1) The finite-difference eikonal equation solver requires a finer grid to obtain more accurate traveltimes; (2) The particular gradient calculation scheme we used also requires a fine grid; (3) The refractors are usually not very deep. The model size is thus 2,161 by 121. Then the model was divided into three parts as discussed in the previous section for large models. Each part is 901 by 121 gridpoints and the inversion was carried out for each part. For the inversion, a 2,000 m/s homogeneous velocity model is used as a starting model and the inversion was considered complete when traveltime residuals stopped decreasing (i.e., after about 50 to 60 iterations). The final result was obtained by combining all three parts. Figure 5.4 shows the result.

Comparing Figure 5.4 and Figure 5.3, we can see that the velocity of the top weathered zone is well reconstructed. The depth and shape of the refractor is also reconstructed except near the left and right edges. The velocity trend of the refractor is depicted in the tomogram and will be a good model for statics corrections. One thing that needs to be mentioned is the vertical strips caused by the downward gradient extrapolation. This is actually a good indication of the refraction ray penetration depth. Anything below this point is not trustworthy.

Although the inversion of eikonal equation traveltimes is successful, there is some doubt about whether high frequency eikonal traveltimes can simulate traveltimes for finite-frequency wave propagation. The following test was designed to make this point clear. In this test, the wave equation was solved by a finite-difference method to calculate seismograms. First arrival traveltimes were then picked from the seismograms. Figure 5.5 shows a common shot gather (CSG) with the shot at an offset of 9000 m. The source wavelet is a 16 Hz Ricker wavelet. To satisfy the dispersion and stability criterion, the grid spacing was set at 10 m and the time step length was 1 ms. The CSG was plotted using a 1.4 second auto-gain-control (AGC) window.

To amplify the first arrival signal for correct picking, an AGC gain with a window of 0.1 s was applied to the seismogram before an automatic picker picked the traveltimes. Figure 5.6 shows the same seismogram as that in Figure 5.5 except for the AGC gain. The picker picks the first arrival time at the point where 5% of the
largest amplitude first occurs in that trace. A time shift is applied which is obtained from the zero offset trace. It is then considered to be the first arrival traveltime.

Figure 5.7 shows the inversion result from the picked traveltime. There are no significant differences between this result and that obtained from the eikonal traveltimes in Figure 5.4. So, we conclude that the first arrival traveltimes from a finite frequency seismic survey do include the refraction information needed for the inversion.

However, we still have a problem. Comparing Figures 5.5 and 5.6, we can see that the first arrivals for some of the refraction signals are very weak. It will be very difficult to pick them without the help of a small window AGC. In the real data case, the weak first arrival might be buried in noise and the AGC gain will not help much. Then we will be in trouble. Suggested solutions include increasing the source energy and source frequency. If necessary, do two different surveys, one for seismic reflections whose aim is to penetrate as deep as possible, the other will be to emphasize the refraction first arrivals.

### 5.4.2 Field Data Example

The method was also tested on a CDP field data set from the Gulf of Mexico. The CDP seismic data were collected from an area where there was an onshore salt intrusion. The length of the seismic line was about 65,340 ft and there were 192 sources with a source interval of 110 ft. Maximum fold of the line is 190 and for each source there were 250 receivers with a receiver interval of 110 ft.

Two thirds of the first arrival traveltimes were picked from the seismograms. For the inversion, the grid spacing used was 55 ft for a grid size of 1188 by 45 grid points. The starting model is a homogeneous model with a velocity of 3000.0 ft/s. The RMS residual decreased from 1.547 s for the starting model to 0.155 s after 20 iterations. Figure 5.8 shows the inverted tomogram after 20 iterations. The salt dome was clearly imaged. This result agrees well with the geologist’s interpretation based on seismic and other information.

For this particular inversion, 30 iterations took about 3 hours running time on an IBM RISC 6000 computer.

### 5.5 Discussion

For the models tested, the refraction tomographic inversion method is very stable and converges to a model that minimizes the traveltime residual. It does not require a good initial model, although a good starting model may accelerate the rate of convergence. The ideal digitization of the model is to discretize the model into very fine grids in order to get accurate finite-difference traveltimes and raypaths; A smoothing operator will mitigate the problems associated with the underdetermined system of equations. To avoid excessive CPU time, the selection of the grid spacing should be based on the size of the model and the capacity of the computer.
Like all nonlinear inversion processes, smaller step lengths seems to give a better result. In our inversion code, we constrained the step length so that, for each iteration, the velocity change did not exceed 10 percent of the largest velocity value in the model.

Although the inversion always converges, there is no guarantee that the final result is the true model. This method can not deal with the nonunique solution problem. It also requires an accurate picking of the first arrival traveltimes. Traveltimes picked from finite frequency seismogram can be used for refraction tomography. However, attention should be paid to the correct picking of the first arrival traveltimes because the first arrival signal may be very weak.

A companion paper (Cai and Qin, 1994) extends this method to three dimensions so that it can be applied to 3-D seismic surveys. A FORTRAN code will be distributed to the sponsors along with its readme file and sample input files.

5.6 Acknowledgements

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5.7 References


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Figure 5.1: Refraction ray paths and the downward extension of the gradient. The gradients in different shadings have similar values.
Figure 5.2: The South America model designed for the seismic reflection study. (Courtesy of Kunhua Chen of Chevron Overseas Petr. Inc.)

Figure 5.3: Top part of the model shown in Figure 5.2 which is used in the following refraction study.
Figure 5.3: Top part of the model shown in Figure 5.2 which is used in the following refraction study.
Figure 5.4: Inversion result using the eikonal equation traveltimes.

Figure 5.7: Inversion result using the picked finite-difference seismogram traveltimes.
Figure 5.5: A finite-difference common shot gather with an AGC window of 1.4 s.
Figure 5.6: The same seismogram as shown in Figure 5.5 except that the AGC window is 0.1 s instead of 1.4 s.
Figure 5.7: Inversion result using the picked finite-difference seismogram traveltimes.
Figure 5.8: Inversion result of the field data set.
6.1 Abstract

I show that the misfit gradient in seismic waveform inversion can be decomposed into both short wavelength and long wavelength components, where the short components are associated with reflection events and the long components are primarily associated with the transmitted arrivals. This decomposition reveals that the shorter wavelength components have much smaller magnitude than the longer wavelength components. This is consistent with earlier studies that demonstrated the slow convergence of reflection waveform inversion in reconstructing the smooth parts of the velocity model. To speed up convergence, the decomposition formula suggests various strategies for inverting the long wavelength components of the velocity field from reflection data.

6.2 Introduction

Numerical experiments by Jannane et al. (1989) suggested that the short ($\lambda_{\text{short}}$) and long ($\lambda_{\text{long}}$) wavelength components of the velocity distribution influence different parts of the seismic data. Synthetic data tests showed that traveltimes are primarily influenced by $\lambda_{\text{long}}$ while amplitudes are influenced by $\lambda_{\text{short}}$. An example of this biased influence was shown by Zhou et al. (1993) where the finite-difference seismograms associated with the smooth Friendswood traveltine tomogram (Chen et al., 1990) were almost completely devoid of significant reflection energy (see Figure...
1a); the traveltime tomogram was too smooth to generate reflections with significant energy. These missing reflections, however, were generated when the detailed parts of the velocity model were included from the traveltime+waveform tomogram (see Figures 1b-1d).

A naive conclusion from the above is that transmission traveltimes can only update the smooth parts of the velocity model, while reflection amplitudes can only update the detailed parts of the model. Therefore, if the traveltime tomogram does not accurately reconstruct \( \lambda_{\text{long}} \) (which is true in practice), then the correct values of the smooth velocity distribution will not be reconstructed by the subsequent waveform inversion. I will show in this paper that, indeed, the long wavelength parts of the velocity model can be updated, in principle, by the subsequent waveform inversion, but we need to "coax" the gradient into doing so.

### 6.3 Waveform Inversion

The strategy for wave equation traveltime+waveform (WTW) inversion (Zhou et al., 1993) is to first update the smooth parts of the velocity model by traveltime inversion, and then update the detailed parts by waveform inversion. For an impulsive source function, the slowness update formula in waveform inversion for a single source (Zhou et al., 1993) is given by

\[
\delta(x)^{(k+1)} = s(x)^{(k)} - \gamma(x),
\]

where the gradient \( \gamma(x) \) is given by

\[
\gamma(x) = \sum_{x_r} G(x|x_s) \otimes G(x|x_r) \ast \delta P(x_r, x_s).
\]

Here \( \delta P(x_r, x_s) \) is the seismogram residual in reverse time for the source at \( x_s \) and the receiver at \( x_r \); \( G(x'|x) \) is the impulse Green's function for a source at \( x \) and a receiver at \( x' \) in a 2-D acoustic medium; and \( \ast \) and \( \otimes \) denote temporal convolution and temporal crosscorrelation, respectively. I have suppressed the notation for the time variables and the temporal derivatives. Setting \( g(x|x_s) = \sum_{x_r} G(x|x_r) \ast \delta P(x_r, x_s) \) simplifies the gradient to

\[
\gamma(x) = G(x|x_s) \otimes g(x|x_s).
\]

The interpretation of this equation is that the slowness field is updated by a temporal crosscorrelation of the backward-propagated residual field \( g(x|x_s) \) with the forward-modeled field \( G(x|x_s) \).

### 6.4 Gradient Decomposition

I will now decompose the gradient field into a sum of transmitted and reflected components; the reflection component can be further divided into forward- and back-propagated parts. The decomposition formula shows that the magnitude of the misfit
gradient associated with the transmitted arrivals (henceforth called the transmission gradient) dominates over the gradient contributions from reflected arrivals (henceforth called the reflection gradient).

I will employ the notation $G_{ij}$ which describes the wavefield along the raypaths that connect the source at $x_i$ with the receiver at $x_j$. For the simple example of a homogeneous medium with a source at $x_0$ and a receiver at $x_2$, the 3-D Green's function $G(x_2 \mid x_0)$ becomes

$$G(x_2 \mid x_0) = \delta(t - |x_0 - x_2|/c)/|x_0 - x_2|,$$  

(6.4)

where $c$ is the medium's velocity. Another example is the single interface model in Figure 6.2a, where the Green's function $G(x_0 \mid x_2)$ can be decomposed into a composite wavefield with support along the transmitted ray $G_{02}$, the downgoing ray $G_{01}$ and an upcoming $G_{12}$, i.e.,

$$G(x_2 \mid x_0) = G_{02} + G_{01} + rG_{12},$$

(6.5)

where $r$ represents the reflection coefficient at the interface.

For a wavefield back-propagated from the receiver array, the Green's $g(x_0 \mid x_2)$ function for a source at $x_2$ can be decomposed into a sum of transmitted $g_{20}$, downgoing $g_{21}$ and upcoming parts $g_{10}$ parts, i.e.,

$$g(x_0 \mid x_2) = g_{20} + rg_{21} + r^2g_{10},$$

(6.6)

where the associated raypaths are shown in Figure 6.2b. Note that the wave downgoing from the receiver well is scaled by $r$ because we assume that this back-propagated reflection started out at the receiver well with an amplitude by $r$. The $g_{10}$ term is multiplied by $r^2$ because it emanates from the downgoing reflected field $rg_{10}$ that reflects a second time from the interface.

### 6.4.1 Transmission Waveform Tomography

Plugging equations 6.6 and 6.5 into equation 6.3 and neglecting all terms in $r$ we get the transmission gradient $\gamma_t(x)$

$$\gamma_t(x) = G_{02} \otimes g_{20},$$

(6.7)

which is, to zeroth-order in $r$, the gradient of the misfit function. For small $r$, this means that the dominant contribution to the velocity update in equation 6.1 is from the transmitted fields. Moreover, the correlation is non-zero along the transmitted raypath so that the velocity update for many source receiver pairs is somewhat distributed throughout the model; it is also a relatively smooth update since equation 6.7 applies nearly equal weighting to the velocity update along the transmitted raypath. This assumes that the geometrical spreading factors have been eliminated by a preconditioning method.
The idea that the transmission gradient updates the smooth components of velocity is consistent with the waveform inversion of Zhou et al. (1993). That is, waveform inversion is first applied to the transmitted arrivals to reconstruct the long wavelength velocity components; after approximately 5 so iterations the direct waves are muted out and waveform inversion is applied to the reflection events to reconstruct the shorter wavelength components.

### 6.4.2 Reflection Waveform Tomography

The previous section derived the transmission gradient formula and showed how it performs a global and smooth update of the velocity distribution. This section will derive the reflection gradient formula, which performs a local and detailed update of the velocity model.

Eliminating the transmitted term $g_{20}$ in equation 6.6 and plugging the result and equation 6.5 into equation 6.3 we get the reflection gradient $\gamma_r(x)$, i.e.,

\[
\gamma_r(x) = (G_{02} + G_{01} + rG_{12}) \otimes (rg_{21} + r^2g_{10}) \\
= r^2G_{02} \otimes g_{10} + rG_{01} \otimes g_{21} + r^2G_{12} \otimes g_{21} + r^3G_{12} \otimes g_{10}. \tag{6.8}
\]

Note that the crosscorrelations only turn on when the forward-propagated rays coincide in both space and time with the back-propagated rays. This implies that $G_{01}$ and $g_{21}$ in Figure 6.2 only coincide at $x_1$ on the reflector boundary; thus, $G_{01} \otimes g_{21} \approx r\delta(x - x_1)$, where geometrical spreading terms have been ignored. It also implies that $G_{02} \otimes g_{21} = G_{02} \otimes g_{10} = 0$ because the direct transmitted ray does not coincide in space and time with the reflected rays. Therefore, the above equation reduces to:

\[
\gamma_r(x) = r\delta(x - x_1) + r^2G_{12} \otimes g_{21} + r^2G_{01} \otimes g_{10} + r^3\delta(x - x_1); \tag{6.9}
\]

neglecting third-order terms in $r$ this becomes

\[
\gamma_r(x) = r\delta(x - x_1) + r^2G_{12} \otimes g_{21} + r^2G_{01} \otimes g_{10}. \tag{6.10}
\]

For small $r$, we see that the reflection gradient is dominated by the part that turns on at the boundary, namely the point $x_1$ where the migrated reflection correlates with the transmitted arrival. Complementary to the transmission gradient, the reflection gradient to first-order in $r$ performs a local and detailed velocity update along the reflecting boundary; and no updates take place along the raypaths. This is probably one of the reasons for the slow convergence of waveform tomography.

### 6.4.3 Kirchhoff vs Reverse Time Migration

Zhou and Qin (1993) applied Kirchhoff and reverse time migration to the Friendswood crosshole data. Their result, shown in Figure 6.3, showed that the reverse time migrated image had more artifacts than the Kirchhoff image. This observation can be
6.5. DISCUSSION

Partially explained by the fact that reverse time migration generates reflected waves both in the forward modeling (i.e., equation 6.5) and in the back-propagation (i.e., equation 6.6). The first- and second-order terms in \( r \) in equation 6.8 contribute to the migrated image along the reflected raypaths, which is undesirable since the goal in migration is to image the reflectors, not the raypaths. This problem is non-existent in Kirchhoff migration because Kirchhoff migration uses high-frequency asymptotic Green’s functions which do not generate secondary reflections (see Figure 6.4). Effectively \( r = 0 \) in equations 6.6 and 6.8 for Kirchhoff migration. A practical remedy is to smooth the velocity field prior to reverse time migration, but this can be a tricky procedure.

6.4.4 Updating Velocities along Raypaths with Reflections

The first term in equation 6.9 must be eliminated in order to primarily update the velocity along the raypaths. This might improve the reconstruction of the velocity distribution’s long wavelength components \( \lambda_{long} \). Some practical strategies that might accomplish this goal are to:

1. calculate \( \gamma_r \) for a given velocity distribution and then compute \( \gamma'_r(x) \) for the smoothed velocity distribution. The associated gradient formula should satisfy

\[
\gamma'_r(x) \approx r \delta(x - x_1) \quad (6.11)
\]

because \( r \approx 0 \) for a smoothed velocity distribution. Subtracting equation 6.10 from 6.11 yields

\[
\gamma'_r(x) - \gamma_r(x) \approx r^2 \delta(x - x_1) + r^2 G_{12} \otimes g_{21} + r^2 G_{01} \otimes g_{10}. \quad (6.12)
\]

which applies equal weighting along the rays and at the reflection points.

2. use two way and one way wave equations to generate the above formula.

3. use an asymptotic Kirchhoff integral method to generate both the reflected and transmitted rays at every point in the medium, similar to the rays in Figure 6.4b. The standard Kirchhoff formula only generates the rays in Figure 6.4a, but it can be extended to generate the rays in Figure 6.4b as well. These different raypaths can be used to calculate equation 6.12.

