UNIVERSITY OF UTAH
TOMOGRAPHY DEVELOPMENT PROJECT

ANNUAL REVIEW

JANUARY 27, 1989
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1989 SUMMARY

TOMOGRAPHY RESEARCH

1989 was the third year of the seismic tomography project and it enjoyed the support of eleven oil/gas companies: Amoco, Arco, British Petroleum, Chevron, Conoco, Exxon, Gas Research Institute, Marathon Oil Co., Mobil, Phillips, and Texaco. The objectives of this project are to develop innovative inversion algorithms and test their effectiveness on geophysical data. We are particularly interested in optimizing the extraction of geological information from seismic and electromagnetic cross-well experiments.

The following is a brief summary of our salient 1989 research results.

**Wave Equation Traveltime Inversion.** A new tomography algorithm is developed which inverts for velocities from traveltimes calculated by a finite difference solution to the wave equation (Paper 1). This proves to be a superior method compared to ray traced tomography for complicated velocity models where the ray tracing approximation is no longer valid. In addition, no travel time picking is necessary; head wave, reflection, diffraction and body wave traveltimes can be used, and the method appears to converge quickly for impedance contrasts as large as 40%.

**Hybrid Traveltime + Waveform Inversion.** The wave equation traveltime tomography method is combined with full wavefield inversion (Paper 2) to produce a hybrid inversion algorithm significantly superior to
standard full wavefield tomography. A problem with the standard method is that it requires the starting model to be within about 10-15% of the actual model. This restriction is eliminated for the new hybrid inversion method.

**Traveltimes By Finite Differencing Of The Eikonal Equation.** We show that a previously published algorithm for finite differencing the Eikonal equation is invalid for models with moderate to large contrasts in velocity. We rectify this problem by developing a new way to finite difference the Eikonal equation (Paper 3). The computed traveltimes appear to be accurate for models with large velocity contrasts. An advantage is that first arrival traveltimes in wave guides and shadow zones can be computed; a disadvantage is that the method is computationally expensive. The fortran code (FRONT.F) for solving the eikonal equation and its documentation (paper 15) is contained in an attached floppy disk in the directory EIK. Occasionally the square root argument becomes negative, but this did not appear to seriously affect the accuracy of our results.

**Inverting The Eikonal Equation.** An inversion algorithm based on the Eikonal equation is theoretically developed (Paper 4). Its development closely parallels that of full waveform inversion. The advantage of this approach is that inversion requires a finite difference solution to a first-order PDE; this can be computationally efficient on a parallel/vector computer.

**Automatic Reflection Picking Algorithm.** An automatic traveltime picking program is developed to pick reflected arrivals in a crosswell
seismogram (Paper 5). Adjacent traces are cross-correlated to so that the reflection traveltimes can picked by the computer. The picking accuracy improves remarkably if the data are F-K filtered.

**Crosswell Attenuation Tomography.** Finite difference simulations suggest that scattering and focusing phenomena may influence seismic amplitudes more than they influence wavelet period (Paper 6). The wavelet periods measured from real crosswell data show erratic changes with depth compared to those from synthetic crosswell data. This might be expected because the synthetics are based on an artificially smoothed velocity model reconstructed from the real traveltime data.

**Static Corrections Using Surface Wave Inversion.** A methodology is developed for using Love and Rayleigh wave data to invert for near-surface velocities (Paper 7). Tests with synthetic data show that the S-velocity is best resolved using simultaneous inversion of both Rayleigh and Love waves. A nine-component seismic experiment was conducted with ARCO (thanks to Nigel Watrous and Jim DiSiena) to verify the effectiveness of this algorithm. Preliminary results show that the velocity model reconstructed from Love waves is in fairly good agreement with the well log shear velocities. The velocity model reconstructed from the Rayleigh wave data does not agree with the well log velocities. This discrepancy is currently being studied.

**Parsimonious Staggered Grid Method.** A new parsimonious staggered grid method is developed which eliminates the need to store stress
components (Paper 8). This can result in a significant savings in computer memory; hence, the parsimonious staggered grid method can accommodate much larger models than the standard staggered grid scheme.

2.5-D Finite Differencing of Maxwell's Equations. An efficient method is developed for solving 2.5-D Maxwell's equations in the time domain by a staggered grid method (Paper 9). This algorithm is at least one, and perhaps two orders of magnitude faster than any other finite difference algorithm. This helps to open the door for seismologists and electrical geophysicists who wish to explore cooperative inversion of seismic and electrical data.

4th-Order Finite Differencing Code. A floppy diskette containing a documented 4th-order finite difference code (in the directory PP4) is included with this report (Paper 10). This algorithm solves the 2-D acoustic wave equation for line sources. There is excellent agreement with an analytic solution. Numerical experiments are performed to assess stability in the presence of absorbing and free-surface boundary conditions. In addition, a 2-2 staggered grid finite difference code which solves the 2-D elastic wave equation (paper 14) is contained in the floppy disk directory PSVR2.

3-D Gravity Inversion of Salt Lake Basin Structure. A 3-D profile of the Salt Lake Basin is reconstructed from gravity data (Paper 11). CDP seismic data and well log data are used to constrain the solution in a pseudo cooperative fashion. New twists to this algorithm include
derivation of an analytic Jacobian, rather than a finite difference derived Jacobian. A documented Fortran code of this algorithm will be given to the sponsors in 1990.

**Preliminary Wave Equation Inversion Of VSP Data.** With less than a week before the January 26 meeting Yi Luo decided to test the wave equation traveltime inversion algorithm on the Bridenstein VSP data (courtesy of ARCO). Preliminary results (using unneeded and perhaps harmful approximations to expedite results for the January 26 meeting) look promising, but show less velocity resolution than that from ray trace traveltime tomography. With careful processing we believe that the reconstructed velocity profile may achieve the same or greater velocity resolution than that from ray tracing tomography.

**News**

Gas Research Institute gave us monies to upgrade our Stellar computer. With matching funds by the University, we will soon upgrade to either the Stardent 2000 or 3000 computer, with 96 Mbytes of memory and either 80 or 200 Mflops of peak speed.

Jinglong Xu recently arrived from PRC to work with us for 18 months as a visiting scholar. His expenses and salaries are completely paid for by the PRC government. Mark Turner (MS, 1990) has accepted a job with ARCO in Houston, Texas and will begin work in May or June, 1990. Hugh Radkins (MS, 1990) accepted a job with Conoco in New Mexico and will begin work in Spring or early summer, 1990. Yi Luo will work at
ARCO research labs in February and March to continue research in poroelastic wave modeling. Kim Olsen will work at the Phillips research lab during the summer and implement his 3-D 4th-order staggered grid code on Phillip's Cray 2 computer.

Mobil Oil Co. donated to us $40,000. in seismic equipment, including a pick-up truck, 1000 geophones and assorted cables. In addition, NSF awarded us a grant to purchase a 24-channel seismic acquisition unit, including three-component phones and cables. This equipment will be used to further our research in tomography, especially in the acquisition of surface wave data.