6.5 Discussion

I show how the waveform gradient can be decomposed into short and long wavelength components. These different components have different weights, and so can give rise to an unbalanced gradient with subsequent artifacts in the reconstructed velocity model. Strategies are suggested to better balance the updating of the long and short wavelength components of the model. Future research will test the effectiveness of these approaches.
Figure 6.1: (a). Synthetic shot gather generated from the smooth travelt ime tomogram associated with the Friendswood crosshole data; note the absence of significant reflections. (b). Synthetic shot gather generated from hybrid traveltime+waveform tomogram associated with the Friendswood crosshole data; note the presence of significant reflections. (c). Field shot gather from the Friendswood data for a source at the same level as that for the synthetic seismograms. (d). Residual field obtained by subtracting the field gather in (c) from the synthetic gather in (b).
Figure 6.2: (a). Forward-propagated rays and (b). Backward-propagated rays.
Figure 6.3: (a). Reverse time migration image and (b). Kirchhoff migration image of the Friendswood crosshole data. The Kirchhoff image has fewer artifacts than the reverse time image because it only has a non-zero correlation at the reflector boundary, not along the ray path.
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Figure 6.4: Rays associated with the (a) Kirchhoff and (b) reverse time migration methods. Note that the forward- and backward- Kirchhoff rays only coincide at the boundary, compared to the correlation along raypaths in the reverse time rays.
6.6 References


Part II

Migration
Report 7

Processing and Kirchhoff Migration of McElroy Crosswell Data

Wenyning Cai and Fuhao Qin

7.1 Abstract

PP and SS reflections are extracted from the McElroy crosswell data set by a sequence of processing steps. The data processing is performed in both the common source and common receiver domains and both PP and SS reflections are migrated to image the interfaces between the two wells. The final migration image is obtained by stacking the migrated sections from both the source and receiver domains. This stacking greatly enhances the image near the source and receiver wells. P-wave and S-wave synthetic seismograms are generated from the sonic logs and a show a good to very good correlation with their associated migrated sections.

7.2 Introduction

A number of seismic imaging methods have been proposed to go beyond the limited resolution of crosswell traveltime tomography, including reflection stacking (Lazaratos et al., 1992), migration (Qin and Schuster, 1993; Mo and Harris, 1993), and waveform inversion (Zhou et al., 1993). The key requirement for successful reflection imaging is to accurately extract the primary reflections without damaging the data.

Different imaging methods may demand different processing procedures. The processing strategy for crosswell data is similar to that of VSP data and typically consists of picking first breaks, bandpass filtering, removing tube waves and direct arrivals, deconvolution, and separating upgoing and downgoing reflections from the complex total wavefield. As Rector et al. (1994) pointed out, wavefield separation of
crosswell data encounters difficulties when the reflection arrivals have moveout similar to that of the direct arrivals. Reflection points near the source well (i.e., sources near the reflector) are characterized by reflections with a moveout similar to that of the direct arrivals in the common source gather domain; this compares unfavorably to the much larger moveout in the common receiver domain. Likewise, reflection points near the receiver well (i.e., receivers near the reflector) are characterized by reflections with a moveout similar to that of the direct arrivals in a CRP gather; this compares unfavorably to the much larger reflection moveout in a CSP gather. Having the reflection moveout differ significantly from the direct wave moveout is very important for separating upgoing and downgoing reflection events when the data is dominated by direct P and PS converted waves, and if the energy of PP reflections are very weak. Performing wavefield separation and muting direct arrivals in the common source domain may destroy reflections from interfaces close to the source well. These reflections will be preserved if the wavefield separation is performed in the common receiver domain where the direct wave removal will not affect the reflection events. Although we can perform wavefield separation in the common offset domain this is equivalent to performing it in both the source and receiver domains, except in the offset domain the number of traces varies with respect to offset. The number of traces decreases with an increase in the offset and it is difficult to process data with only a few traces.

In this paper we present the processing procedure and Kirchhoff migration results for a crosswell data set acquired Jerry Harris at two wells near a west Texas carbonate reservoir. This data set (henceforth called the McElroy data) was used for reflection stack imaging by Lazaratos et al. (1992) and reverse time migration by Mo and Harris (1993). We apply the wavefield separation to both the source and receiver-sorted data, migrate the processed PP and SS reflections, and stack the results from the common source and receiver domains. Our results show a good to very good correlation with the sonic log synthetics.

### 7.3 McElroy Data Processing

The McElroy crosswell data were collected at 2.5 feet intervals in both the source and receiver wells. The distance between the two wells is about 184 feet and the survey depth is about 500 feet. The sweep frequency of the piezoelectric downhole sources ranges from 250 to 2000 Hz and the details of the survey parameters are given by Harris et al. (1992).

Figure 7.1 shows a common receiver gather of raw data. The wave modes are complicated, and are identified as direct P and S waves, PS and SP converted transmission and reflected waves, PP and SS reflections, and tube waves.

We first use the picked first break times to get a tomogram which will be used for the migration velocity model. The tomogram is depicted in Figure 7.2. The following processing steps are applied to both the common receiver and common source sorted
7.3. MCELROY DATA PROCESSING

data to extract the PP reflections:

1. **Preprocessing:** The data are resampled with a smaller time interval (0.05 ms) and a bandpass filter (350-2000 Hz) is applied to the data. The direct S waves are removed by muting all arrivals that are coincident or arrive later than the S direct arrivals. The remaining wave modes include first P arrivals, PP reflected waves, PS reflected waves, and some P to S converted waves.

2. **Direct Wave Removal:** The direct P wave is removed by using a 15 trace median filter followed by the bandpass filter used in the preprocessing step. Median filters typically introduce high frequency noise which must be removed.

3. **Extraction of PP Reflections:** From the P-wave travelt ime tomogram and the direct S arrival times, we find that the P-wave velocity distribution is between 15,000 and 22,000 ft/s, and the S wave velocity ranges from 8,000 to 12,000 ft/s. Because the minimum S velocity (8,000 ft/s) is significantly larger than the tube wave velocity (4,500 ft/s), we decided to remove the PS reflections and PS conversions as well as the tube waves by an f-k fan filter. This f-k filter is
designed to separate the upgoing and downgoing PP reflections and effectively attenuate all other unwanted wave modes.

The extracted upgoing and downgoing reflections from the receiver gather of raw data displayed in Figure 7.1 are depicted in Figures 7.3 and 7.4. The PP reflections are greatly enhanced. A problem with this procedure is that an f-k fan filter can both destroy data and create artificial data. Later we will show that our migrated image compares well with the sonic logs, which suggests that the f-k filter did not damage the data too severely. Nevertheless, proper fan filtering or a viable alternative is a topic for our future research.

7.4 PP Reflection Migration Imaging

The extracted upgoing and downgoing PP reflections are migrated by a constrained Kirchhoff migration method (Qin and Schuster, 1993). In the constrained migration, rays with incidence angles larger than 60 degrees are simply eliminated to alleviate the wavelet stretching and phase change of post-critical reflections. Reflector dip angles are also constrained to be within ±15 degrees to compensate for the limited aperture coverage in the crosswell geometry. Experience suggests that the migration of upgoing and downgoing waves separately will improve the image quality.

The traveltime tomogram is used for the migration velocity. The extracted upgoing and downgoing reflections from both the common source and common receiver sorted data go through the same migration imaging sequence. The upgoing and downgoing migrated sections in the common receiver domain are depicted in Figures 7.5 and 7.6. We have mentioned that imaging for reflections extracted only from the source (or receiver) domain will lose reflector information close to the source (or receiver) well. This can be seen very clearly in the migrated section displayed in Figure 7.7; this image is the result of stacking both the upgoing and downgoing migrated sections in the common receiver domain (Figures 7.5 and 7.6). The image has poor reflector continuity on the side close to the receiver well, as discussed in the previous section.

The final migration result is obtained by stacking the migrated sections from the common source and common receiver domains. Figure 7.8 shows the final imaging result compared with the synthetics from the P sonic logs. Note that there is a very good match between the migration image and the synthetics; this suggests that f-k filtering did not severely damage the raw data. Compared to the reverse time migration result of Mo and Harris (1993) shown in Figure 7.9, it appears that the Kirchhoff migration image is somewhat better resolved.
7.5 SS Reflection Migration Imaging

The SS reflections are extracted in a manner similar to that for extracting PP reflections, except the FK fan filter is designed to capture the SS reflections. Also, the direct S waves and arrivals prior to the transmitted S wave are muted. The S-velocity used for migration is taken from the P-velocity tomogram scaled by a factor of 0.55. This scaling factor was estimated from the P- and S-velocity sonic logs.

Figure 7.10 shows a processed shot gather that contains mostly upgoing SS reflection arrivals. Although there are artifacts introduced by the FK filter, our migration results (to be shown) suggest that the FK filtering did not severely damage the data. Figures 7.11 and 7.12 show the migrated sections obtained from the upgoing and downgoing waves, respectively. Note that the upcoming migrated section provides a good reflectivity image everywhere except at the top part of the model, which is to be expected because of the sparse number of sources and receivers there. Similarly, the downgoing image is well resolved everywhere except at the bottom part of the model. Note that the upgoing and downgoing reflectivity images in the middle part of the model seem to be out of phase in places, or even inconsistent. This might be caused by an incorrect migration velocity, or perhaps by imperfect data processing. Nevertheless, adding the upgoing and downgoing images yields the composite migrated section in Figure 7.13. Comparison with the well log S-wave synthetic seismogram shows a good to very good correlation. However, some of the reflectors lack continuity which is probably due to the phase mismatch in the upgoing and downgoing migrated sections. To ameliorate this phase mismatch, I apply the correction procedure described in Qin (1994, this volume) to both the upgoing and downgoing migrated sections. Figure 7.14 shows the resulting composite migrated section. Here the reflector continuity has been restored in some places, at the minor expense of introducing small fault-like discontinuities. Therefore, to distinguish between the true and false structures will demand correlating the consistent features of Figures 7.11 through 7.14. It should be noted that the S-reflectivity image provides almost twice the spatial resolution of the P-reflectivity image because of the shorter wavelengths associated with S-wave arrivals.

7.6 Discussion

We processed and migrated the McElroy crosswell seismic data. The McElroy data were dominated by complex wave modes, where the PP and SS reflections were much weaker than the direct P arrivals and transmitted PS waves. To unravel these overlapping events, we designed a processing procedure that avoided destroying reflections while extracting the PP and SS reflections from the total wavefield. Multi-domain (i.e., CSP and CRP) data processing and migration imaging stacking is necessary to enhance the imaging result near the source and receiver wells. Migrating the processed data by a constrained Kirchhoff migration method provided PP- and SS-reflection im-
ages that compared well with the sonic log synthetics. The S-reflection images were almost twice the resolution of the P-reflection images; and the Kirchhoff P-reflection image seemed to have a slightly better resolution than the reverse time migration image given in Mo and Harris (1993). Using a slightly higher frequency wavelet in the reverse time migration method would probably provide the same resolution as in the Kirchhoff image.

Our conclusion is that high resolution PP- and SS-reflectivity images can be obtained from crosswell data if careful data processing procedures are used to extract the appropriate reflections. Furthermore, the migration method must use proper constraints in order to reduce the number of artifacts.

7.7 Acknowledgements

We thank Dr. Jerry Harris and Mark Van Schaack for providing the McElroy crosswell data, traveltime picks and well logs. We also thank the members of 1993 University of Utah Modeling and Tomography Consortium for their financial support.

7.8 References


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Report 8

Constrained PS- and SP-Wave Migration

Kim Olsen

8.1 Abstract

PS and SP reflections are extracted from the McElroy crosswell data set and are migrated to the interfaces between the two wells. Compared to PP reflections, migrated converted waves provide enhanced spatial resolution of some reflecting horizons near the source (SP waves) and receiver (PS waves) wells. Compared to migrated SS reflections, converted wave migration near the wells show comparable or improved correlation to synthetic S-wave seismograms generated from sonic logs. Artifacts show up in the migrated image as 'crooked smiles' in areas with weak converted reflections and where the reflector coverage is sparse. The results of migrating converted waves from the McElroy data set suggest that PS and SP waves may complement the migrated images obtained by conventional PP reflections and by SS reflections near the source and receiver wells.

8.2 Introduction

Conventional crosswell imaging is carried out by migrating PP reflections (e.g., Beckey et al., 1991; Stewart and Marchisio, 1991; Lazaratos et al., 1991; Qin and Schuster, 1993). However, the resolution of PP-reflection imaging may be degraded for data with relatively low signal-to-noise ratios, a limited number of source or receiver locations, or artifacts due to the mixed imaging of other reflected modes. In addition, the P-wave images reveal very little information about the corresponding S-wave reflection image. It is therefore important to explore for methods of improving images obtained by the migration of PP reflections.
Few successful improvements in PP-reflection imaging of crosswell data have been published. Takahashi (1993) used an estimate of the expected polarization direction of the scattered waves to obtain an improved migration of PP reflections; the method requires at least two components of the recorded wavefield. Balch et al. (1991) analyzed the use of both PP and SS reflections and PS- and SP-converted waves to image 2 tunnels in a crosswell model; they obtained improved images by stacking partial images for each of the migrated wave modes. However, the multi-mode migration showed severe artifacts due to insufficient separation of the different wave modes.

In this paper I discuss the pro's and con's of using converted waves in cross-borehole imaging; in particular, I study the resolution capabilities and artifacts of PS- and SP-wave migration for a crosswell data set acquired by Jerry Harris of Stanford University at a west Texas carbonate reservoir. This data set was used in stack imaging by Lazaratos et al. (1992) and reverse time migration by Mo and Harris (1993). Processing of the crosswell data includes picking and muting direct P and S waves, muting SS reflections, bandpass filtering, and separating up- and down-going converted phases from the recorded wavefield using f-k filtering. The processed data is migrated using a constrained Kirchhoff technique.

8.3 Problems in Converted-Wave Migration

For a constant velocity model and a single trace, the image points corresponding to a PP reflection are located along an ellipse (e.g., Takahashi, 1993). For sparse receiver or source coverage, migration artifacts will be incompletely cancelled and will appear as elliptical smiles in the migrated section. For PS and SP waves, the image points delineate asymmetric curves because propagation velocity from the source to the image point differs from the velocity between the image point and the receiver. Figure 8.1 shows the relative loci of image points for zero-offset PP, PS and SP reflections in a constant-velocity crosswell model with \( V_s/V_p = 0.55 \). The figure depicts where artifacts are expected for migration of converted waves in areas of limited reflector coverage, namely along 'crooked smiles'; the best coverage is found near the source well (PS partial image) and near the receiver well (SP partial image).

How does the reflector coverage by PS and SP reflections compare to that for PP reflections in crosswell imaging? To answer this question I analyze the coverage of horizontal reflectors by converted waves and PP reflections in a crosswell model with a constant velocity, \( V_s/V_p = 0.55 \) for 201 sources located in the left well and 201 receivers in the right well (similar to the parameters for the McElroy data set), and an evenly layered model with a layer thickness of 1 foot. Figure 8.2 shows the density of reflector coverage using respectively PS and SP waves, as measured by the number of converted wave reflection points per foot along the reflector. The reflector coverage is asymmetric for the converted waves, and the SP-wave (PS-wave) coverage is larger in the half model closest to the source well (receiver well), as expected from Snell's Law. The total reflector coverage of the horizontal reflectors using converted
waves is symmetric (Figure 8.3) and is largest near the top and bottom of the model. Figure 8.3 also shows the coverage by PP reflections, where the best coverage is obtained near the center of the model. The number of PS or SP reflection points per unit length along the reflectors is approximately twice that for PP waves in the leftmost (SP) and rightmost (PS) parts of the model; and in the middle part of the model, PP reflections provide the better coverage. This is further illustrated in Figure 8.4, where the coverage by PP, SP, PS, and PS+SP waves are shown for the reflector at the bottom of the model.

Another factor influencing the quality of the migrated image is the energy of the reflected events; stronger reflections migrate more coherently than weaker ones. The partition of energy is a complex function of incidence angle, source type and velocity and density contrast across the reflector. Figure 8.5 shows the energy partition of an incident P wave for \( \alpha_2/\alpha_1=2.0, \rho_2/\rho_1=0.5 \) for full line and 1.0 for dotted line, \( \alpha_1=0.3 \), and \( \alpha_2=0.25 \); here, 1 (2) depicts incident (reflecting) medium and \( \alpha, \rho \) and \( \sigma \) denote respectively P-wave velocity, density, and Poisson's ratio. The Figure shows that the PS reflection (RS) is much weaker than the PP reflection (RP). This relative strength of the reflected events predicts a better migrated image using PP reflections compared to that obtained by PS reflections.

### 8.4 Field Data Test

The feasibility of migrating converted waves is tested for the McElroy crosswell data set (e.g., Lazaratos et al., 1992). In this section I describe the extraction of converted waves from the data set and the methodology for imaging PS and SP reflections. The results are compared to those obtained by migration of PP and SS reflections, and to synthetic seismograms generated from P-wave and S-wave sonic logs.

#### 8.4.1 Data Processing

The crosswell data were collected at 2.5 feet intervals in both the source and receiver wells. The distance between the two wells is 184 feet and the survey depth is 500 feet. The sweep frequency of the piezoelectric downhole sources ranges from 250 to 2000 Hz. The details of the survey methodology are given by Harris et al. (1992).

Figure 8.6 shows common shot and common receiver gather number 75 from the unprocessed data set. The wave modes are identified as direct P and S waves, PS and SP converted waves, SS reflections, and tube waves.

The following processing steps are applied to the common source (common receiver) gathers to extract PS (SP) reflections:

1. **Preprocessing:** The data is resampled with a smaller time interval (0.05 ms) and a bandpass filter (350-2000 Hz) is applied to the data. The direct S waves are removed by muting out all arrivals after the start of the direct S arrivals.
The remaining wave modes include first P arrivals, PP reflections, and PS and SP converted waves.

2. Direct Wave Removal: The direct P wave is removed by using a 15 trace median filter followed by the bandpass filter used in the preprocessing step.

3. Extraction of converted waves: From the P-wave traveltime tomogram and direct S-wave arrivals, I find that the P-wave velocities range between 15000 and 22000 ft/s and the S-wave velocities range between 8000 and 12000 ft/s. The tube wave velocity is approximately 4500 ft/s. I use an f-k fan-filter to extract reflections with apparent velocities between 8000 ft/s and 20000 ft/s preserving mostly PS waves from the shot gathers and mostly SP waves from the receiver gathers.

The converted waves extracted from the common shot and common receiver gathers shown in Figure 8.6 are depicted in respectively Figures 8.7 and 8.8. As can be seen, many events seem to be artifacts, but it is hoped that after stacking many migrated common shot gathers that the true reflectors will be imaged.

8.4.2 Migration Imaging

Migration of converted waves requires that the propagation velocity from the source to the image points be different than that from the image points to the receiver. The migration P-wave velocities are taken from a tomogram obtained by the waveform inversion of direct P-wave arrivals. The tomogram is shown in Figure 8.9. From analysis of selected shot gathers I take the migration S-wave velocities to be 0.55 times the P-wave velocities.

The extracted up- and down-going reflections are migrated by a constrained Kirchhoff migration method (Qin and Schuster, 1993). In the constrained migration, rays with incidence angles larger than 60 degrees are eliminated to alleviate the wavelet stretching and phase change of postcritical reflections. Reflector dip angles are also constrained to be within 15 degrees to compensate for the limited source-receiver aperture in crosswell geometry.

Figures 8.10 and 8.11 show respectively the migrated partial images of the PS and SP waves for both (top) up-going and (bottom) down-going waves. Note the large number of artifacts ('crooked smiles') in the left half of the PS-wave partial images and the right half of the SP-wave partial images. As discussed in the previous section, these artifacts are expected to be located in areas with sparse coverage and smaller amplitudes of the converted waves.

The genesis of these migration artifacts may include insufficient removal of PP reflections or removal of significant converted wave energy during the processing. However, migration of the f-k filtered wavefield where events with moveout larger than 12000 ft/s were removed failed to eliminate the artifacts. Poorly determined S-wave migration velocities represent another source of artifacts. Images using S-wave
migration velocities of $0.50 \cdot V_p$, $0.55 \cdot V_p$, and $0.60 \cdot V_p$ produced similar artifacts, but more detailed S-wave velocity information may provide an improved converted wave migrated image.

The final imaging result is obtained by stacking the migrated partial images for up-going and down-going PS and SP waves (Figure 8.12). Note that the center part of the image is highly contaminated by the artifacts mentioned above; however, the regions closest to the source and receiver wells show much larger reflector coherence.

Figure 8.13 shows the final converted wave migration image near the source and receiver wells superimposed on the image obtained by migration of PP reflections (Processing and Kirchhoff Migration of McElroy Crosswell Data, this volume); the P sonic logs and synthetic seismograms generated from the logs are plotted next to the image. Note that there is a fair match between the main groups of reflectors obtained by converted wave migration near the wells, the PP-wave migrated image in the middle of the model, and the synthetic seismograms generated from the sonic P logs (near A, B, C, and D). In some areas (E and F), however, the P sonic log information correlate much better to the migrated PP-reflections than to the migrated converted waves. Discontinuity of reflectors from the PP-reflection image to the converted-wave image is in part due to phase-shifts of the converted waves, as pointed out by Balch et al. (1991). The resolution of the converted wave migrated image is finer than that for the PP-reflection migration, as expected from the $V_s/V_p$-ratio (or $\lambda_s/\lambda_p$-ratio) of 0.55.

Figure 8.14 shows the final converted wave migration image near the source and receiver wells superimposed on the image obtained by migration of SS reflections (Processing and Kirchhoff Migration of McElroy Crosswell Data, this volume); synthetic seismograms generated from the S sonic logs are plotted next to the image. In general, the match between the migrated converted waves near the wells and the S sonic log reflections is good at locations A-F. In several areas, the sonic log reflections correlate better to the converted wave image than to the SS-wave image (A, D, and F). Note the discontinuity of reflectors between the SS-reflection image and the converted wave image due to phase-shifted PS and SP waves. The resolution of the converted waves is between that for P waves and S waves.

8.5 Discussion and Conclusions

I have analyzed the reflector coverage from migrating converted waves in a crosswell environment and migrated converted waves extracted from the McElroy crosswell data set. The results suggest that PS and SP waves may help improve migrated images obtained by conventional PP reflections near the source (SP) and receiver (PS) wells due to enhanced coverage, finer resolution and the additional S-wave information. Compared to migrated SS reflections, converted wave migration images can show comparable or improved correlation to S wave sonic logs near the wells. The converted wave migrated image is polluted by artifacts in the center part of the model; these
artifacts may be explained by limited reflector coverage and smaller amplitudes for PS and SP waves. Future work should focus on eliminating these artifacts.

8.6 Acknowledgements

I thank Dr. Jerry Harris and Mark Van Schaack for providing the McElroy crosswell data and well logs. I also thank the members of 1993 University of Utah Modeling and Tomography Consortium for their financial support.

8.7 References


Figure 8.1: The relative loci of image points for zero-offset PP-, PS- and SP-reflections in a crosswell model with constant velocity and $V_s/V_p = 0.55$. 'S' and 'R' depict source and receiver locations, respectively.
Figure 8.2: Coverage of horizontal reflectors for SP (left) and PS (right) waves in a crosswell model with a constant velocity, $V_s/V_p = 0.55$, and an evenly layered model with layer thickness of one foot. The coverage is measured by the numbers of converted wave reflection points per foot along the reflector.
Figure 8.3: Coverage of horizontal reflectors for PP (left) and SP+PS (right) waves in a crosswell model with a constant velocity, $V_s/V_p = 0.55$, and an evenly layered model with layer thickness of one foot. The coverage is measured by the numbers of converted wave reflection points per foot along the reflector.
Figure 8.4: Comparison of the coverage of a horizontal reflector at the bottom of a crosswell model with a constant velocity, $V_s/V_p = 0.55$, and an evenly layered model with layer thickness of one foot for PP (solid), PS and SP (dash-dot) and SP+PS (dashed) waves. The coverage is measured by the numbers of converted wave reflection points per foot along the reflector.
Figure 8.5: Energy partition of an incident P wave for $\alpha_2/\alpha_1=2.0$, $\rho_2/\rho_1=0.5$ for full line and 1.0 for dotted line, $\sigma_1=0.3$, and $\sigma_2=0.25$; here, 1 (2) depicts incident (reflecting) medium and $\alpha$, $\rho$ and $\sigma$ denote respectively P-wave velocity, density, and Poisson's ratio (after Telford et al., 1976).
Figure 8.6: (top) Common shot and (bottom) common receiver gather number 73 from the McElroy data set.

TIME (MS)

DISTANCE (FT)

DISTANCE (FT)

SS
SP
P
S
PS
TUBE WAVES
HEAD WAVES

500
common shot gather number 75 in the MCEIRL data set.

Figure 8.7: (Top) UP-going and (bottom) DOWN-going PS reflections extracted from
Figure 8.8: (top) Up-going and (bottom) down-going SP reflections extracted from common receiver gather number 75 in the McElroy data set.
Figure 8.9: P-wave tomogram of the McElroy data set.
Figure 8.10: (top) Up-going and (bottom) down-going PS-wave partial image of the McElroy data set.
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Figure 8.12: Final converted wave migrated image, obtained by stacking the partial images from the up-going PS and SP waves and the down-going PS and SP waves.
Figure 8.13: Final converted wave image near the source and receiver wells superimposed on the image obtained by PP-reflection migration (Processing and Kirchhoff Migration of McElroy Crosswell Data, this volume). The image is compared to synthetic seismograms generated from P sonic logs at the source and receiver wells, and to the P sonic logs.
Figure 8.14: Final converted wave image near the source and receiver wells superimposed on the image obtained by SS-reflection migration (Processing and Kirchhoff Migration of McElroy Crosswell Data, this volume). The image is compared to synthetic seismograms generated from S sonic logs at the source and receiver wells.
Report 9

A Quasi-Monte Carlo Approach to Efficient 3-D Seismic Image Reconstruction

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9.1 Summary

Woźniakowski\textsuperscript{1} recently achieved a mathematical breakthrough in understanding the tractability of multidimensional integration using nearly-optimal quasi-Monte Carlo methods. Inspired by the new mathematical insights, we have studied the feasibility of applying quasi-Monte Carlo methods to seismic imaging by 3-D pre-stack Kirchhoff migration. This earth imaging technique involves computing a large ($10^9$) number of 3- or 4-dimensional integrals. Our numerical studies show that nearly-optimal quasi-Monte Carlo migration can produce the same or better quality earth images using only a small fraction (one fifth or less) of the data required by a conventional Kirchhoff migration.

9.2 Seismic Migration

In oil and gas exploration, geophysicists image the earth’s subsurface by listening to the seismic echoes from rock boundaries, faults, and other inhomogeneities. Examples of seismic sources are heavy weight drops, explosions, and vibrating trucks. The sources are usually arranged in a grid pattern over a surface area of, say, several...
square kilometers. A line, several lines, or an area distribution of geophones may be active around each source. Seismic migration\(^2\), similar to acoustic holography, is a data processing procedure that images the subsurface layer interfaces by extrapolating the recorded echoes \(S(\vec{x}_r, \vec{x}_s, t)\) to their place of origin, namely their reflection points along a reflecting layer interface. Here \(S(\vec{x}_r, \vec{x}_s, t)\) denotes the seismogram associated with the surface receiver located at the point \(\vec{x}_r = (x_r, y_r, 0)\) and surface source at \(\vec{x}_s = (x_s, y_s, 0)\) for time \(t\). The seismic image \(M(\vec{x})\) of the reflector boundaries at each subsurface point \(\vec{x} = (x, y, z)\) is computed by a 4-D integral (2-D for the source coordinates and another 2-D for the receiver coordinates) of the measured seismograms, i.e.,

\[
M(\vec{x}) = \frac{1}{N^4} \sum_{\vec{x}_r} \sum_{\vec{x}_s} \sum_{t(\vec{x}_r, \vec{x}_s)} S(\vec{x}_r, \vec{x}_s, t(\vec{x}_r, \vec{x}_s, \vec{x})),
\]  

(9.1)

where \(t(\vec{x}_r, \vec{x}_s, \vec{x})\), computed by ray tracing, is the time it takes seismic energy to propagate down from \(\vec{x}_s\) to \(\vec{x}\) and back up to \(\vec{x}_r\). It has been estimated that, for a typical 3-D survey over a 100 km\(^2\) area, a computational effort of \(2 \times 10^{15}\) arithmetic operations with classical methods is required\(^2\), or 25 days of processing on a supercomputer capable of doing \(10^9\) arithmetic operations per second. Thus a more efficient way to compute the 4-D Kirchhoff integral would have significant implications in the oil industry.

### 9.3 Nearly-Optimal Quasi-Monte Carlo Methods

It is known that, for a given error tolerance \(\epsilon\), the computational work required to numerically evaluate a \(d\)--dimensional integral for integrands with bounded derivatives using a regular grid of discretization is on the order \((1/\epsilon)^d\). The integral becomes practically intractable when the \(d\) is large.

Since the 1940's, Monte Carlo methods have been widely used for solving multidimensional problems in various branches of science. In the classical Monte Carlo approach, the discretization points are randomly selected, and the computational work is of the order \((1/\epsilon)^2\) and is independent of \(d\). This makes multivariate integration for large \(d\) tractable, however it does not guarantee \(\epsilon\)-error for every integrand as above. Since then, a number of new discretizations have been proposed\(^3\). Recently, Woźniakowski\(^1,3,4,5,6\) showed that the discretization based on the shifted Hammersley points is nearly-optimal in the average case setting, with the computational work \(H(\epsilon) = O(\epsilon^{-\frac{d}{2}}(\log(\frac{1}{\epsilon}))^q)\), where \(q = (\frac{d-1}{2})\). This means that to guarantee the expected error of the integration to be at most \(\epsilon\) (with respect to a large class of probability measures) one must use at least \(H(\epsilon)\) evaluations, no matter what discretization points are chosen. It has also been shown\(^1\) that for the Hammersley points and for the related Halton points the exponent \(q\) is respectively \(d - 1\) and \(d\).

A key issue is whether the shifted Hammersley points can be used to speed up the computation of practical multidimensional integration problems. In particular, can
the integrals for 3-D pre-stack Kirchhoff migration be computed more efficiently with Hammersley points as compared to regular grid points? Our results imply that 3-D pre-stack Kirchhoff integration can be computed more efficiently (by almost an order of magnitude) on Hammersley points than on regular grid points. This same efficiency gain can also be attained by using other quasi-Monte Carlo integration methods such as Halton points.

9.4 Numerical Results

Several seismic models are used to study the effectiveness of the quasi-Monte Carlo migration. The simplest model has a buried horizontal reflector (Figure 9.1) and provides insights into the computational complexity representative of migrating typical seismograms. The other model is the French model\(^7\) (Figures 9.2 and 9.3) which is a classic 3-D test model for exploration geophysicists. It consists of geometric objects that represent salt domes and a faulted layer.

9.4.1 A Flat Reflector Model

Consider the model in Figure 9.1 with a reflector interface at the depth of \(z = 0.5\) km. The sources and receivers are allowed to take any position within a 1 km x 1 km area. The synthetic seismograms are obtained by a simple raytracing procedure. For a vertical image through the center of the square area, the ideal migration should produce zero image intensities everywhere except near the reflector location at \(z = 0.5\) km. Figures 9.4a, 9.4b, and 9.4c show the computed image intensities plotted as a function of depth \(z\), using various number of regular grid, Halton and Hammersley points respectively. It can be seen that the Halton and Hammersley migrations are much more (one order of magnitude or more) efficient than the regular gridpoint migration for the same quality result. Similar conclusions are drawn for migration using the Hammersley points shifted by various amounts (not shown). This suggests that the Woźniakowski points seem to be no more efficient than the Hammerley or Halton points for the flat reflector integrals with \(d = 4\). It should be pointed out that the Halton points are more convenient to use than Hammersley points because the locations of individual Halton points are not dependent on the total number of points used.

9.4.2 The French Model

The French model\(^7\) is used for testing the efficiency of 3-D migration schemes with realistic source-receiver geometries. Figure 9.2 depicts a perspective view of the earth model characterized by a submerged faulted horizontal plane reflector and two separate reflecting domes. The source locations form a rectangular grid over the model. For each source, an array of 5 receiver lines centered around the source are active,
Figure 9.1: Two-layer reflector model.
Figure 9.2: Perspective view of French model.
Figure 9.3: Plan view of French model. The line AB represents the intersection of the surface with the reconstructed vertical cross-sections shown in Figures 5-7.
Figure 9.4: Migration results for the flat reflector model in Figure 1. Image of reflector boundary for (a). regular grid point, (b). Halton point, and (c). Hammersley point migration for various number of grid points. Here, kmax is the total number of grid points and the ideal reflector image should be that for kmax=810,000 points.
where there are 9 receivers per line. The receivers use the same grid points as the sources and the synthetic data is again generated by a raytracing method.