Students

Students supported in part or full by the 1989 tomography development project include (graduation dates):

1). Mark Turner (MS, 4/90, ARCO)
2). Yi Luo (PhD, '91)
3). Kim Olsen (PhD, '91)
4). Fuhao Qin (PhD, '92)
5). Hugh Radkins (MS, 5/90, Conoco)
6). Chris Calnan (MS, 3/89, Shell)
7). Mary Murphy (MS, 4/89, Shell)
8). Axel Quintus-Bosz (MS, '91, research support starts Fall '90)
9). Bill Stephenson (MS, '91, summer help)

Former Students in Tomography Group:

1). Ed Salo (MS, 12/86, Shell)
2) Dan Johnson (MS, '87, Chevron)
3) Zak Ahmed (MS, '87, U of U)
4) Chris Calnan (MS, 3/89, Shell)
5) Mary Murphy (MS, 4/89, Shell)

Former Schuster students:
1). Tony Robertson (MS, '89, Shell)
2). Julie Hill (MS, '88, Mobil)

1990 Research

Tomography research in 1990 will extend some of the current projects as well as initiate new projects, including

1). Wave Equation Traveltime Inversion of Reflection Traveltimes.
2). Anisotropic Parameter Inversion.
3). Electromagnetic and Seismic Analysis of Reservoirs.
4). Traveltime Inversion Using the Eikonal Equation.
5). Statics Corrections Using Surface Wave Inversion.
6). Wave Equation Traveltime + Waveform Inversion of Real Crosswell Data.
8). Anisotropic Seismic Modeling
References

1989 Final Reports

1. Wave Equation Traveltime Inversion.
2. Hybrid Traveltime + Full Waveform Inversion.
4. Inverting The Eikonal Equation.
5. Automatic Picking of Reflected Seismic Arrivals: Improvement by Contour-Slice F-K Filtering.
7. Static Corrections Using Surface Wave Inversion.
9. 2.5-D and 3-D Finite Differencing of Maxwell's Equations.
11. 3-D Gravity Inversion of Salt Lake Basin by Using Right Rectangular Prisms and Damped Least Squares.
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14. PSVR2 Program Description.
15. EIK Program Description.
WAVE EQUATION TRAVEL TIME INVERSION

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ABSTRACT

This paper presents a new seismic inversion method which implements travel time inversion based on the wave equation. In this new method, designated as wave equation travel time inversion (WT), seismograms are computed by any full wave forward modeling method (we use a finite difference method). The velocity model is perturbed until the traveltimes from the synthetic seismograms are best fitted to the observed traveltimes in a least squares sense. A gradient optimization method is used and the Frechet derivative (perturbation of traveltimes with respect to velocity) is derived directly from the wave equation. No traveltme picking or ray tracing is necessary, and there are no high frequency assumptions about the data. Body wave, diffraction, reflection and head wave traveltimes can be automatically incorporated into the inversion without their explicit identification. In the high frequency limit WT inversion reduces to ray based traveltme tomography. It can also be shown that WT inversion is approximately equivalent to full wave inversion when the starting velocity model is "close" to the actual model.

Numerical simulations show that WT inversion succeeds for models with up to 80% velocity contrasts compared to the failure of full wave inversion for some models with no more than 8% velocity contrast. It is also shown that the WT method succeeds in inverting the Langan velocity model where ray tracing fails to work. The disadvantage of this method is that it has less resolution compared to full wave inversion, but this problem can be remedied by a hybrid traveltme+full wave inversion method (Luo and Schuster, 1990).
INTRODUCTION

Seismic inversion algorithms span the range between two extremes: traveltime inversion (Dines and Lytle, 1979; Paulsson et al., 1985; Ivansson, 1987; Bishop et al., 1985; Lines, 1988; Justice et al., 1989; and many others) and full wave inversion (Tarantola, 1987 and others). Traveltime inversion uses ray tracing to compute both the traveltimes and Frechet derivative (perturbations of traveltimes with respect to velocities). While computationally efficient, traveltime inversion assumes a high frequency approximation to the data and can therefore fail when the earth's velocity variations are nearly the same wavelength as the source wavelet. On the other hand, the misfit function to be minimized (normed squared error between observed and calculated traveltimes) can be shown to be pseudo-linear with respect to the normed difference between the starting and actual velocity model. This means that a gradient optimization algorithm (e.g., conjugate gradients) can make rapid progress in searching for the correct velocity model. Hence, successful inversion can be achieved even if the starting model is far from the actual model.

Attempts to bridge the gap between the extremes of traveltime inversion and full wave inversion include Born inversion (Clayton and Stolt, 1981; Weglein, 1982; Keys and Weglein, 1983; Bleistein and Gray, 1985; Carrion and Foster, 1985) and other amplitude methods which are subject to some approximation in the data. These intermediate methods can be very successful for some data sets but will usually not suffice for data with strong contrasts in impedance. Intermediate methods also include surface wave inversion (Wattrus, N., 1989) and diffraction tomography (Lo et al., 1988).

Full wave inversion overcomes limitations imposed by the high frequency
approximation of traveltime inversion and the weak scattering approximation of Born methods by perturbing the velocity model until the synthetic seismograms match the observed seismograms. No approximations are necessary and the synthetic seismograms are usually computed by a finite difference solution to the wave equation. In addition, the Frechet derivatives are elegantly computed by reverse time migration of the seismogram residuals. The problem with full wave inversion, however, is that the misfit function (normed difference between observed and synthetic seismograms) is highly non-linear with respect to the actual and assumed velocity models. Gauthier (1986) showed that full wave inversion will fail for a large model (several wavelengths in dimension) with more than 8% velocity contrast. A reason for this failure is that the misfit function is highly non-linear with respect to velocity perturbations in the model. In this case a gradient method will tend to get stuck in local minima if the starting model is moderately far from the actual model.

Can one borrow the best characteristics of traveltime inversion (pseudo-linear misfit function) and full wave inversion (no approximations to the data) to create an inversion method free from approximations, robust in the presence of data noise, and quickly convergent for starting models far from the actual model? The answer is that traveltime inversion achieves this goal if the wave equation, rather than the approximate method of ray tracing, is used to compute traveltimes and Frechet derivatives. This paper describes the derivation of a new velocity inversion method, wave equation traveltime inversion (WT), which minimizes traveltime residuals using traveltimes and Frechet derivatives computed from the wave equation. The merits of WT inversion are that it can invert for some velocity models with more than 80% contrast in impedance, its misfit function is largely independent of density variations, it can invert for complicated velocity
models where ray tracing fails, and no traveltime picking or event identification is needed. The first part of this paper will derive the WT method, and the second part will present results from synthetic tests of the method.

THEORY

This section will derive the wave equation traveltime inversion (WT) method. The key steps are to define an misfit function which is a function of the traveltime residual (normed difference between observed and synthetic travel-times), and to derive the perturbation of this misfit function (Frechet derivative) using the wave equation.

The following analysis assumes that the propagation of seismic waves honors the 2-D acoustic wave equation. Let \( p(x_r, t; x_s)_{\text{obs}} \) denote the measured (observed) pressure seismograms at receiver location \( x_r \) due to a source excited at time \( t=0 \) and at location \( x_s \). For a given model, \( p(x_r, t; x_s)_{\text{cal}} \) denotes the computed seismograms which honors the acoustic wave equation,

\[
c(x)^2 \frac{\partial^2 p(x_r, t; x_s)}{\partial t^2} - \rho(x) \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla p(x_r, t; x_s) \right] = s(t; x_s), \tag{1}
\]

where \( \rho(x) \) is the density, \( s(t; x_s) \) is the source function and \( c(x) \) is the wave speed.

Traveltime Residual. The degree in which the synthetic and observed seismograms match each other can be estimated by the cross-correlation function:

\[
f(x_r, \tau; x_s) = \int dt \frac{p(x_r, t+\tau; x_s)_{\text{obs}}}{A(x_r; x_s)_{\text{obs}}} p(x_r, t; x_s)_{\text{cal}},
\]
where, \( A(x_r; x_s)_{obs} \) is the maximum amplitude in \( p(x_r; t; x_s)_{obs} \) and \( \tau \) is the shift time between synthetic and real seismograms. The divisor \( A(x_r; x_s)_{obs} \) normalizes the observed seismograms to a maximum amplitude of 1.

We seek a \( \tau \) in which a synthetic seismogram must be shifted so that it "best" matches the observed seismogram. The criteria for "best" match is defined as the traveltime residual \( \Delta \tau \) which maximizes the cross-correlation function, \( f(x_r; \tau; x_s) \), i.e.,

\[
f(x_r; \Delta \tau; x_s) = \max \left\{ f(x_r; \tau; x_s) \mid \tau \in [-T, T] \right\},
\]

where \( T \) is the estimated maximum travel time difference between the observed and calculated seismograms. It is easy to see that the derivative of \( f(x_r; \tau; x_s) \) with respect to \( \tau \) should be zero at \( \Delta \tau \) unless its maximum is at an endpoint \( \Delta \tau = T \) or \( \Delta \tau = -T \):

\[
f_{\Delta \tau} = \left[ \frac{\partial f(x_r; \tau; x_s)}{\partial \tau} \right]_{\tau = \Delta \tau}
\]

\[
= \int dt \left( \frac{\partial}{\partial t} \frac{p(x_r; t + \Delta \tau; x_s)_{obs}}{A(x_r; x_s)_{obs}} \right) p(x_r; t; x_s)_{cal} = 0,
\]

where, \( \dot{p} = \frac{\partial p(x_r; t; x_s)}{\partial t} \). Equation (3) will be used to compute the misfit function of summed squared traveltime residual.