In typical field experiments, seismic data are collected for a regular grid of source-receiver locations, which are mostly non-coincident with the quasi-Monte Carlo points. Therefore, use of a quasi-Monte Carlo method for the Kirchhoff migration requires some type of approximation to the quasi-Monte Carlo points. In the following tests we approximate the quasi-Monte Carlo point with the nearest regular grid point. The point will be discarded if the corresponding receiver at this grid point is not active.

Figures 9.5a-9.5d show the results of regular grid 3-D pre-stack migration using, respectively, 8,100 seismograms, 32,400 seismograms, 129,600 seismograms and 518,400 seismograms. Here we show a vertical image section cutting through the fault plane and the center of one dome. The image artifacts seen in Figures 9.5a-9.5c are mainly caused by under-sampling of the integrand on a regular grid, and are mostly eliminated in the Figure 9.5d image.

Figures 9.6 and 9.7 are the same as Figure 9.4 except Halton points were used to produce Figure 9.6 and Hammersley points were used to generate Figure 9.7. For a given number of points used, the Halton and Hammersley migration images have smaller and fewer coherent errors than the regular grid migration. Apparently the errors in under-sampling the Kirchhoff integrand are somewhat suppressed by the quasi-random ordering of the grid points. This is important because coherent noise in the migration image can lead to erroneous interpretation of the subsurface geology. Calculation of the model errors for the image intensities away from the reflector image indicate that more than 6-9 times more points are required for regular grid migration to produce similar model errors as in the Halton or Hammersley migration. For synthetic data with additive random noise, our results (not shown) suggest that 5 times more traces are required by the regular grid migration relative to the Quasi-Monte Carlo method.

As a final application we examine the use of Hammersley points with 3-D post-stack migration. In the oil industry 3-D post-stack migration is much cheaper than 3-D pre-stack migration because it is applied to a much smaller subset of data; this subset is obtained from, roughly speaking, a source which shoots into only one receiver that is coincident with the source location. In other words, the four-fold summation in equation 9.1 is replaced by a double summation over the source indices. The penalty with this cheaper form of migration is a loss of imaging accuracy. Nevertheless, in our results for the French model suggest that 3-D post-stack migration using Hammersley points showed no significant efficiency gain relative to regular grid migration. This is not surprising because post-stack migration involves calculating discrete 2-D integrals rather than the discrete 4-D integrals of 3-D pre-stack migration; and the efficiency of the Quasi-Monte Carlo approach as compared to the regular grid integration increases with dimension.
Figure 9.5: Migration results for the French model in Figures 2 and 3. Image of reflector boundary along AB profile (see Figure 3) for regular grid point migration with (a) 8,100 points, (b) 32,400 points, (c) 129,600 points, and (d) 518,400 points. The desired image is similar to that in Figure 5d.
Figure 9.6: Same as Figure 5 except Halton points are used.
Figure 9.7: Same as Figure 5 except Hammersley points are used.
9.5 Discussion

We have demonstrated that for a given 3-D pre-stack data set calculated for a regularly spaced source-receiver grid, quasi-Monte Carlo methods may be used to deterministically select a subset of the data for migration. 3-D pre-stack migration of this data subset can reduce the computational work load by a factor of 5 or more while producing an image with less coherent errors. Apparently the regular grid migration coherently amplifies the errors in under-sampling the integrand while the quasi-Monte Carlo migration seems to attenuate these errors. This seems reasonable because a regular grid summation tends to promote coherent error patterns due to constructive interference while a quasi-random sampling reduces the error by preventing constructive error build-up. Despite our discovery that 3-D pre-stack migration with shifted Hammersley points is not significantly more efficient than migration with Halton points, this research suggests that 3-D Kirchhoff migration of some data sets may be much more tractable on current supercomputers if a quasi-Monte Carlo approach is used.

9.6 Acknowledgments

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9.7 References


Report 10

Conjugate Gradient Migration of Crosswell Seismic Data

Tamas Nemeth

10.1 Abstract

The conventional migration technique is the zeroth iterate of a linear iterative migration. In this paper I apply an iterative conjugate gradient (CG) migration algorithm to crosswell seismic data and compare CG migration images with images from a standard migration. The results show that the CG images are superior to those from a standard migration method. In addition, the conjugate gradient migrated sections can be used as earth models to generate synthetic seismograms that are much closer to the observed seismograms, compared to the ones obtained from the conventional migrated sections. It is shown that wrong events can be filtered out by using a regularization term. The possible applications of this technique are also discussed.

10.2 Introduction

In recent years numerous migration algorithms have been developed. There is a very extensive literature, but I mention just a few papers: Stolt (1978), Schneider (1978), Gazdag (1978), Berryhill (1979), Judson et al. (1980), Schultz and Sherdwood (1980), Larner et al. (1981), Kosloff and Baysal (1983), McMechan (1983), Baysal et al. (1983). Although they are very different in their implementations, their common denominator is that that the transpose (adjoint for the Fourier-transform based algorithms) of the forward modeling operator is applied to the observed data in order to estimate the reflectivity model of the earth. Although standard migration is the main imaging tool in exploration seismology, this approach is a very restricted one from the optimization theory point of view. According to this theory, the standard migration procedure can be viewed as the zeroth iterate in a CG or least-squares
solution to the linearized Lippmann-Schwinger equation and therefore can produce serious artifacts in the migrated section.

A more general approach to migration is to solve this linear optimization problem with not just one CG iterate but with a satisfactory number of iterations. We can also naturally introduce constraints under the general framework of optimization theory. From this general point of view the migrated section can be seen as the velocity perturbation (or reflectivity) section which minimizes the objective function. Here the objective function is defined as a linear combination of the seismogram misfit function and regularization terms.

In this paper I apply CG migration to crosswell seismic data (see also Schuster, 1993 and Nemeth, 1993). In the first section I present the theory for least-squares migration which describes the specific features of the conjugate gradient optimization scheme and the asymptotic Kirchhoff forward and inverse modeling schemes. In the next section the application of this technique is analyzed for the examples of optimizing a prestack migrated section, optimizing a number of neighboring sections together and optimizing the whole data set at a time.

### 10.3 Theory

Assume that the earth's velocity field is decoupled into a long wavelength part (background velocity) and a short wavelength part (velocity perturbations or reflectivity distribution). The background velocity governs the propagation of the transmitted waves and the reflectivity distribution generates the reflected waves (Bleistein, 1984). In this case the linearized Born-approximation to the scattered field \( p_o \) due to the slowness perturbation \( \Delta s \) about a background slowness \( s_o \) is given by:

\[
L_0 \Delta s = p_o ,
\]

where \( L_0 \) is the forward modeling operator for the background slowness, \( \Delta s \) is the slowness perturbation vector and \( p_o \) is the seismogram vector, containing the single reflected events. A seismogram, which contains only the primary reflections and excludes the direct and multiple reflection arrivals, can be generated from equation 10.1. In practice we can extract the primary reflections by dip filtering of the field data.

Equation 10.1 also can be written in integral equation form (Miller et al., 1987):

\[
p_o(x_r,t|x_s,0) = 2 \int \Delta s(x) \ G(x_r,t|x,0) \ast G(x,t|x_s,0) \ast W(t) \ dx ,
\]

where \( \ast \) denotes temporal convolution, \( G(x,t|x_s,0) \) is the 3-D acoustic Green's functions for a source at \( x_s \) and a receiver at \( x \), \( \Delta s(x) \) is the reflectivity distribution at \( x \) and \( W(t) \) is a source wavelet function. Using the first-order asymptotic Green's function

\[
G(x,t|x',0) = \frac{\delta(t - \tau_{xx'})}{A_{xx'}} ,
\]

where ...
where $A_{x'x} = |x - x'|$ is the amplitude term for a 3-D homogeneous medium, and in inhomogeneous media $A_{x'x}$ is computed from the transport equation. Substituting the Green’s function into equation 10.2 yields:

$$p_o(x_r, t|x_s, 0) = 2\int \Delta s(x) \frac{W(t - \tau_{sx} - \tau_{sx})}{A_{ss}A_{sr}} \, dx,$$

(10.4)

where $\tau_{sx}$ is the traveltime for wave propagation from the source to the scattering point and $\tau_{sr}$ is the traveltime for wave propagation from the scattering point at $x$ to the receiver. Equations 10.2 and 10.4 are also used for Kirchhoff forward modeling.

Similarly we can get the adjoint of the Kirchhoff modeling operator (after Schuster, 1993):

$$\Delta s(x) \approx 2\sum_r \int p_o(x_r, t|x_s, 0) \frac{W(t - \tau_{sx} - \tau_{sr})}{A_{sr}A_{sr}} \, dt.$$

(10.5)

Equation 10.5 can be rewritten in matrix notation as

$$\Delta s \approx L_o^T p_o,$$

(10.6)

where $L_o^T$ is the transpose to the forward modeling operator. Here we can recognize the well known fact that the migration operator is the transpose of the forward modeling operator (Claerbout, 1992).

A more general approach to finding an estimate for $\Delta s$ from equation 10.1 is to find $\Delta s$ that minimizes the parametric functional $\epsilon$:

$$\epsilon = \| W_d L_o \Delta s - W_d p_o \|^2 + \alpha \| C \Delta s - C D_{\text{appr}} \|^2.$$

(10.7)

The first term on the right-hand side of equation 10.7 is the data misfit function and the second term is a regularizing functional, where $W$ is the data weighting matrix, $C$ denotes a linear operator acting on $\Delta s$ and $\alpha$ is a Lagrange-multiplier. With this particular choice of the regularizing functional we assume some knowledge of an a priori slowness perturbation $D_{\text{appr}}$.

To find the solution that minimizes equation 10.7, calculate the first variation of the parametric functional and set it zero, resulting in

$$\Delta s = (L_o^T W_d^T W_d L_o + \alpha C^T C)^{-1} (L_o^T W_d^T W_d p_o + \alpha C^T C D_{\text{appr}}).$$

(10.8)

Equation 10.8 represents the regularized least-squares solution to equation 10.1.

10.4 Methodology of the CG Migration

The solution $\Delta s$ cannot be practically found by direct inverse matrix computation because the dimension of the matrix is too large. Instead, I use the indirect method of an iterative regularized CG technique, which is given in the following box.
\[ \Delta s_0 = 0; \quad d_\alpha = \begin{pmatrix} P_\alpha \\ C \Delta s_{\text{apr}} \end{pmatrix} \quad l_0^\alpha = \begin{pmatrix} L_\alpha \\ \alpha C \end{pmatrix}^T \begin{pmatrix} P_\alpha \\ C \Delta s_{\text{apr}} \end{pmatrix} \quad \tilde{l}_0^\alpha = l_0^\alpha \]

and for \( n = 0, 1, 2, \ldots \)

\[ f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} L_\alpha \\ C \end{pmatrix} \tilde{l}_n^\alpha \]

\[ \tilde{\kappa}_n^\alpha = \frac{||l_n^\alpha||^2}{||f_1||^2 + \sigma ||f_2||^2} \]

\[ \Delta s_{n+1} = \Delta s_n + \tilde{\kappa}_n^\alpha \tilde{l}_n^\alpha, \quad d_{n+1} = d_n - \tilde{\kappa}_n^\alpha f \]

\[ l_{n+1}^\alpha = \begin{pmatrix} L_\alpha \\ \alpha C \end{pmatrix}^T d_{n+1} \]

if \( l_{n+1}^\alpha = 0 \) then quit

\[ \beta_n^\alpha = \frac{||l_{n+1}^\alpha||^2}{||l_n^\alpha||^2} \]

\[ \tilde{l}_{n+1}^\alpha = l_{n+1}^\alpha + \beta_n^\alpha \tilde{l}_n^\alpha \]
10.5. Numerical Results

In this section I apply CG migration to three types of data, namely, a CSP shot gather, ten CSP gathers and the entire data set together. The description of the results follows this order. These data are taken from the Friendswood data set (Chen et al., 1990).

10.5.1 Single CSP Gather Migration.

I now apply the CG migration to a shot gather (source depth is 720 ft) from the Friendswood crosswell data set. Figure 10.1 depicts the observed seismogram used the tomogram obtained by traveltome tomography (Figure 10.2) for the migration velocity. Using this velocity model I performed the standard Kirchhoff migration, which is the zeroth iterate of the conjugate gradient scheme. The result is depicted in Figure 10.3, and Figure 10.4 shows the calculated seismogram using this migrated section as a reflectivity model. Analyzing the Figures 10.3 and 10.4 reflections reveals the following problems.

1. The relative amplitude of reflections is incorrect as evidenced by the mismatch between Figure 10.1 and Figure 10.4. Clearly more iterations are needed to
eliminate this artifact.

2. The migrated section in Kirchhoff migration is composed of stacked point scatterer images, resulting in the reflectivity distribution. We know also the velocity distribution from tomography. The velocity distribution can be used to constrain the reflectivity distribution, i.e. filter out the point scatterers which are associated with non-Snellian raypaths. To eliminate these scatterers we use a regularization term. Considering the basic layered structure of the velocity field I choose in this particular case $C = \frac{\partial}{\partial x}$ where the transpose will be $C^T = -\frac{\partial}{\partial x}$ (Tarantola, 1987).

Applying the iterative conjugate gradient algorithm without the regularization term yields Figure 10.5 after 20 iteration; and Figure 10.6 depicts the calculated seismogram using this migrated section as the reflectivity model. As we can see, the amplitude ratio between the different reflectors is adequately reconstructed in the least squares sense, since this reflectivity model predicts the observed data much better than the one obtained from the zeroth iteration of the CG migration. However, the scatterers due to non-Snellian scattering events are still present (Figure 10.5).

Now I use the regularization term $C = \frac{\partial}{\partial x}$. The basic problem here is the proper choice of the regularization parameter $\alpha$. I use the following adaptive regularization parameter:

$$\alpha = \alpha_0 \gamma^{\text{iteration}-1},$$

where $\alpha_0$ and $\gamma$ needs to be determined. To determine these unknowns I use the following condition. For the unregularized iterations the terms $\tilde{\kappa}_1^\alpha = \frac{||\tilde{H}\tilde{\kappa}||^2}{||L\tilde{\kappa}||}$ and $\tilde{\kappa}_2^\alpha = \frac{||\tilde{C}\tilde{\kappa}||^2}{||C\tilde{\kappa}||}$ have different orders of magnitude. I choose $\alpha_0$ from the condition that $\tilde{\kappa}_n^\alpha \approx \tilde{\kappa}_2^\alpha$ for the first iterations, which can be determined from the nonregularized (but calculated) iterations. Then I choose $\gamma$ such that $\tilde{\kappa}_n^\alpha \approx \tilde{\kappa}_1^\alpha$ for the last iterations. With this strategy I calculate 20 conjugate gradient iterations. The migrated section after 20 iteration is shown in Figure 10.7. We can see that the scatterers associated with non-Snellian raypaths are mostly eliminated. Figure 10.8 depicts the calculated seismogram using this migrated section. As expected, the calculated seismograms for the regularized case does not differ too much from the one for the unregularized case.

The next step is to stack the individual migrated sections to form a composite migrated section. Figure 10.9 depicts the composite migrated section using the standard Kirchhoff migration with no constraints applied. As we can notice, the migration ellipses are not cancelled completely after stacking 98 prestack migrated sections and the reflectors are not everywhere continuous. At some places they are overlapped with scattered energy associated with non-Snellian raypaths. Attempts were made to incorporate constraints into the migration, beyond the framework of the optimization theory. Qin (1993) analysed the artifacts in crosowell migration and applied ad hoc constraints in the migration, such as the dipping angle constraint which supresses
scattered energy associated with non-Snellian raypaths and the incidence angle constraint which eliminates the large incidence angle events.

Figure 10.10 shows the migrated section obtained by Qin (1993) using constrained Kirchhoff migration. For comparison, Figure 10.11 shows the composite migrated section using all 98 prestack migrated sections after twenty iterations of the regularized conjugate gradient algorithm. The migrated section in Figure 10.11 is superior to the one in Figure 10.9 but it is not better than the one depicted in Figure 10.10. The reason for this is that the standard constrained Kirchhoff migration successfully simulated the iterative migration by a careful choice of constraints. Another possible reason is that the parametric functional ε is not an adequate choice for multigather data, since it does not account for the inherent nonlinearity of the seismic inverse problem.

Cai (1994) found that better migration imaging was obtained by separately processing the up- and downgoing waves. Figure 10.12 depicts the composite migrated section for the upcoming waves, while Figure 10.13 shows the composite migrated section for the downgoing waves. As expected the composite migrated section in Figure 10.11 is mostly free of the non-Snellian scattered energy and as a result, the reflectors are continuous and their amplitudes are more evenly balanced. On the left hand side of the picture (near the source well) the migrated section is not continuous and several artifacts are present. This abrupt behaviour is due to preprocessing when reflections close to the direct waves were muted out or altered by filtering. Rearranging the data into common receiver gathers and preprocessing will eliminate these artifacts near the source well, meaning that the migrated image can be further improved by migrating both the source and receiver gathers and combining them together as demonstrated in Cai (1994).

10.5.2 Multiple Gather CG Migration.

In the previous example each seismogram was migrated independently to produce a prestack migrated image, then these sections were stacked to give the composite migrated section. This order of operation is based on the assumption that each CSP gather comes from an independent model but we will later combine the reflectivity models together. However, our goal is to find a model which predicts all of the observed seismograms or a subset of them. The next series of calculations carries out this idea.

First, I group every 10 neighboring CSP gathers to form a total of 10 groups. I then migrate each group of 10 gathers to find a model which predicts the seismograms of the group. Figure 10.14 shows the migrated section for the seismograms of 10 source gathers using the standard migration where the 10 sources are evenly distributed along the first 100 feet of the source well. The typical migration ellipses are dominating the upper part of the model and the amplitudes of different reflectors are misbalanced. Figure 10.15 depicts the same prestack migrated section after 20
iterations of the conjugate gradient algorithm and with constraints, described above for the previous experiments. The migration ellipses mostly disappear and the reflectivity model predicts the seismograms better than the one in Figure 10.14.

After calculating the other groups of iterative migrated sections, I stacked them together to produce the composite migrated section depicted in Figure 10.16. Although this migrated section looks very similar to the one in Figure 10.9, obtained by migrating each seismogram separately, it lacks some of the important details presented in the other migrated sections (see, for example, the dipping reflector at 430 feet in Figure 10.9.) The possible reason for this is that the exact background velocity is not known, so each seismogram produces a slightly different migrated section. Optimizing a group of seismograms together produces a common migrated section which predicts each of the seismograms in the least squares sense but it might mean the loss of some details. This phenomenon is even more pronounced in Figure 10.17 where the migrated section predicts all the seismograms at the same time. As we can see on the migrated section, the typically high energy middle part (500-1000) of the model has small amplitudes, possibly because the reflectors are stacked incoherently due to the incorrect velocity. Also notice that as we optimize more and more seismograms together, more iterations are needed to obtain an adequate result, the large energy reflectors are still present near the surface and the 500-1000-ft-depth section of the model has anomalously small amplitudes. These results suggest that the best strategy for migrating the data set with an iterative algorithm is to migrate each seismogram separately and later combine them together. However, there are still artifacts in the migrated image which demand a remedy.

The conjugate gradient algorithm can significantly enhance the amplitude-noise ratio by systematically filtering out the unwanted events and adjusting the amplitudes of the reflectors in such a way that they are consistent with the observed seismograms. However, we can obtain a good signal-to-noise ratio in the composite migrated section by combining many prestack migrated sections together. In this case the random noise is greatly diminished and the amplitude of the coherent signal is enhanced. Having stacked 98 migrated sections in the case of the Friendswood data enhances the signal-to-noise ratio of the composite migrated section compared to using a standard migration algorithm.

In the case of sparse data sets it is not always the case that the random noise is greatly diminished. To simulate this situation, I stacked every tenth prestack migrated section using both the standard and the conjugate gradient algorithms. The results are depicted on Figure 10.18 and Figure 10.19, respectively. As we can see, the migrated section using the conjugate gradient algorithm provides a much better signal-to-noise ratio.
10.6 Discussion

A conjugate gradient migration algorithm was studied, which computes a migrated section that predicts the observed seismogram in the least squares sense. Asymptotic forward and inverse (transpose) modeling methods were used for computational efficiency. We found the following advantages of this technique:

1. It is straightforward to incorporate constraints into the migration procedure. These constraints suppress many of the aperture and coherent noise artifacts seen in standard migrated images.

2. It adjusts the amplitudes of the reflectors in such a way that they will be consistent with the observed seismograms.

3. The disadvantage of the CG migration is that it requires much more CPU time compared to standard migration. Each CG iteration for the Friendswood data (98 source gathers with 96 receivers per gather) with a 121 by 651 model required about 2-3 hours on a Sparc10 station. Further accuracy can be achieved (at the expense of more CPU time) by using more accurate forward and inverse modeling algorithms.

Although this method performs well on the Friendswood data set, the migrated images are not superior to the one obtained by a constrained standard Kirchhoff migration. The reason is that for simple cases the effect of the iterative migration can be simulated with a careful choice of constraints in the standard Kirchhoff migration. In more difficult situations the iterative migration may be superior to standard migration but this is still to be demonstrated. Recent papers (Chavent, 1993) suggest that the formulation of the multigather migration as an optimization might yield a different parametric functional $\epsilon$.

Future research will include applying this technique to multimode data (PP, PS, SS, SP) as well as to CDP (poststack, prestack) data and difficult data sets. The ability of CG migration to produce amplitude balanced data suggest its use in AVO analysis. An important task is to find the right formulation of the multigather optimization problem and experiment with different parametric functionals.

And finally, there is no substitute for a better velocity model to give better migrated sections; our goal is to use this algorithm in conjunction with iterative velocity updates to achieve an optimal reconstruction of the reflectivity distribution.

10.7 Acknowledgements

I thank Exxon Production Research for providing the Friendswood data. I also thank the members of the 1993 University of Utah Modeling and Tomography Consortium for their financial support. And finally, I am grateful to Jerry Schuster, Fuhao Qin, Changxi Zhou and Wenyeng Cai for helpful discussions.
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Figure 10.1: A common source gather of the Friendswood data. Source depth is 720 ft.
Figure 10.2: The traveltime tomogram of the velocity field used for the migration.
Figure 10.3: The standard prestack migrated section.
Figure 10.4: The calculated shot gather using the standard prestack migrated section.
Figure 10.5: The prestack migrated section after 20 iterations. No regularization was applied.
Figure 10.6: The calculated shot gather of primary reflections using the prestack migrated section after 20 iterations. Compare to Figure 4.
Figure 10.7: The migrated section after 20 iterations with regularization applied. Compare to the unregularized migrated section in Figure 3.
Figure 10.8: The calculated shot gather using the regularized migrated section after 20 iterations.
Figure 10.9: The composite migrated section applying the standard Kirchhoff migration. No constraints were applied.
Figure 10.10: The composite migrated section applying standard Kirchhoff migration with constraints. (after Qin, 1993)
Figure 10.11: The composite migrated section after 20 iterations optimizing each seismogram individually. Compare to Figures 9 and 10.
Figure 10.12: The resulting upcoming wave migrated section after 20 iterations optimizing each seismogram individually.
Figure 10.13: The resulting downgoing wave migrated section after 20 iterations optimizing each seismogram individually.
Figure 10.14: The migrated section after 1 iteration for the upper 10 seismograms.
Figure 10.15: The standard migrated section after 20 iterations for the upper 10 seismograms.
Figure 10.16: The composite migrated section combining all the prestack migrated sections using a CG migration. The prestack migrated section were obtained by migrating every 10 seismograms together. Compare to Figure 11.
Figure 10.17: The composite migrated section optimizing all the seismograms together using 20 iterations of CG migration. The loss of the middle part of the migrated section is due to incoherent stacking during the iterative migration.
Figure 10.18: The composite migrated section using every tenth prestack migrated section. A standard Kirchhoff migration was used.
Figure 10.19: The composite migrated section using every tenth prestack migrated section. Twenty iterations of CG migration were applied.
Report 11

Migration and Coherency Stacking of the Friendswood Reverse VSP data

Tamas Nemeth

11.1 Abstract

In this paper I apply two new processing methods to reverse VSP data: iterative conjugate gradient migration and coherency stacking. Iterative conjugate gradient migration helps to eliminate the migration artifacts and produce a reflectivity distribution which is consistent with the observed seismogram. The second method, the coherent stacking, helps to coherently stack the prestack migrated sections to produce a better composite migrated section. Numerical results show that these procedures are capable of eliminating a fault artifact from the conventionally processed VSP migrated section.

11.2 Introduction

In many oil and gas exploration plays it is too expensive to perform a crosswell study. A cheaper alternative is a VSP or offset VSP survey, which is used to image the near borehole subsurface structure. Sometimes a variant of offset VSP, called reverse offset VSP (i.e. RVSP) is used. For an RVSP experiment the source is located in the borehole and the geophones are located on the surface. This variant is relatively inexpensive and it is feasible to extend to a 3-D RVSP survey. VSP reflections can provide high resolution images of horizons at and below the well, with a resolution comparable to that from a crosswell reflection survey.

In this paper we show that processed RVSP data set can image subsurface details comparable to the images obtained by crosswell imaging (Nemeth, 1994). Two new
processing steps are applied, iterative conjugate gradient migration and coherency stacking. Iterative conjugate gradient migration helps to eliminate the migration artifacts and produce a reflectivity distribution which is consistent with the observed seismogram. The coherent stacking procedure helps to stack the prestack migrated sections coherently to produce a better composite migrated section. The first part of this paper describes the processing sequence and the second part presents the result of processing the Friendswood RVSP data.

11.3 Methodology

In this section I describe the main processing steps in the processing sequence. The reverse VSP data were acquired near Friendswood, TX and the description of the experiment is given in Chen et al. (1990).

11.3.1 Preprocessing

The data preprocessing included the following steps:

1. Bandpass filtering (90-600 Hz) to remove both the high and low frequency noise.

2. Pick first arrival traveltimes which are to be used for direct wave removal and travelttime tomography.

3. Direct wave removal by aligning the direct waves and applying a 7-point median filter (Cai, 1993) to them.

4. Trace amplitude normalization and reflection amplitude balancing.

After processing, the resulting seismogram mainly contains the upcoming primary reflections and the outgoing multiple reflections. Figure 11.2 shows a seismogram after the preprocessing. Offset from the source well is 400 ft.

11.3.2 Background Velocity Distribution Estimation

The background velocity or migration velocity can be estimated either from the sonic log or from the direct waves by traveltime inversion. Both options have advantages and disadvantages.

The advantage of using the sonic log is that it provides an accurate velocity distribution along a vertical well. However, the arrival times in a VSP experiment are greatly influenced by the upper low velocity zone, which is usually not manifested in the sonic log. In this case an independent shallow subsurface experiment is needed
to provide this information. Having obtained the shallow subsurface velocity distribution, we still need to match the combined sonic log + shallow velocity distribution so that it will be consistent with the measured data.

Estimating the background velocity distribution from direct wave traveltime inversion always provides a velocity distribution which is somewhat consistent with the data. Figures 11.3 and 11.4 show the background velocity distributions using the sonic logs and the traveltime inversion, respectively. Below 1000 ft an average velocity layer was appended to estimate the subwell reflections. Note that the lower triangle part on the right hand side in Figure 11.4 is an extension of the surrounding left hand side velocity distribution in this direction, since the rays do not pass this area and therefore it is not updated during the inversion.

These two velocity distributions yield almost the same migrated sections. In the following sections I show the results obtained by using the velocity distribution in Figure 11.4.

11.3.3 Migration of the Reflection Seismograms

A finite-difference or Kirchhoff migration is used for VSP migration (Amundsen et al., 1993; Dillon, 1988; Dillon, 1990; Kohler and Koenig, 1986). I use both a standard and an iterative conjugate gradient Kirchhoff method for migration (Nemeth, 1994). Figure 11.5 depicts a prestack migrated section using standard Kirchhoff migration and Figure 11.6 shows the calculated seismogram based on this migrated section. Although this migrated section provides much detail about the subsurface structure, I applied an iterative conjugate gradient Kirchhoff migration with horizontal damping constraints to eliminate the elliptic migration artifacts and to enhance the reflector continuity. Figure 11.7 shows the migrated section after 20 iterations and Figure 11.8 depicts the calculated seismogram from this reflectivity distribution. Comparing Figure 11.8 to Figure 11.2 we notice that the calculated seismogram is still not matched with the observed one. The reason for this is that even after 20 iterations the solution did not converge in the least square sense because the distance from the source to the receivers varies significantly with depth in the VSP geometry.

Then I applied the iterative conjugate gradient Kirchhoff migration algorithm to the entire data set (a total of 23 receiver gathers) and obtained the prestack migrated sections which form the image cube. Figure 11.11 shows the composite migrated section by stacking all migrated shot gathers together. There are several artifacts in this section which are the result of incoherent stacking of the migrated shot gathers. For example, see the reflector images at about 300 ft. To eliminate these artifacts and enhance the coherence on common image gathers, a residual coherent stacking procedure was applied to the prestack migrated sections.
11.3.4 Coherent Stacking of the Prestack Migrated Sections

Since the migration velocity distribution is not the correct one, the migrated reflectivities on the common image gathers will not be aligned. Figure 11.9 illustrates this effect. For example, the reflector images at about 300 ft are stacked incoherently, resulting in only one peak on the output trace, instead of two. A residual algorithm is needed to correct these mispositionings. The residual moveout correction algorithm for the VSP geometry is described in the Appendix. Applying this algorithm to the image cube we obtain the result in Figure 11.10. The reflector images in Figure 11.10 are aligned and they result in coherent stacking.

Now I apply the coherent stacking to the entire image cube. The resulting migrated section is depicted in Figure 11.12. Comparing it to Figure 11.11, we notice that many of the artifacts at about 300 ft are corrected and the reflectors correspond to the synthetic reflectivity model obtained from the sonic log.

11.4 Discussion

I have presented a description of the reverse VSP data processing. In addition to the conventional processing sequence (i.e. preprocessing, background velocity distribution estimation, migration) I added an extra step, the coherent stacking procedure to the processing sequence.

Numerical results show that after coherent stacking the migrated images can be stacked coherently to enhance the quality of the composite migrated section and a fault artifact was removed from the composite migrated section.

Future work should be aimed at finding a better background velocity distribution by combining the sonic log and travelt ime tomogram information and also to include iterative velocity analysis techniques into the processing sequence in order to increase the coherency in the common image gathers.

11.5 Acknowledgements

I thank Exxon Production Research for providing the Friendswood RVSP data. I also thank the members of the 1993 University of Utah Modeling and Tomography Consortium for their financial support. And finally, I am grateful to Jerry Schuster, Fuhao Qin, Changxi Zhou and Wenying Cai for helpful discussions.

11.6 References

11.7 Appendix

In this section the basic formula for the VSP residual moveout equation is given. The residual moveout equation is the basis for coherent stacking of prestack migrated sections.

For simplicity, consider the two-layer geometry depicted in Figure 11.1. The travelt ime from the source to the receiver is given by

\[ t^2 = \frac{x^2}{V^2} + \frac{x^2}{(x - x_1)^2 V^2}, \quad (11.1) \]

where \( t \) is the travelt ime from the source to the receiver, \( x \) is the offset between the well and the source, \( x_1 \) is the offset between the well and the reflection point, \( z \) is the reflector depth and \( V \) is the average velocity.

The travelt ime \( t \) can be also satisfied with the migration velocity \( V_m \) and the migrated reflector depth \( z_m \). Equating the two variants of equation 11.1 and expressing \( z_m \) yields:

\[ z_m^2 = (\gamma^2 - 1)(x - x_1)^2 + \gamma^2 z^2, \quad (11.2) \]
Figure 11.1: The geometry of the VSP experiment where S is the source, P is the scattering point on the reflector, R is the receiver, x is the source-well offset, \( x_1 \) is the scattering point-borehole offset and z is the reflector depth.

where \( \gamma \) is the ratio between the migration and the correct velocities \( \gamma = \frac{V_p}{V} \). Equation 11.2 is the VSP residual normal moveout equation and it describes a hyperbola which is a function of the source offset in common image gathers.