**Misfit Function.** The WT method attempts to determine a velocity model \( c(x) \) which predicts seismograms \( p(x_r; t; x_s)_{cal} \) that minimize the following misfit
function:

\[
S = \frac{1}{2} \sum_s \sum_r \Delta \tau(x_r, x_s)^2
\]  \hspace{1cm} (4)

where \( \Delta \tau \) is defined by equation (2) and the factor 1/2 is introduced for subsequent simplifications. This criterion, of course, can be generalized to take into account the estimated observation errors or a priori information in model space.

A gradient method method can be used to solve equation (4). For simplification we will discuss the steepest descent method here, and in Appendix B give the formula for the conjugate gradient method. To update the velocity model, the steepest descent method gives,

\[
c(x)_{k+1} = c(x)_k + \alpha_k \gamma(x)_k
\]  \hspace{1cm} (5)

where \( \gamma(x)_k \) is the steepest descent direction of the misfit function \( S \), \( \alpha(x)_k \) is simply a constant scaling factor (Appendix B gives a method to estimate it), and \( k \) represents the kth iteration. The central problem is how to get \( \gamma(x) \) based on the wave equation. To obtain \( \gamma(x) \), construct the Frechet derivative of \( S \) with respect to the velocity model \( c(x) \):

\[
\gamma(x) = -\frac{\partial S}{\partial c(x)} = -\sum_s \sum_r \frac{\partial (\Delta \tau)}{\partial c(x)} \Delta \tau(x_r, x_s).
\]  \hspace{1cm} (6)

Using equation (2) and the rule for an implicit function derivative we get

\[
\frac{\partial (\Delta \tau)}{\partial c(x)} = - \left[ \frac{\partial (f^{\Delta \tau})}{\partial (c(x))} \right] / \left[ \frac{\partial (f^{\Delta \tau})}{\partial (\Delta \tau)} \right]
\]
\[
= \frac{1}{E} \int \, dt \, \dot{p}(x_r,t+\Delta t; x_s)_{obs} \frac{\partial p(x_r,t; x_s)_{cal}}{\partial c(x)}
\]  

(7a)

where

\[
E = - \int \, dt \, \dot{p}(x_r,t+\Delta t; x_s)_{obs} \, p(x_r,t; x_s)_{cal}
\]

\[
= \int \, dt \, \dot{p}(x_r,t+\Delta t; x_s)_{obs} \, \dot{p}(x_r,t; x_s)_{cal}
\]  

(7b)

In Appendix A, we show that the Frechet derivative of the pressure field \( p(x_r,t; x_s)_{cal} \) is:

\[
\frac{\partial p(x_r,t; x_s)_{cal}}{\partial c(x)} = \frac{2}{c(x)^3} \, \dot{g}(x,t; x_r,0) * \dot{p}(x,t; x_s),
\]

(7c)

where, \( g(x,t; x',t') \) is the Green's function for equation (1), that is, the pressure field at point \( x \) and time \( t \) due to the impulse source \( \delta(x-x_r)\delta(t-t') \). The symbol \(*\) represents time convolution. Substitute equation (7c) into equation (7a),

\[
\frac{\partial(\Delta t)}{\partial c(x)} = \frac{2}{c(x)^3} \int \, dt \, \dot{g}(x,t; x_r,0) * \dot{p}(x,t; x_s) \frac{\dot{p}(x_r,t+\Delta t; x_s)_{obs}}{E},
\]

(7d)

substitute equation (7d) into (6)

\[
\gamma = \frac{1}{c(x)^3} \sum_s \sum_r \int \, dt \, \dot{g}(x,t; x_r,0) * \dot{p}(x,t; x_s) \delta \tau(x_r,t; x_s),
\]

(8a)

where \( \delta \tau \) is the pseudo-travel time residual,
\[ \delta \tau(x_r, t; x_s) = -\frac{2}{E} \dot{p}(x_r, t+\Delta t; x_s)_{\text{obs}} \cdot \Delta \tau(x_r, x_s). \] 

(8b)

Using the identities

\[ \int dt \ [f(t) \ast g(t)]h(t) = \int dt \ g(t)[f(-t) \ast h(t)], \]

and

\[ g(x, -t; x', 0) = g(x, 0; x', t) \]

\[ \gamma = \frac{1}{c(x)^3} \sum_s \int dt \ \dot{p}(x, t; x_s) \sum_r \left[ \dot{g}(x, 0; x_r, t) \ast \delta \tau(x_r, t; x_s) \right] \]

one can rewrite equation (8a) as

\[ \gamma = \frac{1}{c(x)^3} \sum_s \int dt \ \dot{p}(x, t; x_s) \dot{p}'(x, t; x_s), \] 

(9)

where

\[ p'(x, t; x_s) = \sum_g g(x, t; x_r, 0) \ast \delta \tau(x_r, t; x_s), \]

where, \( p(x, t; x_s) \) is the pressure field calculated for the current velocity model \( c(x) \) and \( p'(x, t; x_s) \) is the field computed by reverse time propagation of the pseudo residual \( \delta \tau(x_r, t; x_s) \) acting as sources at receiver location \( x_r \). This result is the same as that of full wave inversion except \( \delta \tau \) is used instead of \( \delta p \). In full wave inversion \( \delta p \) is defined as
\[ \delta p = p(x_r, t; x_s)_{obs} - p(x_r, t; x_s)_{cal} \] (10)

Combining equation (9) and (5) yields an iterative method to invert for a velocity model \( c(x) \) from traveltime residuals. In Appendix B, we describe the computer implementation of this theory.

**RELATIONSHIP TO RAY TRACING TRAVEL TIME TOMOGRAPHY AND FULL WAVE INVERSION**

This section shows that in the high frequency limit and under a liner perturbation assumption the WT method reduces to ray trace traveltime tomography. It is also shown that the WT method is approximately equivalent to full wave inversion if the starting model is close to the true model.

According to ray theory,

\[ p(x_r, t; x_s)_{cal} = A(x_r; x_s)_{cal} \delta (t - \tau(x_r; x_s)_{cal}) \] (11)

where, \( \tau(x_r; x_s)_{cal} \) is the travel time computed along rays for a given velocity model. Plugging equation (11) into (7a) and (7b) yields:

\[ \frac{\partial (\Delta \tau)}{\partial c(x)} = \frac{1}{E} \int dt \dot{p}(x_r, t + \Delta \tau; x_s)_{obs} \left[ \frac{\partial A(x_r; x_s)_{cal}}{\partial c(x)} \delta (t - \tau(x_r; x_s)_{obs}) \right. \]

\[ - A(x_r; x_s)_{cal} \delta (t - \tau(x_r; x_s)_{cal}) \frac{\partial \tau(x_r; x_s)_{cal}}{\partial c(x)} \] (12a)
\[
E = \int dt \dot{p}(x_r, t+\Delta t; x_s)_{obs} \cdot A(x_r; x_s)_{cal} \hat{\delta}(t - \tau(x_r; x_s)_{cal}) . \tag{12b}
\]

Ray tracing tomography also makes a linear perturbation assumption, i.e. when the velocity \(c(x)\) is perturbed, the ray path remains unchanged to first order variation in velocity. An unperturbed ray path leads to an unperturbed amplitude with a constant ray tube. Therefore, setting the derivative of \(A(x_r; x_s)_{cal}\) with respect to \(c(x)\) be zero in equation (12a), and substituting (12b) into (12a), we get

\[
\frac{\partial(\Delta t')}{\partial c(x)} = -\frac{\partial \tau(x_r; x_s)_{cal}}{\partial c(x)} = \frac{\partial(\Delta t)}{\partial c(x)} \tag{12c}
\]

where \(\Delta t' = \tau(x_r; x_s)_{obs} - \tau(x_r; x_s)_{cal}\) and \(\tau(x_r; x_s)_{obs}\) is the travel time picked from real data. This result shows that the Jacobian matrix in ray tracing tomography is a special case of our Jacobian operator in equation (7) for the high frequency and linear perturbation assumptions.