The mispositioning error of the reflector images on the common image gathers is described by the residual full prestack migration. It can be split into three parts, namely, into the residual migration, the residual moveout correction and the residual dip moveout correction. From these three procedures the residual normal moveout correction is the most important and mostly it controls the coherent stacking. However, the effects of both the residual migration and residual dip moveout are to be studied. The validity of the approximation of the residual full prestack migration with the above mentioned three residual processes also should be addressed.
Figure 11.2: A processed common receiver gather of the Friendswood RVSP data. Source offset is 400 ft.
Figure 11.3: The velocity distribution based on the sonic log.
Figure 11.4: The velocity distribution obtained from the traveltime tomogram.
Figure 11.5: A standard prestack migrated section. The source offset is 400 ft.
Figure 11.6: The calculated shot gather using the standard prestack migrated section in the previous figure.
Figure 11.7: The prestack migrated section after 20 iterations of conjugate gradient migration, using horizontal damping constraints. Compare to Figure 3.
Figure 11.8: The calculated shot gather of primary reflections using the prestack migrated section in the previous figure. Compare to Figure 4.
Figure 11.9: A common image gather (source offset is 150 ft). The rightmost two traces show the stacked trace. The two strong reflector images in the window are stacked incoherently, resulting in the cancellation of one of the images. Traces 1-12 are not shown, since they are closer to the well than the source so they do not contribute to the reflector images at this offset.
Figure 11.10: A common image gather (source offset is 150 ft) after the residual correction. The rightmost two traces show the stacked trace after correction, while traces 31-32 show the stacked trace before the correction. The reflector images in the window are stacked coherently, resulting in large amplitude images on the output stacked trace.
Figure 11.11: The resulting migrated section without the residual correction. The events in the window are stacked incoherently and they give the impression of a fault.
Figure 11.12: The resulting migrated section after the residual correction. The events in the window are stacked coherently and they show a layered structure which is consistent with the results of crosswell migration (Nemeth, 1994) and waveform inversion (Zhou et al., 1993).
Part III

Data Processing
Report 12

Maximum Energy Stacking of Migrated Cross-well Seismic Sections

Fuhao Qin

12.1 Abstract

I propose a stacking method that can produce better migration images than obtained by standard migration. In the proposed stacking method, each trace in the pre-stack migrated sections is shifted slightly up or down to achieve maximum cross-correlations with the corresponding trace in a master section. The master section can be a combination of parts from different migrated CSG sections or simply be the stacked migrated image. The shifted migrated sections are stacked together, so that the stacked section can then be used as a new master section to further improve the final image. The new stacking scheme can alleviate the migration inaccuracy caused by local errors in the migration velocity. The drawback, however, is that a good master section is required as a starting model. If the master image is far from the true model then the result might be an incorrect migrated image.

12.2 Introduction

Cross-well seismic migration, just like all other migration methods, is sensitive to the migration velocity. Although first arrival travelt ime tomograms or well logs provide a good migration velocity for cross-well migration, there are still many artifacts in the migrated image. These artifacts might be caused by bad traveltime picks and limited ray angular coverage. Inaccurate migration velocities will cause reflections from different shot and receiver combinations to be migrated to their incorrect spatial locations. This will cause the stacked image to be somewhat blurred.
The best solution to the above problem is, of course, to find a better velocity by either waveform inversion (Zhou et al., 1993) or some kind of reflection traveltime migration. However, they are either infeasible or not cost efficient.

In this report, we propose a maximum energy stacking method to stack migrated CSG (CRG) sections. This stacking method is aimed to modify the migrated CSG sections that are slightly affected by the local inaccuracy of the migration velocity, namely, the depth shift of migrated CSG images from their "actual" position (master section).

The "actual" image position is not readily available but, an approximation to it can always be obtained by a combination of the reliable pieces of the image from different CSG sections or, simply, an ordinary stacking of all the CSG sections.

Each trace in the CSG sections will be compared with the corresponding trace in the master migrated image. The optimum shift is found by finding the maximum cross-correlation between the two. Usually, the whole trace should not be corrected with just a single depth shift, the trace should be separated into several parts each with its own shift. The shifted CSG sections are stacked to get the updated master section and this procedure can be iterated until convergence. Iteration stops when there is no obvious improvement over the previous iteration result.

12.3 Procedure

- Step 1. Stack all migrated CSG sections to obtain the first iterate of the master migrated image.

- Step 2. Process the master image to remove obvious migration artifacts or any recognizable noise. The processing may include gain control, band pass filtering, f-k filtering etc.

- Step 3. Crosscorrelate each trace in the CSG sections with its corresponding trace in the master section to find the best shift distance for it. The trace is divided into one or two segments for the first couple of iterations to stabilize the stacking and may be further divided into more segments for later iterations.

- Step 4. Stack all the shifted sections to get the master image at the next iteration.

- Step 5. Repeat steps 2-4 until a satisfactory image is obtained or result stalls to improve.

12.4 Numerical Results

In this section, we use the Friendswood data set (Chen, et al., 1990) as an example to examine how the migration result is improved by the stacking procedure.
12.4. NUMERICAL RESULTS

All 96 CSG gathers are migrated by the constrained Kirchhoff integral migration method (Qin and Schuster, 1993). Figure 12.1 shows a migrated CSG section with the source located at a depth of 580 ft in the left well. Figure 12.2 shows the result after stacking all of the CSG sections. Figure 12.2 shows better interface continuity and looks more geologic. However, we can see several good layer images on the left part of Figure 12.1 near the 600 ft mark which we can not see or at least can not see clearly in Figure 12.2. Parts of the pre-stack CSG images disappear in the stacking process because the same image in different prestacked CSG sections are not all in phase. Upon stacking they almost cancel each other out. That is exactly the reason why we need to do the maximum energy stacking.

At the first iteration, the ordinary stacked section in Figure 12.2 was used as the master migrated section (after bandpass filtering and AGC). Each trace was divided into 2 segments and they allowed to shift up and down in a plus and minus 20 ft depth range. The optimum shift was found based on the cross-correlation with the master trace. All the sections were stacked after the shifts are made and Figure 12.3 shows the stacked result. It can be seen that the image quality is improved compared to the original result (Figure 12.2), e.g. the images near the depth of 600 ft.

To see if the images can be further improved, I did another 3 iterations where each iteration used the stacked section of the previous iteration as an updated master section. The shift range was still plus and minus 20 ft. Each trace was divided into 6, 15 and 30 segments for iteration numbers 2, 3 and 4, respectively. Figure 12.4 shows the final result. Compared to the first iteration result, we can see that some reflector images are further enhanced, especially between the depth range of 400 ft and 600 ft.

To further evaluate the effect of the stacking technique, Figures 12.5 and 12.6 show common image gathers at the lateral offset of 545 ft from the source well before and after the shift. A common image gather is a collection of migrated traces sharing the same spatial location. Events in Figure 12.6 are much better aligned than those in Figure 12.5. That is why it provides a better stacking image than the original unshifted traces.

It is very interesting to note that, in Figures 12.2 through 12.4, the layer images near the left edge (source well) are not as continuous as those near the right edge (receiver well). The image near the left edge is also very noisy. The reason is that the data processing is performed in the CSG domain where the reflection events from the source well have moveout similar to that from direct waves; thus reflections can easily be destroyed by the processing that removes first arrivals through f-K filtering or median filtering. The reflections that originate near the source well can be further weakened by the f-k filtering to separate up- and down-going waves. Thus, a more reasonable way to do the migration might be to process the data in the CSG domain, generate migrated CSG sections near the receiver well, and then process the data set in the CRG domain to migrate CRG sections near the source well. The maximum energy stacking method can then be applied to all those unstacked migrated sections to get the optimal image.
Throughout the report, no f-k filter was used during the process to avoid false structures introduced by the f-k filter. However, for some very noisy data, the f-k filter may have to be used to get a better master section.

12.5 Discussion

My numerical results show that the maximum energy stacking method can enhance the migrated image and alleviate the effects of some local migration velocity errors. It seems to preserve the images contained in the master sections but does not introduce long wavelength changes to the images. Therefore, some kind of inversion is still necessary to modify the migration velocity and update the migration image.

It is obvious that the shifted distance of each trace contains information about the actual velocity distribution. Future work will explore the possibility of inversion based on the maximum energy stacking technique.

12.6 References


Figure 12.1: Migrated CSG section with a source at depth 600 ft
Figure 12.2: Stacked result of all the CSG sections
Figure 12.3: Result of the first iteration of the maximum energy stacking.
Figure 12.4: Result of the fourth iteration of the maximum energy stacking.
Figure 12.5: Common image gather for the image at an offset of 100 ft before shift.
Figure 12.6: Common image gather for the image at an offset of 100 ft after shift.
Report 13

Coherent Stacking of Prestack Migrated Sections

Tamas Nemeth

13.1 Abstract

I present a coherent stacking method that estimates and partially corrects the mispositioning errors of migrated reflections in common image gathers. Such errors can result in an incoherent summation of images and a subsequent deterioration in the quality of the stacked migrated section. The technique is based on the migration residual moveout equation and it sums up the prestack images along lines of maximum coherency. Numerical results show that the coherent stacking can partially correct the mispositioning errors of images and, therefore, it can be a useful tool in subsurface imaging.

13.2 Introduction

Crosswell seismic migration is usually performed by applying Kirchhoff or reverse time migration to the observed data in order to obtain a depth migrated section for each individual shot gather. This is a prestack depth migration process and as such, it is very sensitive to errors in the migration parameters, such as the background velocity field.

On the other hand, the acquired data contains redundant information about the reflector geometry by virtue of the many independent shot gathers. This redundancy allows us to study (and adjust) the mispositioning errors of reflectors caused by the use of incorrect migration parameters.

Prestack depth migration of different shot gathers allows for the construction of many independent migrated sections. We can rearrange these migrated shot gathers into common image gathers (CIGs) which contain the images of the subsurface points.
from different migrated sections (Figure 13.1). These CIGs provide a useful means to access the reliability of the migrated images. If the migration is applied correctly, the images from the same reflector point will be located at the same depth on the CIGs. If this is not the case, then either the migration parameters (mostly the applied velocity distribution) were incorrect or some algorithmic errors occurred. An example of the latter is the failure of the eikonal equation in computing the reflection traveltimes at large incidence angles.

We can adjust the migration parameters by estimating the reflector positioning errors on the CIG; this leads to either an iterative prestack depth migration + velocity analysis procedure or an ad hoc procedure to correct these mispositioning errors. The latter involves moving the corresponding images to a 'master' depth which is supposedly close to the correct depth. This correction procedure is based on the approximation of the first iterate of the migration velocity analysis (Nemeth, 1993), i.e. on the migration residual moveout equation.

In this paper a correction procedure I call coherent stacking is described and applied to crosswell seismic data. First, the behavior of image points on the CIG is studied then the coherent stacking correction is applied to the Friendswood data (Chen et al., 1989). The coherent stacking is based on the global or long wavelength variations of the image point positions on CIGs. A companion paper (Qin, 1994) describes a technique he calls the maximum crosscorrelation stacking based on the local variations of the CIG image point positions.
Figure 13.2: CIG as a function of $\beta$ for low velocity estimates. (a) CIG near the receiver well; (b) CIG in the middle of the source-receiver well distance; (c) CIG near the source well.

13.3 Theory

The perturbation of CIG image points with respect to changes in the background velocity was studied by Nemeth (1993). For a homogeneous velocity the mispositioned reflector depth $z_m$ is described by the migration residual moveout equation:

$$z_m = z_s + \sqrt{(\gamma^2 - 1) \frac{x^2}{(\beta + 1)^2} + \gamma^2 (z - z_s)^2},$$

(13.1)

where $z_s$ is the source depth, $z$ is the reflector depth, $x$ is the source-receiver offset, $\gamma$ is the ratio between the migration and correct average velocities ($\gamma = \frac{V_m}{V}$) and $\beta$ is the ratio between the distance of the reflection point to the receiver well to the distance of the reflection point to the source well.

The perturbations of the velocity distribution with respect to the smooth background velocity distribution result in second-order effects of the migration residual moveout equation (equation 13.1) and therefore their effects can be neglected for moderate velocity variations. Thus equation 13.1 represents the first-order (global) mispositioning errors in the reflector positions and it is used for the coherent stacking. The perturbation of the CIG image points with respect to changes in the background velocity for a two-layer homogeneous model is demonstrated in Figures 13.2 and 13.3. For a detailed description see Nemeth (1993).

The next figures show the CIGs for the Friendswood data. Figure 13.4 depicts a CIG near the source well. Migrated reflections in the upper triangle are formed from the downgoing field and migrated reflections in the lower triangle are from the
Figure 13.3: CIG as a function of $\beta$ for high velocity estimates. (a) CIG near the receiver well; (b) CIG in the middle of the source-receiver well distance; (c) CIG near the source well.

migrated upcoming field. Any migrated reflections that coincide with the diagonal line from (0,0) to (100,1000) are associated with source-receiver pairs at the same depth level. The migrated reflections near the source well are concentrated near the diagonal line and are along straight lines. The curves near the 300 ft and 600 ft depth levels close to the diagonal line are large incidence angle artifacts from the eikonal equation. The trace at the right hand side shows the output trace after stacking the upcoming waves.

Figure 13.5 shows a CIG located midway between the source and receiver wells after applying standard Kirchhoff migration; Figure 13.6 shows the same CIG after applying iterative conjugate gradient Kirchhoff migration (Nemeth, 1994). The reflections in Figure 13.6 show better continuity, amplitude balance and vertical spatial resolution. Therefore, iterative migration is used in this paper to generate the CIG’s. Figure 13.6 also reveals that the migrated reflections are mostly continuous in each individual migrated section. Also the events are along straight lines, except near the separation line where artifacts from the the eikonal equation are extant.

Figure 13.7 depicts a CIG close to the receiver hole. The migrated reflections behave similarly to those in Figure 13.6. The curved reflector image near the 1000 ft depth level is caused by the use of an incorrect migration velocity. Figure 13.8 shows a CIG near the receiver well. The reflector images are continuous but they are not necessarily along straight lines and this curvature can be due both to eikonal equation artifacts and incorrect velocities.

The CIG images in Figures 13.4-13.8 show features very similar to those in Figures 13.2 and 13.3 which show that the use of equation 13.1 is robust and allows the use of mispositioning corrections.
Figure 13.4: A common image gather near the source well. (Offset from the source well is 150 ft.) The rightmost 2 traces represent the stacked trace for the upcoming waves. The diagonal line (0,0)-(100,1000) is the upcoming-downdgoing separation line between the upcoming and downdgoing migrated events. Reflector images above this line are due to the downdgoing waves and reflector images below this line are due to the upcoming waves.
Figure 13.5: A common image gather at the midpoint between the source and receiver wells, offset from the source well is 300 ft. A standard Kirchhoff migration was applied and the rightmost 2 traces represent the stacked trace for the upcoming migrated images.
Figure 13.6: The same as the previous figure except an iterative Kirchhoff migration is used. The reflector point images, shown in the window, have better vertical spatial resolution than the ones close to the separation line.
Figure 13.7: A common image gather near the receiver well (offset is 450 ft). The reflector images above the diagonal with coordinates (0,0)-(100-1000) are from the downgoing waves while reflector images below the diagonal line are from the upcoming waves. The rightmost 2 traces represent the stacked trace computed by stacking the migrated upcoming waves.
Figure 13.8: A common image gather near the receiver well (offset is 550 ft). The rightmost 2 traces represent the stacked trace for the migrated upcoming waves. The curved reflector images shown in the windows are stacked incoherently.
Using the migration residual moveout equation, the migrated CIG images near the source well and in the middle of the model are along straight lines; The depth level of the image points belonging to these lines converges to the correct depth level as the source approaches the reflector. Reflector images near the receiver well for high velocity estimates are also along straight lines and the straight line points toward the correct depth of the image point. For low velocity estimates near the receiver the images become curved and the depth level of the image points diverges from the correct depth level as the source approaches the reflector, away from the correct depth. This simplified scheme (which is an approximate solution of a general velocity analysis algorithm) is used to construct a mispositioning correction algorithm called coherent stacking. The algorithm works as follows.

1. Rearrange the prestack migrated sections into CIG's.

2. Stack the events along the straight lines of maximum coherency.

3. Place the stacked value at the depth determined by the master trace. This depth is determined by the intersection of the line of maximum coherency with the diagonal line that separates the up- and down-images.

4. Apply second-order curve fitting corrections to straight line fitting if necessary (near the receiver well).

### 13.4 Numerical results

The coherent stacking method was tested on the Friendswood data (Chen et al., 1989). For the migration I used a smoothed version of the velocity distribution (Figure 13.9) obtained by Zhou et al. (1993). An iterative Kirchhoff migration was used (Nemeth, 1994) with 20 iterations that reconstructed a CIG image that was more detailed than the one obtained by a standard Kirchhoff algorithm.

The reflector images on the CIG's are continuous, mostly horizontal but in many cases dipping, resulting in destructive stacking. For example, in Figure 13.7 the reflector image for the upcoming waves at 800 ft is slightly dipping downward away from the separation line. Also, curving events are summed up incoherently in many cases, resulting in mispositioning and the loss of resolution. For example, see the windowed reflector images in Figure 13.8 at about 1000 ft.

The coherent stacking technique was tested to correct the mispositioning errors in Figure 13.8. Figure 13.10 shows the CIG after the linear correction for the upcoming waves. Note that the dipping events at 800 ft are now horizontal. The location of the maximum signal amplitude is slightly changed and the output trace (rightmost 2 traces) corresponds to the reflectors much better than the one on the original output trace (traces 106-107). Also we notice that the curving events at 1000 ft are still not aligned. To align these events, we apply a correction to the best fit quadratic
Figure 13.9: The background velocity field used for migration.
line. The result is depicted in Figure 13.11. Notice that the events at 1000 ft are aligned with one another and the spatial resolution of the output trace (rightmost 2 traces) is better than the one in the original output trace (traces 106-107) because of coherent stacking.