To establish the relationship between WT and full wave inversion, assume \(p(x_r, t; x_s)_{obs}\) is the seismogram for the true velocity model \(c(x)\). If the current velocity model is \(c(x) + \delta c(x)\) and \(\delta c(x)\) is small, then the calculated seismogram \(p(x_r, t; x_s)_{cal}\) differs by only a time shift from \(p(x_r, t; x_s)_{obs}\),

\[
p(x_r, t; x_s)_{cal} \approx p(x_r, t+\Delta t; x_s)_{obs}
\]

This assumes that the amplitude differences are negligible. Therefore,

\[
\dot{p}(x_r, t+\Delta t; x_s)_{obs} = \frac{p(x_r, t+\Delta t; x_s)_{obs} - p(x_r, t; x_s)_{obs}}{\Delta t}
\]
\[ \approx \frac{p(x_r,t; x_s)_{cal} - p(x_r,t; x_s)_{obs}}{\Delta t} \]

Substituting the above equation into (8b) yields

\[ \delta t(x_r,t; x_s) \approx \frac{2}{E} \left[ p(x_r,t; x_s)_{obs} - p(x_r,t; x_s)_{cal} \right] = \frac{2}{E} \delta p. \quad (13) \]

This result shows that \( \delta t \) equals \( \delta p \) except for a factor of \( 2/E \); the WT inversion method is equivalent to full wave inversion when the current velocity model is close to the true model. In practice, however, it is difficult to determine \( \Delta t \) by cross-correlation (equation 2) when \( \Delta t \) is small. This difficulty will result in less velocity resolution compared to the full wave inversion method. An optimal method might be to use the WT method to reconstruct a moderately fine velocity model, and use full wave inversion for the finer velocity details. This hybrid strategy is successfully exploited in Luo and Schuster (1990a).

**NUMERICAL EXAMPLES**

The WT method is tested for three different models; a camembert model, the Langan velocity model, and a Langan velocity-density model. The camembert model is used to verify that WT inversion is more robust than full wave inversion for incorrect starting models. The Langan velocity model is used to show WT inversion succeeds when ray tracing fails. And the Langan velocity-density model is used to confirm that WT inversion can invert velocities without
knowing densities. For all of the three models, a Ricker wavelet is used with a peak frequency of 25 hz.

Camembert Model

Gauthier (1986) presented a camembert model (Figure 1a) composed of a cylindrical velocity perturbation 5 wavelengths in diameter superimposed on a homogeneous medium with velocity 2500 m/s. The starting model is assumed to be a homogeneous medium with velocity 2500 m/s. In the finite difference modeling, we use $\Delta x=\Delta z=5m$, $\Delta t=0.8ms$, the number of time steps is 900 (720 ms) and the model is digitized into a mesh of $201 \times 201$ grid points. Two sources are used per edge for a total of eight sources and 380 receivers are spaced every two grid points along an edge. Figure 1c shows the CSP seismograms recorded on the edge opposite to the source edge for the Figure 1a model; $e$, the velocity contrast, is equal to 20%. The velocity is assumed to be known in a band of width $2\lambda$ around the source and receiver locations. Hence, the velocity values in a $160 \times 160$ grid are inverted for using a conjugate gradient method. The velocity contrast $e$ is defined as $e = \frac{\Delta c}{c}$, where $\Delta c$ is the difference between the background and cylinder velocity.

Figure 2 shows the full wave inversion results where the right column of figures depicts the reconstructed velocity cross-section along the center of the model. The full wave inversion method is successful at inverting models with 5% velocity contrast (Figure 2a) but fails for 10% contrast models (Figure 2b). This is similar to the result in Gauthier (1986). The reason for this failure is illustrated in Figure 3. Figure 3b is the normed seismogram misfit function calculated from equation 10 plotted against increasing velocity contrast in the model.
Figure 1
Figure 2
It is obvious that beyond 10%, the full wave inversion method will fail. Figure 3a is the normed travelt ime misfit function calculated from equation (4), and figure 3c is the corresponding normed pseudo-travel time misfit function defined by equation (8b). Both graphs suggest that the normed traveltime misfit function has a pseudo-linear behavior with respect to increasing velocity contrast. For comparison, we combine Figure 3b (solid line) and Figure 3c (dashed line) in Figure 3d and take the logarithm. In Figure 3d, we can see that full wave inversion will be as effective as the WT method for velocity contrasts less than 5%. Beyond 10% velocity contrasts, the full wave inversion will fail while the WT inversion should succeed for models with 40% velocity contrast. The numerical result in Figure 4 confirm this prediction. In Figure 4, the reflection events were muted to prevent interference with the transmitted arrivals. For the 40% case, the sides of the model were ramped to avoid severe Gibbs phenomena in the reconstructions (figure 1b). Also shown in Figure 4 is the successful reconstruction for a cylinder model with 80% velocity contrast.

Langan Velocity Model

Figure 5 (courtesy of R. Langan) shows the failure of ray tracing for a complicated velocity model. Synthetic seismograms are computed for the acquisition system shown in Figure 6a and a digitized approximation to the 1-D Langan velocity in Figure 6b. Eight sources are in the source well, and 83 receivers with a 4.8m spacing are located in the receiver well. In this case, \( \Delta x = \Delta z = 2.4 \text{m}, \Delta t = 0.5 \text{ms} \), and 700 time steps are calculated. Figure 6c shows the seismograms for the source at depth 168 m. To avoid aperture problems, the velocity is assumed to be known from 0-48m and 350-400m. Figure 7 shows the reconstructed velocity obtained by the WT inversion method using a steepest gradient method. The high frequency velocity components are not reconstructed
Figure 3
$\epsilon = 5\%$

after 5 iterations

$\epsilon = 10\%$

after 5 iterations

$\epsilon = 20\%$

after 7 iterations

$\epsilon = 40\%$

after 10 iterations

Figure 4a
Figure 4b. Reconstruction of model with an 80% velocity contrast in the perturbed velocity. The starting model is assumed to be a homogenous medium with velocity of 2.5 km/s. The dashed curve in the right hand side figures is the reconstructed velocity model, and the solid curve represents the actual velocity model.
Well Separation = 700'
even after 40 iterations. This is a defect of the WT inversion method, which is remedied in Luo and Schuster (1990).

Langan Velocity/Density Model

In the previous two models, the density is kept constant \((2.5 \times 10^{-4} m^3/kg)\) for both forward modeling and inversion. In the Langan velocity/density model, the acquisition configuration and velocity is the same as that in the Langan velocity model (Figure 6a), except the the density profile is that in Figure (8a). This density function was computed with a formula derived from well log measurements (Gardener, 1974)

\[
\frac{\rho}{\rho_o} = \left[ \frac{c}{c_o} \right]^{\frac{1}{4}}
\]

(14)

where, \(c\) is the velocity (Figure 6b), \(c_o = 2000 m/s\), and \(\rho_o = 2.5 \times 10^{-4} m^3/kg\).

In the inversion, the incorrect lightness profile in Figure (8b) is used. Although the density is incorrect, after 20 iterations the WT method still achieves a good velocity reconstruction (Figure 8c).

CONCLUSION

A new seismic inversion method is presented which reconstructs velocities from traveltimes based on the wave equation. No approximations are needed, traveltime picking and event identification are unnecessary, velocities are decoupled from densities, and the computer time is no more than that of full wave inversion. Synthetic tests show that successful reconstructions can be achieved
Figure 6
Figure 7

Initial Velocity Model

After 10 iterations

After 20 iterations
with model velocity contrasts over 80%. This is an improvement over full wave inversion which fails for similar models with little more than 8% velocity contrast.

Future work will research the applicability of the WT method to reflection data, anisotropic data and real crosswell data.
APPENDIX A

FRECHET DERIVATIVE

To obtain the Frechet derivative of the pressure field with respect to the velocity (equation 7c), introduce the pressure field \( p(x,t; x_s) \) propagating in the medium. Then,

\[
\frac{1}{c(x)^2} \frac{\partial^2 p(x,t; x_s)}{\partial t^2} - \rho(x) \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla p(x,t; x_s) \right] = s(t; x_s). \tag{A-1}
\]

\[ p(x,0; x_s) = 0 \quad \dot{p}(x,t; x_s) = 0. \]

The corresponding Green’s function obeys,

\[
\frac{1}{c(x)^2} \frac{\partial^2 g(x,t; x',t')}{\partial t^2} - \rho(x) \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla g(x,t; x',t') \right] = \delta(x - x')\delta(t - t') \tag{A-2}
\]

\[ g(x,t; x',t') = 0 \quad \dot{g}(x,t; x',t') = 0 \quad \text{for} \ (t \leq t') \]

A perturbation of velocity \( c(x) \rightarrow c(x) + \delta c(x) \) will produce a field \( p(x,t; x_s) + \delta p(x,t; x_s) \) defined by

\[
\frac{1}{[c(x) + \delta c(x)]^2} \frac{\partial^2 [p(x,t; x_s) + \delta p(x,t; x_s)]}{\partial t^2} - \rho(x) \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla [p(x,t; x_s) + \delta p(x,t; x_s)] \right] = s(t; x_s) \tag{A-3}
\]
\begin{align*}
p(x,0; x_s) + \delta p(x,0; x_s) = 0 & \quad \dot{p}(x,t; x_s) + \delta \dot{p}(x,0; x_s) = 0. \\

\text{Using}

\frac{1}{(c(x) + \delta c(x))^2} \approx \frac{1}{c(x)^2} - \frac{2 \cdot \delta c(x)}{c(x)^3},

\text{and subtracting equation (A-2) from (A-3) gives,}

\frac{1}{c(x)^2} \frac{\partial^2 \delta p(x,t; x_s)}{\partial t^2} - p(x) \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla \delta p(x,t; x_s) \right] = \\
\frac{\partial^2 p(x,t; x_s)}{\partial t^2} \frac{2 \cdot \delta c(x)}{c(x)^3} + O(\delta c(x)^2),

(A-4)

\delta p(x,0; x_s) = 0 \quad \delta \dot{p}(x,0; x_s) = 0.

\text{Using the Green's function, the solution of equation (A-4) can be written,}

\delta p(x,t; x_s) \approx \int dv(x') g(x_r,t; x',0) * \ddot{p}(x',t; x_s) \frac{2 \cdot \delta c(x')}{c(x')^3} \quad (A-5)

\text{where, " * " denotes time convolution. Since the perturbation occurs only at one special point, set}

\delta c(x') = \Delta c \delta( x' - x )

\text{Then equation (A-5) becomes,}

\delta p(x,t; x_s) \approx g(x_r,t; x,0) * \ddot{p}(x,t; x_s) \frac{2 \Delta c}{c(x)^3}
Divided by $\Delta c$ on both sides, we get equation (7c)

$$\frac{\partial p(x_r, t; x_s)_{cal}}{\partial c(x)} = \frac{2}{c(x)^3} \hat{g}(x, t; x_r, 0) * \hat{p}(x, t; x_s), \quad (A-6)$$

Here, we use reciprocity to allow us to make the exchange $x \leftrightarrow x_r$. 
APPENDIX B

IMPLEMENTATION OF WT INVERSION

FORWARD MODELING

In principle, one can use any forward modeling scheme which simulates wave propagation; we use a staggered grid finite-difference scheme (Virieux, 1984). To use this scheme, we rewrite equation (1) as

\[
\frac{\partial p(x,t; x_s)}{\partial t} = c(x)^2 \rho(x) \nabla \cdot (w(x_r, t; x_s)) + c(x)^2 (s(t; x_s)), \quad (B-1)
\]

\[
\frac{\partial w(x_r, t; x_s)}{\partial t} = \frac{1}{\rho(x)} \nabla p(x,t; x_s),
\]

and \(w\) is the particle velocity vector and the initial conditions are given as

\[
p(x_r,0; x_s) = 0; \quad w(x_r,0; x_s) = 0. \quad \text{for} \quad (t \leq 0)
\]

Here, \(\tilde{s}\) is,

\[
\tilde{s}(t; x_s) = \int_0^t dt \ s(t; x_s)
\]

where \(s(t; x_s)\) is the source term in the second order wave equation.

From this modeling, we can get \(p(x_r, t; x_s)_{cat}\) and \(p(x,t; x_s)\) which will be used in the time correlation with reverse time propagation field \(p'(x_r, t; x_s)\). As for the time correlation in equation (9), we need to multiply the field \(p(x,t; x_s)\) and \(p'(x_r, t; x_s)\) at the same time. We can either choose to store in the computer
memory the entire history of field $p(x,t;x_s)$, or to recalculate it, backward in
time, simultaneously with the calculation of the field $p'(x_r,t;x_s)$ (Gauthier et al
1986). It is this last option that we have chosen. For the recalculation of the
field $p(x,t;x_s)$, we need to store in the computer the history of the field of
$p(x,t;x_s)$ at the boundaries, and, of course, the final two states of the field.

**BACKWARD PROPAGATION**

From equation (9), $p'(x_r,t;x_s)$ should satisfy,

$$
\frac{1}{c(x)^2} \frac{\partial^2 p'(x_r,t;x_s)}{\partial t^2} - \rho(x) \nabla \left( \frac{1}{\rho(x)} \nabla p'(x_r,t;x_s) \right) = \delta \tau(x_r,t;x_s).
$$

In the time correlation of equation (9), the field is $\dot{p}'(x_r,t;x_s)$ not $p'(x_r,t;x_s)$, so
that the time derivative must be taken on both sides of the above equation,

$$
\frac{1}{c(x)^2} \frac{\partial^2 \dot{p}'(x_r,t;x_s)}{\partial t^2} - \rho(x) \nabla \left( \frac{1}{\rho(x)} \nabla \dot{p}'(x_r,t;x_s) \right) = \dot{\delta \tau}(x_r,t;x_s). \quad (B-2)
$$

Again, to use a staggered finite-difference scheme, rewrite (B-2) as,

$$
\frac{\partial \dot{p}'(x_r,t;x_s)}{\partial t} = c(x)^2 \rho(x) \nabla \cdot ( \dot{\nu}'(x_r,t;x_s) ) + c(x)^2 \delta \tau(x_r,t;x_s), \quad (B-3)
$$

$$
\frac{\partial \nu'(x_r,t;x_s)}{\partial t} = \frac{1}{\rho(x)} \nabla \dot{p}'(x_r,t;x_s),
$$

with initial condition,
\[ \dot{p}'(x_r, T; x_s) = 0; \quad \dot{w}'(x_r, T + \frac{1}{2} \Delta t; x_s) = 0, \]

where, \( \Delta t \) is the discretized time step length used in the finite difference method, and \( T \) is the total recording length. The field \( \dot{w}' \) is set to zero at \( T + \frac{1}{2} \Delta t \) because \( \dot{w}' \) and \( \dot{p}' \) are staggered in time. The pseudo-residual \( \delta t \) is calculated from equation (8b) and the \( \Delta t \) is obtained from equation (2). Since this initial condition is an approximation, we need to damp the end of each trace to make this approximation more reasonable.

**DIRECTION OF UPDATING THE MODEL**

Instead of using a steepest gradient direction, we can use some modified direction for updating the model. In general this update scheme can be expressed as

\[ c(x)_{k+1} = c(x)_k + \alpha_k \cdot \phi_k \quad \text{(B-4)} \]

where the steep descent method is given as

\[ \phi_k = \gamma(x)_k \]

where \( \gamma(x) \) is the negative gradient of the misfit function \( S \) given by equation (9). Another modification is the use of a preconditioned gradient direction,

\[ \phi_k = \beta(x)_k = \gamma(x)_k \cdot ||x_i - x_s||^{1/2}. \]
This preconditioning compensates for geometrical expansion. Of course, one can use the well known conjugate gradient direction,

\[ \phi_k = \beta(x)_k + \lambda (\phi_{k-1}) \quad k \geq 2 \]

where,

\[ \lambda = \frac{[\beta_k]' \cdot [\gamma_k]}{[\beta_{k-1}]'(\gamma_{k-1})} \]

and superscript "t" means transpose matrix.