Now I apply the coherent stacking to the entire Friendswood data set. Figure 13.12 shows the original migrated section using all seismograms. An AGC was applied to enhance the amplitudes near the flanks. Note that the incoherent stacking of events between 500-1000 ft destroys the image of the presumably horizontal reflectors.

I applied the method separately to the upcoming and downgoing sections, then I stacked them. The result is shown in Figure 13.13. Notice that now the events between 500-1000 ft are mostly horizontal and continuous. It is also possible to use different subsets of the prestack migration data to enhance the resolution. As we can see in Figures 13.6 and 13.7, images near the separation line have less horizontal continuity and vertical resolution than far away from it. This is explained by the change in migrated wave shape at large incidence angles and from the phenomenon of wavelet stretching during migration. So, one might want to stack only the events which are further away from the separation line, i.e., events with small incidence angles. Figure 13.14 shows a portion of the upcoming migrated section using small incidence angles. For reflection points near the source well there are no reflections with small incidence angles. As we can see, the vertical resolution is much enhanced and several reflectors appeared which fill in the space in Figure 13.13. Also some reflectors are not continuous even in this picture, for example the one at about 650 ft. This means that the spatial resolution is still insufficient to recover that part of the reflector.

13.5 Discussion

A technique I call coherent stacking was applied to align dipping images in the CIGs so that they could be summed coherently. The technique is based on the first iterate of the migration velocity analysis technique (i.e., the migration residual moveout equation) which describes the global (robust) features of the reflector image mispositioning curve in the CIGs.

The numerical results show that after correction the migrated images can be stacked coherently to significantly enhance the quality of the composite migrated section. In this way the mispositioning due to incorrect velocities and algorithmical inaccuracies can be partially corrected. Different subsets of the prestack migrated sections contain images with different resolution and quality. Choosing a subset with higher resolution and better quality for stacking can enhance the quality of the composite migrated section.

The coherent stacking method is based on the robust features of the reflector image positioning curves in the CIGs. For the Friendswood data this robustness holds true, but there might be cases when the maximum coherency line is very different from
13.6 ACKNOWLEDGEMENTS

a straight line or from a second-order curve. In this situation the extracted average information can be inadequate.

Future research will include making this technique more robust with respect to mispositioning errors in the CIGs and combine it with a local correlation stacking technique.

13.6 Acknowledgements

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13.7 References


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Figure 13.10: A common image gather near the receiver well (offset is 550 ft) after linear correction of the events. The rightmost 2 traces represent the stacked trace after the linear correction while traces 106-107 display the stacked trace before the correction. The reflector image shown in the upper window is stacked coherently, but the reflector image in the lower window is still stacked incoherently.
Figure 13.11: A common image gather near the receiver well (offset is 550 ft) after nonlinear correction of the migrated events. The rightmost 2 traces represent the stacked trace after the nonlinear correction while traces 106-107 display the stacked trace before the correction. The reflector image in the lower window is stacked coherently.
Figure 13.12: The original migrated section using both the upcoming and downgoing waves. The reflectors shown in the window are stacked incoherently, resulting in a pinchout-like structure.
Figure 13.13: The corrected migrated section using both the upcoming and downgoing waves. The reflectors shown in the window are stacked coherently, recovering a horizontally layered structure.
Figure 13.14: The corrected migrated section using the upcoming reflected waves that have small incidence angles. The migrated section is overlapped with a synthetic reflectivity section calculated from sonic log. The spatial resolution of this migrated section is comparable to the one obtained from the sonic log.
Report 14

Wavelet Filtering of Tube and Direct Waves in Seismic Data

Xu Ji

14.1 Abstract

Schuster and Sun (1993) used wavelet filtering to eliminate coherent events in cross-well data, and for the Phillips data set it was shown that wavelet filtering produced less artifacts than median filtering. In this report, we use this wavelet filtering to eliminate tube waves and direct waves in the McElroy crosswell data and compare its performance with that of a 7-point median filter. My results are consistent with those from the Phillips data.

14.2 Introduction to Wavelet Theory

A discrete wavelet can be represented in the following form:

$$\varphi_{jk} = 2^{j-1/2} \varphi(2^{j-1} x - k),$$  \hfill (14.1)

where $j$ is the dilation parameter, and $k$ is the translation parameter. The wavelet transform of a signal $f(x)$ is:

$$\begin{align*}
    f(x) & = \sum_j c_j^k \varphi_{jk}(x) \\
    c_j^k & = < f(x), \varphi_{jk}(x) >,
\end{align*}$$  \hfill (14.2)

where $c_j^k$ is the wavelet transform coefficient of $f(x)$. An important quality of a wavelet function is its dilation property, and this make it a useful tool in decomposing a signal into different resolution (frequency) scales.

Suppose we have a signal $f(x)$ at its highest resolution $f_N$, then it can be decomposed in the following way:

$$f_N = f_{N-1} + e_{N-1},$$  \hfill (14.3)
where \( f_{N-1} \) can be taken as the low frequency part of \( f_N \), and \( e_{N-1} \) the high frequency part. By repeating this process, we have:

\[
f_N = e_{N-1} + e_{N-2} + e_{N-3} + \ldots + e_{N-M} + f_{N-M}.
\] (14.4)

Equation (4) is a wavelet decomposition, where \( f_{N-M} \) is the desired lowest level of resolution. The signal can be reconstructed by recursively finding \( e_i \) from equation (3). The functions \( f_j \) and \( e_j(j = N, \ldots, N - M) \) have the following representations:

\[
\begin{align*}
  f_j(x) &= \sum_k c_j^k \phi(2^j x - k) \\
  e_j(x) &= \sum_k d_j^k \varphi(2^j x - k),
\end{align*}
\] (14.5)

where \( \phi(2^j x - k) \) is called the scaling function, and \( \varphi(2^j x - k) \) is called the wavelet function. As both \( f_{j-1} \) and \( e_{j-1} \) are generated from \( f_j \), which has a representation in terms of \( \phi(x) \), so we should have:

\[
\begin{align*}
  \phi(x) &= 2 \sum_k h_k \phi(2x - k) \\
  \varphi(x) &= 2 \sum_k g_k \varphi(2x - k),
\end{align*}
\] (14.6)

where \( g_k = (-1)^k \tilde{h}_{1-k} \). Equation 6 forms the reconstruction relations, where \( h_k \) is some low pass filter to get \( f_i(i = N, \ldots, N - M) \), and \( g_k \) is some high pass filter to get \( e_i(i = N, \ldots, N - M) \). As we have \( f_j = f_{j-1} + e_{j-1} \), so we should have:

\[
\begin{align*}
  \phi(2x) &= \sum_k [h_{-2k} \phi(x - k) + g_{-2k} \varphi(x - k)] \\
  \phi(2x - 1) &= \sum_k [h_{1-2k} \phi(x - k) + g_{1-2k} \varphi(x - k)],
\end{align*}
\] (14.7)

which can be written as one equation:

\[
\phi(2x - l) = \sum_k [h_{l-2k} \phi(x - k) + g_{l-2k} \varphi(x - k)],
\] (14.8)

where \( l = 0, 1 \). This is called the decomposition relation.

Wavelet filtering is a kind of multiresolution analysis, which means the signal is decomposed into different resolution (frequency) scales, and then reconstructed from the desired frequency components. From the discussion above, the decomposition and reconstruction algorithm is the following:

- The decomposition algorithm is:

\[
\begin{align*}
  c_{j-1}^k &= \sum_i h_{i-2k} c_j^i \\
  d_{j-1}^k &= \sum_i g_{i-2k} c_j^i = \sum_i (-1)^i \tilde{h}_{1-i-2k} c_j^i,
\end{align*}
\] (14.9)
and the graph can be shown as:

\[
\begin{align*}
    c_N & \rightarrow d_{N-1} \rightarrow d_{N-2} \rightarrow \cdots \rightarrow d_{N-M} \\
    & \rightarrow c_{N-1} \rightarrow c_{N-2} \rightarrow \cdots \rightarrow c_{N-M},
\end{align*}
\tag{14.10}
\]

It can be seen that both \(c_{j-1}\) and \(d_{j-1}\) are obtained by moving average schemes, using decomposition sequences \(h_i\) and \(g_i\) as weights, with the exception that these moving averages are sampled only at the even integers, which is also called downsampling. So, this process is a moving average followed by a downsampling at even indices.

- The reconstruction is based on the desired frequency part. The algorithm is just a reverse of the decomposition:

\[
\begin{align*}
    c_j^k &= \sum_i [h_{k-2i} c_{j+1}^i + g_{k-2i} d_{j-1}^i] = \sum_i [h_{k-2i} c_{j+1}^i + (-1)^i h_{1-(k-2i)} d_{j-1}^i], \\
    c_N &\leftarrow d_{N-1} \leftarrow d_{N-M+1} \leftarrow d_{N-M},
\end{align*}
\tag{14.11}
\]

Again, \(c_j\) is obtained from \(c_{j-1}\) and \(d_{j-1}\) by two moving averages, using the reconstruction sequences as weights, with the exception that an upsampling is required before the moving averages are performed.

- In this report, we use a B4 wavelet transform, where the decomposition is:

\[
\begin{align*}
    \{f(j, k) &= \sum_i h(i - 2j) f(i, k + 1) \\
    c(j, k) &= \sum_i (-1)^i h(1 - i + 2j) f(i, k + 1),
\end{align*}
\tag{14.13}
\]

where \(f(j, k)\) is the signal at the \(k\)th level resolution, which can be derived from the \(k - 1\)th level \(f(j, k-1)\). The signal at the highest level \((k = 0)\) of resolution is the input data itself, and \(e(j, k)\) is the \(k\)th level detail (high frequency part) of the \(k - 1\)th signal part. This process is performed for \(k = -1, -2, -3, \ldots\), until the desired level of resolution is reached or until \(k\) is somewhat smaller than \(-\log_2(N)\), where \(N\) is the number of points in the input data.

The reconstruction process using the same wavelet is:

\[
f(j, k) = \sum_i \{2h(i - 2j) f(i, k + 1) + 2(-1)^i h(1 - i + 2j) e(i, k + 1)\},
\tag{14.14}
\]
This reconstruction process is continued until \( k = 0 \). In the lowest resolution, \( f_i \) is set to zero, so that the constructed signal will lack the low frequency components that are present in the input data. This is what is called wavelet filtering.

### 14.3 Wavelet Filtering of Coherent Events

To use a wavelet transform to filter out a coherent event (such as tube wave or direct waves), we go through the following steps:

- **Lineup the event**: all the traces are shifted in time so that, say the tube waves, are time aligned with each other. That is, the events occur at the same time from trace to trace.

- **Window the events and apply decomposition**: The events are windowed both in time and in the trace coordinate, and the wavelet decompositon is applied to the windowed data to get the wavelet coefficients for different levels \( n = -1, -2, -3, \ldots, -M \), where there are \( 2^N \) traces, and \( M < N \). \( M \) is chosen so that the low frequency components below this level belong to the coherent events that need to be eliminated.

- **Filtering and reconstruction**: Zero out \( f_{N-M} \) so that the coefficients lower than this level primarily represent the contribution from the coherent arrivals. Experience suggests that \( M \) equals to 3 or 2 usually gives better results.

- **Dealign in time**: dealign the filtered data by applying the same time shift as before, except in opposite polarity.

- **Separate upgoing and downgoing waves**: After removing the direct waves and tube waves by wavelet and median filters, I separated the up-going and down-going waves by an FK filter.

### 14.4 Numerical Results

We apply the B4 wavelet filter and a 7-point median filter to eliminate tube and direct waves in crosswell data. The test data is a source gather from the McElroy data set,
14.5. CONCLUSION

which is shown in Figure 14.1.

- Tube wave suppression: We use both a B4 wavelet transform and a median filter to eliminate the tube waves. The tube wave velocity is about 4500 ft./s. Among several wavelet offset levels, it is found that \( n = -3 \) or \(-2\) usually gives better results (Figure 14.2 (b) and (c)). The tube waves filtered by the median filter is shown in Figure 14.2 (a). Figure 14.3 (a) gives an expanded view of the tube waves in the original data, and Figures 14.3 (b) (c) (d) give an expanded view of Figures 14.2 (b) (c) ,and (a). The results show that a wavelet filter generates fewer artifacts than the median filter. And elimination of tube waves by a wavelet filter makes other coherent events (such as reflections) more pronounced.

- Direct wave suppression: Both wavelet and median filters are used to remove the direct waves in the same gather. The wavelet results are shown in Figure 14.4 (b) and (c), while the median filter result is shown in Figure 14.4 (a). Again, by wavelet filtering, the removal of direct waves enhance the reflections and generate fewer artifacts, while the median filter result has some distortion in the reflections.

- Upgoing and downgoing waves separation: We use an FK filter to separate upgoing and downgoing waves after wavelet and median filtering. Figures 14.5 (a) (b) and (c) show the upgoing waves after applying a 7-point median filter, and wavelet filter at levels 3 and 2, respectively. Figures 14.6 (a) (b) and (c) show the downgoing waves after applying a 7-point median filter, and wavelet filter at levels 3 and 2, respectively.

14.5 Conclusion

Wavelet filtering is used to eliminate tube and direct waves in the McElroy crosswell data, and was shown to generate fewer artifacts than generated by a 7-point median filter. The disadvantage of wavelet filtering is that it requires much more CPU time than required by a median filter.

14.6 Future Work

- Compress the wavelet code and make the wavelet filtering faster.
- 2-D wavelet transform to separate upgoing and downgoing waves.
14.7 Acknowledgements

I would like to thank W. Cai and C. Zhou for their help.

14.8 References


Figure 14.1: A source gather from the McElroy data set.
Figure 14.2: (a). Tube waves removed by using a 7-point median filter.
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Report 15

Numerical Verification of Converting 3-D Data to 2-D Data

Jinlong Xu

15.1 Abstract

A filtering operation is applied to 3-D data to convert them to 2-D data. The 3-D data are calculated from the Exxon Friendswood model and compared to synthetic 2-D data. The results show that the conversion is almost error free.

15.2 Introduction

2-D waveform tomography methods require that the 3-D data be converted to 2-D data. Therefore, the 3-D to 2-D filtering operation is needed. How accurate is this filtering operation? This paper demonstrate that it is very accurate for Exxon Friendswood model.

15.3 Theory

Assume that seismic waves honor the acoustic wave equation.

\[
(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})p = -f,
\]

(15.1)

where the \( p \) is pressure and the \( f \) is a source function.

Taken the Fourier transform of both sides, equation (15.1) becomes the Helmholtz equation below

\[
(\nabla^2 + k^2)P = -F,
\]

(15.2)
\[ P = \int_{-\infty}^{+\infty} pt e^{-i\omega t} dt, \quad (15.3) \]
\[ F = \int_{-\infty}^{+\infty} ft e^{-i\omega t} dt, \quad (15.4) \]

where \( k^2 = \omega^2/c^2 \).

The Green's function in the frequency domain is the solution of the following equation
\[ (\nabla^2 + k^2)G = -\delta(x - x_0), \quad (15.5) \]

where the solution of equation (15.2) can be expressed as
\[ P = GF \quad (15.6) \]

The \( P^{(2)} \) and \( G^{(2)} \) respectively denote solutions of equations (15.2) and (15.5) in two dimensions; and \( P^{(3)} \) and \( G^{(3)} \) respectively denote solutions of equations (15.2) and (15.5) in three dimensions. Thus we have
\[ P^{(2)} = G^{(2)}F, \quad (15.7) \]
\[ P^{(3)} = G^{(3)}F. \quad (15.8) \]

In a homogeneous medium, we know that
\[ G^{(2)} = \frac{i}{4\pi r} H_0^{(1)}(kr), \quad (15.9) \]
\[ G^{(3)} = \frac{1}{4\pi r} e^{ikr}, \quad (15.10) \]

where \( k = \omega/c, r = \sqrt{||x||^2} \), and \( H_0^{(1)} \) the Hankel function of the first-kind and zero-order.