**CALCULATION OF THE STEP LENGTH**

Pica et al (1989) gives a formula for the estimation of step length \( \alpha_k \) in equation (B-4). The final formula is

\[ \alpha_k = \frac{[\phi_k]'\gamma(x)}{[F\phi_k]'F\phi_k} \]  \hspace{1cm} \text{(B-5)}

where

\[ [\phi_k]'\gamma(x) = \sum_x [\phi_k \gamma(x)_k] \]

and

\[ [F\phi_k] = \frac{g[c(x) + \phi_k] - g[c(x)]}{\varepsilon} = \frac{\delta p(x_r,t;x_g)}{\varepsilon}. \]
where, \( g[ c(x) ] \) means forward modeling to get seismograms for the velocity model \( c(x) \).

\[
\left[ F \phi_k \right] \left[ F \phi_k \right]' = \sum_r \int dt \left[ \frac{\delta p(x_r,t;x_s)}{\varepsilon} \right]^2,
\]

where \( \varepsilon \) is estimated by

\[
\max \left\{ \varepsilon \cdot \phi_k \right\} \leq \frac{\max \left\{ c(x)_k \right\}}{100}
\]
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WAVE EQUATION TRAVEL TIME AND WAVE FORM INVERSION

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ABSTRACT

A seismic inversion method is presented which minimizes the misfit functions for both travel times and wave forms using a least-squares criterion. This method, designated as WTW inversion, is a hybrid combination of wave equation travel time inversion (Luo and Schuster, 1990) and full wave inversion (Tarantola, 1987). The WTW method retains the advantages of both full wave inversion and traveltime inversion; i.e., it is characterized by rapid convergence that is somewhat independent of the initial model and it can resolve fine features of the velocity model. The computational cost of the WTW method is only 5-10% more than that for full wave inversion.

INTRODUCTION

There are three criteria which are sometimes used to evaluate the merits of an inversion method; rejection of data noise, degree of model resolution, and convergence rate with respect to different starting models. Among the various inversion methods there are two extremes, travel time inversion ((Dines and Lytle, 1979; Paulsson et al., 1985; Ivansson, 1987; Bishop et al., 1985; Lines, 1988; Justice et al., 1989 and many others) and full wave inversion (Tarantola, 1987; and others). Travel time inversion usually assumes a high frequency approximation of the data and can therefore fail when the earth’s velocity variations are nearly the same wavelength as the source wavelet. In addition, the model resolution of
travel time inversion is less than that of full wave inversion. On the other hand, Luo and Schuster (1990) showed that the traveltime misfit function (normed squared error between observed and calculated travel times) can be shown to be pseudo-linear with respect to the normed difference between the starting and actual velocity model. Hence, successful inversion can be achieved even if the starting model is far from the actual model. Moreover, traveltime inversion is more stable than amplitude inversion for media with random impedance perturbations (which always exist in the real earth).

The model resolution characteristics of full wave inversion are almost complementary to that of traveltime inversion. While very sensitive to the choice of starting model or noisy amplitudes, full wave inversion can sometimes achieve a very high resolution of the model. This is because there are no approximations to the data, and all seismic events are included in the minimization of the misfit function. The problem with full wave inversion, however, is that its misfit function (normed difference between observed and synthetic seismograms) is highly non-linear with respect to the actual and assumed velocity models (Gauthier et al., 1986; Luo and Schuster, 1990). In this case, a gradient method will tend to get stuck in local minima if the starting model is moderately far from the actual model.

Both traveltime and full wave inversion methods have complementary strengths and weaknesses. To exploit these strengths and eliminate the weaknesses, this paper presents a hybrid inversion method which minimizes a weighted combination of traveltime (Luo and Schuster, 1990) and seismogram (Tarantola, 1987) misfit functions. The main benefits are a
convergence rate which is somewhat insensitive to the starting model, high model resolution, no approximations to the data and a robustness in the presence of data noise. Synthetic tests show this new method, designated wave equation traveltime and wave form inversion (WTF), is significantly superior to standard full wavefield inversion.

THEORY

The following analysis assumes that the propagation of seismic waves honors the 2-D acoustic wave equation. Let the pressure observed at the receiver location \( x_r \) (\( r=1,2,...,Nr \)) due to a source at \( x_s \) (\( s=1,2,...,Ns \)) be denoted by \( p(x_r,t; x_s)_{obs} \). The source is always assumed to be initiated at time \( t=0 \). For a given model, \( p(x_r,t; x_s)_{cal} \) denotes the computed seismograms which honor the wave equation

\[
\frac{1}{K(x)} \frac{\partial^2 p(x_r,t; x_s)}{\partial t^2} - \nabla \cdot \left[ \frac{1}{\rho(x)} \nabla p(x_r,t; x_s) \right] = s(t; x_s), \tag{1}
\]

where, \( \rho(x) \) is the density, \( K(x) \) is the bulk modulus, and \( s(t; x_s) \) is the source function at \( x_s \). The inverse problem can be defined as finding the velocity model \( c(x) \) which predicts seismograms \( p(x_r,t; x_s)_{cal} \) that minimize the misfit function:

\[
S = \frac{1}{2} \sum_s \sum_r (\Delta t(x_r,x_s))^2 + \frac{1}{2} \sum_r \sum_s \int dt \delta p \cdot \delta t \cdot \delta p. \tag{2a}
\]
where $\delta p$ is the seismogram residual

\[
\delta p = p(x_r, t; x_s)_{\text{obs}} - p(x_r, t; x_s)_{\text{cal}},
\]

and $\Delta \tau$ is the travel time residual defined as (Luo and Schuster, 1990)

\[
f(x_r, \Delta \tau; x_s) = \max \left\{ f(x_r, \tau; x_s) | \tau \in [-T, T] \right\}.
\]

where $f(x_r, \tau; x_s)$ is defined as

\[
f(x_r, \tau; x_s) = \int \frac{p(x_r, t+\tau; x_s)_{\text{obs}}}{A(x_r; x_s)_{\text{obs}}} \frac{p(x_r, t; x_s)_{\text{cal}}}{p(x_r, t; x_s)_{\text{cal}}}
\]

The $w$ is a weighting factor used to balance out these two residuals and the factor $1/2$ is introduced for subsequent simplifications.

For simplicity we choose a steepest descent method to minimize equation (2c), with the understanding that a conjugate gradient method is used in practice. To update the velocity model, the steepest descent method gives,

\[
c(x)_{k+1} = c(x)_k + \alpha_k \cdot \gamma_k
\]

where $\gamma(x)_k$ is the direction of the steepest descent for the misfit function $S$, $\alpha(x)_k$ is a constant scaling factor, and $k$ denotes the $k$th iteration.

Represent the first term in equation (2a) as $S_1$, the second term as $S_2$, and take the Frechet derivative of $S$ with respect to velocity,
\[
\gamma(x) = -\frac{\partial S}{\partial c(x)} = -\frac{\partial S_1}{\partial c(x)} - \frac{\partial S_2}{\partial c(x)} = \gamma_1 + \gamma_2. \tag{4}
\]

From Luo and Schuster (1990)

\[
\gamma_1 = \frac{1}{c(x)^3} \sum s \int dt \dot{p}(x,t; x_s) \dot{p}_1'(x,t; x_s), \tag{4b}
\]

\[
p_1'(x,t; x_s) = \sum g(x,t; x_r,0) \ast \delta \tau(x_r,t; x_s).
\]

Keeping the density constant, the \(\gamma_2\) term in equation (4a) becomes (Tarantola, 1987)

\[
\gamma_2 = \frac{1}{c(x)^3} \sum s \int dt \dot{p}(x,t; x_s) \dot{p}_2'(x,t; x_s), \tag{4c}
\]

\[
p_2'(x,t; x_s) = \sum g(x,t; x_r,0) \ast \delta \dot{p}(x_r,t; x_s).
\]