When \( kr \gg 1 \), we have
\[ H_0^{(1)} \approx \left( \frac{2}{k^2 r} \right)^{1/2} e^{i(kr - \pi/4)}. \quad (15.11) \]

Plugging (15.11) into (15.9), we have
\[ G^{(2)} = \left( \frac{2\pi r c}{\omega} \right)^{1/2} e^{i\pi/4} e^{ikr} = \left( \frac{2\pi r c}{\omega} \right)^{1/2} e^{i\pi/4} G^{(3)}. \quad (15.12) \]

From equations (15.7) and (15.8), we have
\[ P^{(2)} = G^{(2)}F = \left( \frac{2\pi r c}{\omega} \right)^{1/2} e^{i\pi/4} G^{(3)}F = \left( \frac{2\pi r c}{\omega} \right)^{1/2} e^{i\pi/4} P^{(3)}. \quad (15.13) \]

The solution of equation (15.1) is
\[ p^{(2)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( \frac{2\pi r c}{\omega} \right)^{1/2} e^{i\pi/4} P^{(3)} d\omega. \quad (15.14) \]

This is the formula which we use to convert 3-D data to 2-D data.
15.4 Numerical Results

We use the 2.5-D finite-difference code (Xu, 1993) to generate 3-D data. The 2-D data are generated by a 2-D finite-difference code (Schuster and Xu, 1993).

First we have to verify our algorithm for a homogeneous model. Figure 15.1 shows a homogeneous crosswell model.

<table>
<thead>
<tr>
<th>Source</th>
<th>Ricker Wavelet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Frequency</td>
<td>250 Hz</td>
</tr>
<tr>
<td>Step Length in Space</td>
<td>2 ft</td>
</tr>
<tr>
<td>Step Length in Time</td>
<td>0.000125 sec.</td>
</tr>
<tr>
<td>Time Steps</td>
<td>2048</td>
</tr>
<tr>
<td>$V_p$</td>
<td>6500 ft/s</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2.2 g/cm$^3$</td>
</tr>
</tbody>
</table>

Table 15.1: 2.5-D and 2-D code parameters

Table 15.1 lists input parameters for the 2.5-D and 2-D finite-difference codes. Figure 15.2 shows that the 3-D and 2-D seismic traces are different from one another. After conversion of 3-D data to 2-D data using equation 15.14, the differences between them are very small (Figure 15.4). This result is expected from the theory in the above section. But the question is how well does it work for in heterogeneous media?

To answer this question and to verify the results of real data inversion by the WTW (Zhou, 1993), we choose the Exxon Friendswood velocity model (Figure 15.4). For the 52nd shot and the 40th receiver, the 3-D data and 2-D data are different as expected (Figure 15.5). After conversion of the 3-D data to 2-D data, most of the wave trains are matched. Figures 15.7, 15.8, 15.9, 15.10, and 15.11 show the seismograms associated with the 52nd shot. It is hard to see the difference between the 3-D and 2-D seismograms (Figures 15.7 and 15.8), but the differences between them are obvious with the subtraction of the 2-D seismogram from the 3-D seismogram (Figure 15.9). Figure 15.10 shows the seismogram which is the 3-D to 2-D conversion of the 3-D seismogram, and the differences between them are small (Figure 15.11). In this case, we chose $r$ and $c$ in equation 15.14 to be equal to the shot-receiver distance and 6500 ft/s respectively.

15.5 Conclusions

The 3-D to 2-D filtering operation is shown to be valid for the Exxon crosswell model. Therefore, 2-D waveform inversion can be applied to 3-D data sets after application of the 2-D conversion method; this assumes an approximate 2-D earth model.
15.6 References


Figure 15.1: 2-D crosswell model. The 3-D crosswell model is the rotation of the 2-D model about the vertical source axis well. The source in the source well is located at the depth of 520 ft. There are 98 receivers evenly spaced in the receiver well with 10 ft intervals.

Figure 15.2: Comparison between 3-D and 2-D data for a homogeneous model
Figure 15.3: Comparison between 2-D and converted 3-D to 2-D data for a homogeneous model.

Figure 15.4: The Exxon model. The distribution of the source and the receivers are the same as in Figure 15.1.
Figure 15.5: Comparison between 3-D data and 2-D data for the Exxon Friendswood model.

Figure 15.6: Comparison between 2-D and converted 2-D data. The source is at the depth of 520 ft. The receiver is at the depth of 400 ft.
Figure 15.7: 3-D synthetic seismogram with the shot at the depth of 520 ft.

Figure 15.8: 2-D synthetic seismogram with the shot at the depth of 520 ft.
Figure 15.9: 3-D synthetic seismogram data minus 2-D synthetic seismogram with the shots at the depth of 520 ft.

Figure 15.10: Converted 2-D seismogram from the 3-D synthetic seismogram.
Figure 15.11: 2-D synthetic seismogram minus converted 2-D seismogram with the shot at the depth of 520 ft.
Figure 15.9: 3-D synthetic seismogram data minus 2-D synthetic seismogram with the shots at the depth of 520 ft.

Figure 15.10: Converted 2-D seismogram from the 3-D synthetic seismogram.
Figure 15.11: 2-D synthetic seismogram minus converted 2-D seismogram with the shot at the depth of 520 ft.
Report 16

Data Filtering by Inverse Migration

T. Nemeth and G.T. Schuster

16.1 Abstract

We present a method for filtering different types of wave modes from seismic data. The idea is to invert a shot gather for the best-fit reflectivity model. We find that if the reflectivity model has many parameters so that the problem is underdetermined, then it is possible to reconstruct synthetic seismograms that closely resemble the field data. The reconstructed reflectivity model may not be correct, but importantly the reconstructed seismograms are almost correct. The reflectivity model is then used to generate seismograms that contain only the filtered modes of interest, such as PP reflections, or PS reflections, etc.. Preliminary results with synthetic and field data are encouraging. Possible applications of this method include automatic event identification and S-wave traveltme/waveform tomography.

16.2 Introduction

Unlike surface seismic data, crosshole seismic data abounds with a rich variety of converted wave modes. As an example, Figure 16.1 depicts a shot gather from the McElroy crosshole survey in West Texas. The well offset was only 180 feet yet there are large amplitude SP- and PS-transmitted waves, SS-reflected waves and, not shown, PS- and SP-reflected waves. These modes are rich in information about the interwell geology, but can raise havoc in migration and waveform inversion algorithms. A data set with too many types of wave modes can confuse a waveform inversion method and render it ineffective, or the different wave modes can introduce significant artifacts into the migrated, say PP, section. For example, PS-reflected waves can have a
Figure 16.1: A common source gather of the McElroy data.
moveout similar to that for SS-reflected waves so that an SS-migration algorithm will image both SS and PS reflections.

To eliminate these problems with modal confusion, a "divide and conquer" strategy is followed. The idea is to extract a specified type of wave mode from the data, and then invert or migrate those arrivals separate from others. Zhou and Hassanzadeh (1994) and Cai (1994) both used this idea to invert and migrate the McElroy data. Unfortunately, they used an FK-fan filter, which always has the potential to severely damage the data and/or introduce false arrivals. An alternative mode extraction method is needed.

In this paper, we explore the potential for replacing the FK-fan filter with a model-based filtering method. The idea is to apply a least squares migration method to obtain a reflectivity model that best fits the data. Then, the forward modeling operator that corresponds to the mode of interest is selected, and applied to the reflectivity model to generate the specified arrivals. In principle, the direct S arrivals or converted PS-transmitted arrivals can be extracted and used for traveltime or waveform tomography. We call this filtering method "inverse migration" because we first least squares migrate the data to get the reflectivity distribution, and then forward model (i.e., inverse migrate) to get the reconstructed/filtered data. The assumptions are that we have a fairly good, but not perfect, estimate of the background velocity model.

We first present the theory of this filtering method and then show preliminary numerical results for both synthetic and field data.

16.3 Theory

Assume that the earth's velocity field is decoupled into a long wavelength part (background velocity) and a short wavelength part (velocity perturbations or reflectivity distribution). The background velocity governs the propagation of the transmitted waves and the reflectivity distribution $\delta s$ generates the reflected waves $p$ (Bleistein, 1984). We will define three types of problems associated with seismic exploration: the forward problem, the inverse problem, and the forward-inverse problem. It is the solution to the forward-inverse problem that will provide the means to do model-based filtering.

16.3.1 Forward Modeling Problem

The forward modeling problem is defined as: given the reflectivity $\delta s$ and background slowness $s_0$, find the data $p$. For a weak reflectivity in an acoustic medium, we solve for the data by assuming a linearized Born-approximation and using the Lippmann-Schwinger equation to get

$$p_c(x_r, t|x_s, 0) = 2 \int \Delta s(x) \frac{W(t - \tau_{ax} - \tau_{xt})}{A_{ax} A_{xt}} dx,$$  \hspace{1cm} (16.1)
where \( W(t) \) is the source wavelet time history, \( \tau_{xx} \) is the traveltime for wave propagation from the source point \( x_s \) to the scattering point \( x \), and \( \tau_{xr} \) is the traveltime for wave propagation from the scattering point \( x \) to the receiver point \( x_r \). Here, \( A_{xx'} = |x - x'| \) is the geometrical spreading term for a 3-D homogeneous medium, and in inhomogeneous media \( A_{xx'} \) is computed from the transport equation (Miller et al., 1987). Equation 16.1 is used for Kirchhoff forward modeling.

Equation 16.1 can be recast in matrix-vector notation as

\[
\mathbf{L}\Delta\mathbf{s} = \mathbf{p}_o , \tag{16.2}
\]

where \( \mathbf{L} \) is the forward modeling operator for the background slowness, \( \Delta\mathbf{s} \) is the slowness perturbation vector and \( \mathbf{p}_o \) is the seismogram vector, containing the single reflected events. This equation can be extended to elastic media by redefining \( \mathbf{L} \) as a composite of forward modeling operators, i.e.,

\[
\mathbf{L} = \mathbf{L}_P + \mathbf{L}_{PP} + \mathbf{L}_S + \mathbf{L}_{SS} + \mathbf{L}_{PS} + \mathbf{L}_{SP} + \mathbf{L}_{PST} + \mathbf{L}_{SPS} + \ldots, \tag{16.3}
\]

Here the subscript, say \( PP \), indicates to the \( PP \) reflections generated by \( \mathbf{L}_{PP} \) operator. Specifically, the modes are: \( PS = \) converted \( PS \) reflected waves, \( PSt = \) converted \( PS \) transmitted waves, \( P = \) direct \( P \) wave, etc.

In practice, the different wave types can easily be calculated by solving the eikonal equation for first arrival traveltimes (Qin et al., 1992) for sources at the receiver wells and at the source wells. The traveltimes are computed for both the \( P \)-velocity and \( S \)-velocity distributions. For example, the eikonal equation can be solved for the \( P \)-arrival traveltime \( \tau_{xx}^P \) from the source to the scatterer, and the \( S \)-arrival traveltime \( \tau_{xx}^S \) from the scatterer to the receiver. Thus, \( \mathbf{L}_{PST} + \mathbf{L}_P \) can be computed so that equation 16.2 can be used to generate the seismograms that only contain the \( PS \) and \( PSt \) arrivals. For the Steepbank velocity model shown in Figure 16.2, the common shot seismograms generated from \( P \) and \( S \)-arrival eikonal traveltimes and equation 16.3 are shown in Figure 16.3. In this figure, the amplitude terms were approximated by using inverse distance calculations, and the source wavelet is a Ricker wavelet.

### 16.3.2 Inverse Problem

The inverse problem is defined as: given the data \( \mathbf{p} \) and background slowness \( s_o \), find the reflectivity model \( \Delta\mathbf{s} \). In the context of standard migration, the reflectivity model is approximated by (Claerbout, 1992):

\[
\Delta\mathbf{s} \approx \mathbf{L}^T\mathbf{p} , \tag{16.4}
\]

where the migration operator \( \mathbf{L}^T \) is the transpose to the forward modeling operator.

A more precise reflectivity estimate is the least squares solution (Schuster, 1993)

\[
\Delta\mathbf{s} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T\mathbf{p}. \tag{16.5}
\]
Figure 16.2: From left to right: Steepbank tomogram and Steepbank PP migrated section. Figures courtesy of Fuhao Qin.
Figure 16.3: Synthetic seismograms generated from the Steepbank velocity model by an asymptotic Greens function. The following modes types are included: PP, SS, SP, PS, S, P reflections and PS and SP transmitted waves.
Nemeth (1994) found that the slowness models obtained by applying equation 16.5 to field data were superior to the reflectivity models obtained from the standard migration operation in equation 16.4.

16.3.3 Forward-Inverse Problem

The forward-inverse problem is defined as: given the field data, find the synthetic data that best-fit the field data. The solution to this problem is to take the least squares reflectivity model in equation 16.5 and generate the synthetic data \( p' \) with the forward model operator \( L \), i.e.,

\[
p' = L(L^T L)^{-1} L^T p. \tag{16.6}
\]

Nemeth (1994) calculated equation 16.6 by an iterative conjugate gradient method to generate synthetic seismograms \( p' \) from the Friendswood data \( p \), and found that they were very similar to the field seismograms. Figure 16.4 shows a field shot gather from the Friendswood crosshole data set (Chen et al., 1990), alongside the PP shot gather generated from equation 16.5. Note the similarities in the moveouts associated with the PP reflections.

This good match, at first glance, was surprising since the background velocity model (i.e., the traveltime tomogram) was just an approximation to the actual velocity model. Apparently, generating somewhat accurate synthetic seismograms for one shot gather is not too difficult if there are many degrees of freedom in the slowness model. This is analogous to generating ray-traced traveltimes that exactly fit a few observed travelt ime picks, as long as there are many adjustable slowness cells, i.e., an underdetermined inverse problem.

16.3.4 Identification and Filtering of Wave Modes

The above result suggests that equation 16.6 can be used as a means to filter out or identify certain wave modes. For example, the direct S-wave may be buried in the data, but using \( L_S \) in equation 16.6, i.e.,

\[
p' = L_S(L_S^T L_S)^{-1} L_S^T p \tag{16.7}
\]

will yield \( p' \), the approximate direct S-wave seismograms. \( p' \) could then be used to identify the S-wave mode in the field data \( p \), or it could be used as the starting data for S-wave traveltime or waveform tomography. The filtered data can be updated by equation 16.7 as the velocity estimate is improved. Other wave modes can be extracted by using one or a combination of operators in equation 16.3.
Figure 16.4: From left to right: Friendswood shot gather and reconstructed shot gather of PP reflections. Background velocity is taken to be from traveltime tomogram.
16.4 Numerical Results

This section shows the results of applying equation 16.6 to both synthetic and field crosshole data.

For the Steepbank velocity model (Zhou et al., 1994) shown in Figure 16.2, the synthetic data are generated by equation 16.3 to give the shot gather in Figure 16.3. In this case, the P-, S-, PS-, and SP-transmitted waves are intermingled with the PP-, SS-, SP-, and PS-reflected waves. Figures 16.5, 16.6 and 16.7 show the theoretical reconstructed shot gathers for P waves and S waves, respectively. The reconstructions are for the 20th conjugate gradient iterate. In all cases, the wave modes are reconstructed well and are readily identified. Figure 16.7 shows the reconstructed P-wave gathers when the background velocity is scaled by 10 percent from the actual background velocity. Even with this incorrect velocity, the reconstructed gathers are quite similar to the actual gathers.

Finally, equation 16.6 is used to extract the S-direct and SS-reflected waves from the McElroy data. Time limitations prevented us from using the tomogram velocity so we used a homogeneous background velocity. The field shot gather and reconstructed SS+S' shot gather are shown in Figure 16.8. The reconstructed shot gather is the 20th iterate of the conjugate gradient solver and shows a fair resemblance to the actual shot gather. This is encouraging because a homogeneous background velocity is quite far from the actual tomogram velocity. There would have been a much better correlation if the actual S-wave tomographic velocity had been used.

16.5 Discussion

We presented an inverse migration procedure for filtering different wave modes from seismic data. The synthetic results suggest that this method can be used to identify the type of waves in complicated records. We hope to eventually use this method to facilitate the divide and conquer strategy of inversion, i.e., separate the different wave modes and invert each type separately. Then, starting with the updated velocity, combine these modes together and simultaneously invert them for the actual velocity model. As the model gets updated the data may need to be refiltered. Much work remains to achieve this goal.

16.6 References

Cai, W., 1994, Processing and migration of McElroy data, University of Utah Modeling and Tomography Development Project, 1994 Annual Report, this volume

Figure 16.5: From left to right: the theoretical direct P and reflected PP waves, the reconstructed reflected PP waves, and the reconstructed direct P waves. The reconstructed data are computed from the Steepbank shot gather in the previous figure.
Figure 16.6: From left to right: the theoretical direct S and reflected SS waves, the reconstructed reflected SS waves, and the reconstructed direct S waves. The reconstructed data are computed from the synthetic Steepbank shot gather.
Figure 16.7: Same reconstructions of the direct P wave and reflected PP waves as before, except the background velocity was scaled by 10 percent from the true background velocity.
Figure 16.8: From left to right: McElroy shot gather and reconstructed shot gather of SS+S reflections. Background velocity is an obviously incorrect homogeneous velocity.