Substituting equations (4b) and (4c) into equation (4a) yields

\[
\gamma = \frac{1}{c(x)^3} \sum s \int dt \dot{p}(x,t; x_s) \dot{p}'(x,t; x_s), \tag{5a}
\]

\[
p'(x,t; x_s) = \sum g(x,t; x_r,0) \ast [\delta \tau(x_r,t; x_s) + \delta \dot{p}(x_r,t; x_s)],
\]

where \(\delta \tau\) is the pseudo-travel time residual,

\[
\delta \tau(x_r,t; x_s) = -\frac{2}{E} \dot{p}(x_r,t+\Delta \tau; x_s)_{obs} \cdot \Delta \tau(x_r,x_s), \tag{5b}
\]
with

\[ E = \int dt \dot{p}(x_r, t + \Delta \tau; x_s) \cdot \Delta \tau \cdot \dot{p}(x_r, t; x_s)_{\text{cal}}, \]

and

\[ \delta \dot{p} = \omega \cdot \delta p. \]  \hspace{1cm} (5c)

Luo and Schuster (1990, equation 13) proved, for small \( \Delta \tau \), that

\[ \delta \tau(x_r, t; x_s) \approx \frac{2}{E} \delta p. \]  \hspace{1cm} (5d)

so that \( \omega \) is \( 2/E \). Introducing a hybrid residual, \( \delta h \),

\[ \delta h = \Delta \tau + \frac{2}{E} \delta \dot{p}, \]

we rewrite equation (5a) as,

\[ \gamma = \frac{1}{c(x)^3} \sum_s \int dt \dot{p}(x, t; x_s) \dot{p}'(x, t; x_s), \]  \hspace{1cm} (6a)

\[ p'(x, t; x_s) = \sum_g g(x, t; x_r, 0) * \delta h \]

where, \( p(x, t; x_s) \) is the calculated pressure field for the current velocity model \( c(x) \), and \( p'(x, t; x_s) \) is the field obtained by reverse time propagation of the hybrid residuals \( \delta h \) acting as sources at receiver location \( x_r \). \( g(x, t; x', t') \) is the Green’s function for equation (1) which is the pressure
field at point $x$ and time $t$ due to the impulsive source $\delta(x-x_p)\delta(t-t')$ at
time $t'$ and location $x_p$. The symbol $*$ represents time convolution. In
practice, we use $\delta h$ as,

$$\delta h = a \cdot \Delta \tau + b \cdot \frac{2}{E} \delta \dot{\rho}, \quad (6b)$$

where weights $a$ and $b$ are used to emphasize either $\delta \tau$ or $\delta \rho$. The criteria
we used are

$$a = 1 \quad b = 0 \quad \text{for} \quad \Delta \tau > T/4, \quad (6c)$$

$$a = 0 \quad b = 1 \quad \text{for} \quad \Delta \tau \leq T/4,$$

where $T$ is the period corresponding to the peak frequency of the wavelet.

More sophisticated choices of $a$ and $b$ may improve the inversion results.

When $\Delta \tau$ is large, we emphasize the travel time inversion for which only
normalized seismograms are needed. When the current velocity model is
close to the true one (i.e., $\Delta \tau$ small) we can ignore the changes of amplitudes. Hence, we only use the normalized seismograms for WTW inver-
sion. In the following numerical tests, we always use the normalized seismograms (unless stated otherwise).

**NUMERICAL EXAMPLES**

The WTW method will now be applied to the four velocity models in
Figure 1. The background velocity for all four models is 2.5 km/s, and the
velocity contrasts across steps are 15% in models A and B,
Velocity Models

Figure 1
and range between 40% and 15% in models C and D. The objective of these exercises is to demonstrate the high resolution of the WTW method for low velocity zones (Model B and D) and for sharp interfaces (models A and C). In all cases the starting velocity is a homogeneous model with velocity 2.5 km/s.

Figure 2 demonstrates the failure of full waveform inversion and the success of WTW for Model A. The ability to resolve sharp interfaces is demonstrated in Figure 3, where the top figure resulted from unnormalized amplitude inversion while the bottom figure resulted from normalized amplitude inversion.

Even shadow zones created by the low velocity zone in Model B do not prevent a successful reconstruction in Figure 4. The combination of a sharp and smooth zone in Model C is also successfully reconstructed in Figure 5. And finally, the low velocity zone embedded in a smoothed velocity perturbation is reconstructed in Figure 6.

The Langan model, bane to many ray tracing codes, is given in Figure 7. The cross-well source-receiver is given at the top left of Figure 7, with evenly spaced 8 sources in the source well and 83 evenly spaced receivers in the receiver well. The 1-D velocity profile is given at the top right and the seismograms for a source located at depth 300 m is at the bottom center. For the starting model given at the top left in Figure 8, the WTW method reconstructs a good approximation to the actual Langan model. Figure 9 shows that a more accurate starting model provides a more accurate velocity reconstruction.

CONCLUSION
Comparison of WTW inversion and Full Wave Inversion

Figure 2
Reconstricted Velocity Model A

Figure 3
Reconstructed Velocity Model B

Figure 4
Reconstructed Velocity Model C

Figure 5
After 15 iterations

After 30 iterations

Reconstructed Velocity Model D

Figure 6
Reconstructed Langan's Velocity Model

Figure 8
Reconstructed Langan's Velocity Model

Figure 9
We have presented a velocity inversion method which incorporates the advantages of both travel time and full wave inversion methods. This method makes the success of full wave inversion much less dependent upon the initial velocity model and it is valid when the ray tracing method fails in a complex velocity medium. Moreover, we only use normalized seismograms. All of these advantages make the WTW inversion a feasible method for inverting seismic data and significantly more effective than standard full wavefield inversion. Future studies will apply this method to a real crosswell data set.

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SOLUTION OF THE EIKONAL EQUATION BY A FINITE–DIFFERENCE METHOD

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ABSTRACT

An improved finite difference method is presented which solves the eikonal equation for first arrival times of seismic waves. In this method, the solution region progresses along an "expanding wavefront" rather than an "expanding square", and therefore honors causality. This method is stable and accurate for a variety of velocity models with moderate to large velocity contrasts.
INTRODUCTION

Seismic tomography requires the traveltine calculation of the seismic waves. In most cases, these traveltimes are efficiently and accurately computed by a ray-tracing method. Raytraced traveltimes, however, may sometimes not honor the actual traveltimes of first arrivals. This situation can arise in, e.g., either a wave guide or a shadow zone. Low velocity zones may be characterized by shadow zones where rays fails to emerge; and the first arrivals in a wave guide may be a head wave or due to a complex interference phenomena. In either case, a raytracing method will either fail to find the correct first arrival ray or, in doing so, will require large computational costs. Hence there is a need to develop a traveltine calculation method which is both faster and more accurate than conventional raytracing.

Vidale (1988) presented a method which solves for first arrival travel times by a finite difference solution to the eikonal equation. Rather than solving for traveltimes along a ray path, the eikonal equation solution gives the traveltimes along the wavefronts. Wavefronts will penetrate shadow zones and also account for head waves and interference phenomena in waveguides; hence the eikonal equation solution should be more robust than that from the raytracing method.

In Vidale’s method, the solution region expands along an expanding square. It will be shown that this procedure is both unstable and inaccurate for moderate to large velocity contrasts. This report will present an improved scheme that solves the eikonal equation by a finite difference solution along an expanding wavefront. The new method is stable and accurate for a variety of computed models. The penalty, however, is that the improved scheme is computationally expensive and not optimally suited for vector processors.
FINITE–DIFFERENCE SOLUTION OF THE EIKONAL EQUATION

The 2-D eikonal equation is given by,

\begin{equation}
\left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 = s(x,z)^2, \quad (1)
\end{equation}

where \(s\) is slowness, \(T\) is the travel-time of a wavefront emanating from a certain source and \(x,z\) are the spatial coordinates.

To solve equation (1) by finite-differences, a velocity model is discretized on a rectangular net of evenly spaced grid points. A finite difference approximation to equation (1) is then used to solve for the traveltimes at the grid points. The finite difference approximations to equation (1) are given for the following two cases:

**Scheme 1.**

Suppose the arrival times \(T_a, T_b, \text{ and } T_c\) at points, respectively, \(A, B, C\) are known (Figure 1). The only unknown is \(T_d\) at point \(D\). We construct a new coordinate system, such that the \(Z\) axis intersects points \(B\) and \(C\), and the \(X\) axis intersects points \(A\) and \(D\). A second-order centered finite-difference approximation to equation (1) can be used,

\begin{equation}
\frac{\partial T}{\partial z} = \frac{(T_c - T_b)}{\sqrt{2}h}, \quad (2)
\end{equation}

\begin{equation}
\frac{\partial T}{\partial x} = \frac{(T_d - T_a)}{\sqrt{2}h}, \quad (3)
\end{equation}

where \(h\) is the mesh spacing. Substituting (2) and (3) into (1), and solving for \(T_d\) gives
\[ T_d = T_a + \sqrt{2 \cdot (h \cdot \bar{s})^2 - (T_b - T_c)^2} \]  \hspace{1cm} (4)

where \( \bar{s} = (s_a + s_b)/2 \) is the average slowness between points A and B. The justification for this choice will be shown later.

**Scheme 2.**

Consider the situation in Figure 2 where the arrival times \( T_a, T_b, T_c \) at points A, B, C are known, and the only unknown arrival time is \( T_d \) at point D. A non-centered finite-difference approximation to equation (1) is used to give

\[
\frac{\partial T}{\partial x} = \frac{(T_c - T_b)}{2 \cdot h}, \hspace{1cm} (5)
\]

\[
\frac{\partial T}{\partial z} = \frac{(T_d - T_a)}{h}. \hspace{1cm} (6)
\]

Substituting equations (5), and (6) into equation (1), and solving for \( T_d \) gives

\[
T_d = T_a + \sqrt{(h \cdot \bar{s})^2 - .25 \cdot (T_b - T_c)^2} \hspace{1cm} (7)
\]

Again \( \bar{s} = (s_a + s_b)/2 \) represents an averaged slowness between points A and B. Equations (4) and (7) can now be used to solve the eikonal equation.

**VIDALE'S METHOD, SOLUTION ALONG AN EXPANDING SQUARE**

Vidale (1988) proposed the following scheme to solve equation (1):

**Step 1. (Figure 3)**

Assume that the source is located at point A, to find the arrival times at the four points \( B_1, B_2, B_3 \) and \( B_4 \) use the product of distance and average slowness.
Then calculate the arrival times at the four corner points in Figure 3 using equation (4),

\[
T_{B_i} = h \cdot \frac{(s_{B_i} + s_A)}{2}
\]  

(8)

\[
T_{C_i} = T_A + \sqrt{2 \cdot (h \cdot \bar{s})^2 - (T_{B_{i+1}} - T_{B_i})}
\]  

(9)

when \( i = 4, T_{B_{i+1}} = T_{B_1} \)

\[
\bar{s}_i = \frac{1}{2} (s_a + S_{C_i})
\]

Step 2. (Figure 4)

The grid points in Figure 3 are timed and now the ring of points shown in Figure 4 as filled circles are about to be timed. The hollow circles indicate points that have had their travel time calculated in Step 1.

The points along an edge are identified which are associated with a relative minimum travel-time. A relative traveltime minimum is assumed if there is a traveltime minimum along the previously timed adjacent row. To time the identified relative minimum points, finite difference scheme 2 (equation 7) is used. Starting from the minimum time point, the points are sequentially timed along the edge using finite difference scheme 1 (equation 4) until either a corner point or a relative maximum is encountered. After all four edges are timed, the times of the four corner points can be calculated. Hence, the traveltimes are sequentially calculated along the "expanding squares" until traveltimes at all grid points inside the model are known.

The problem with this "expanding square" strategy is that it is invalid for models with moderate to large velocity contrasts. It is because causality, "that is,
the time for the part of the ray path leading to a point must be known before the time of the point can be found (Vidale, 1988)", is violated in some cases. This can lead to negative values inside the square root (equation 4 and equation 7) resulting in completely erroneous travel times.

To illustrate this point, consider the two layer model in Figure 5 where the source is located within the low velocity layer. The actual first arrival at point D is the head wave transmitted along the path A-B-C-D. However, the expanding square scheme will time point D before the edge of the square reaches the interface. Hence the contribution of the high velocity layer is not included. This results in an incorrect time at point D as well as at many other points. Figure 6 is a sketch to show the difference between the actual wavefronts and those calculated by the "expanding square" method where the mis-calculation of the head waves is clearly seen.

NEW METHOD, SOLUTION ALONG AN EXPANDING WAVEFRONT

To ensure that causality is preserved, a new method is presented which solves the eikonal equation along an expanding wavefront rather than an expanding square. This strategy insures that any point about to be timed will have had its associated ray completely timed up to that point.

Step 1

The first step in this strategy is the same as that in Vidale's scheme (equation 8 and equation 9). Then, after all eight grid points are timed (Figure 3), all times along the outer perimeter of timed points are stored in an "perimeter" array for use in the next step.
Step 2

In this step, the global minimum travel-time point along the outer perimeter of timed points (stored in the array mentioned in the above paragraph) is found. This is the point along the perimeter at which the wavefront first reaches; thus the solution region should be expanded from this point first. To do so, all unknown time points adjacent to this point are timed by using either finite difference scheme 1 (equation 4) or scheme 2 (equation 7). The choice of scheme depends on the situation consistent with the two cases shown in Figure 1 and Figure 2. Thus a new solution region is formed by expanding the perimeter at this point to its neighbors. The procedure is repeated until all the points inside the model are timed.

This last step can be seen in Figure 7. Suppose that the points shown as hollow and filled circles (Figure 7a) form the solution region at a certain time and the actual wavefront is shown as the dashed curve. All times at filled circles along the perimeter are stored in a perimeter array, including the minimum travel time point shown as a filled circle inside a hollow circle. The points adjacent to the minimum traveltime point, shown as hollow squares in Figure 7b, are then timed by using finite difference scheme 1 (equation 4 for the left and right side square points) and scheme 2 (equation 7 for the top square point). Thus, a new solution region is formed and the new outer perimeter points are used to refresh the "perimeter" array. A new minimum travel time point is found among the "perimeter" array elements and the same procedure is repeated (Figure 7c). Applying the above procedure iteratively, all the points in the model can be timed. Thus the eikonal equation is solved using a finite difference method which honors causality.

Sometimes, there may be a situation in which neither equation (4) nor
equation (7) can be used when expanding the solution away from a minimum traveltime point. This is the case when some of the previous values as \( T_a, T_b, \) or \( T_c \) in Figure 1 or Figure 2 are not known. Fortunately, the necessary values can be calculated from times at other points, and this procedure is not a serious violation of causality. When a grid point at the model boundary is timed, it will not be considered as a new point along the solution perimeter; the solution should stop at the boundary.

The average slowness used in equation (4) or equation (7) is \((S_a + S_d)/2\) (see Figure 1 and Figure 2). This is reasonable because point A in Figure 1 or Figure 2 is usually the global minimum travel-time point. As an approximation, we assume the wave is transmitted to point D along the direction of A-D, so that the slowness along \( \overline{AD} \) is more important than that along \( \overline{BC} \). Hence, an average slowness weighted along \( \overline{AD} \) is used; the slowness weight at points B and C equal zero and that at point A and D equal 0.5.

One of the most frequent problems encountered with the "expanding square" scheme is that the square root arguments in equation (4) or equation (7) can become negative at interfaces. This occurs because, first, as shown in the above section (Figure 6), this method may give incorrect traveltimes when the edge of the "expanding square" reaches the high velocity boundary. Second, for a large velocity contrast, the average slowness may be greatly decreased. In some cases, the positive part (\( 2(h \cdot \vec{\nabla})^2 \) in equation (4) or \( (h \cdot \vec{\nabla})^2 \) in equation (7)) may become smaller than the corresponding negative part. In the new "expanding wavefront" scheme the square root argument going negative is largely eliminated. It appears that the local problem of a negative square root is alleviated if the global problem is solved.
LAYERED LANGAN MODEL

The "expanding wavefront" scheme is applied to a layered model approximated from the sonic log data in Langan (1988). The velocity distribution is shown in Figure 8 and varies from 1500 to 2600m/s. The grid is 401 by 401 with a mesh spacing of 1.0 m and the source is located at the grid point coordinates (241, 231).

Figure 9 is the contour map of the travel-times calculated from the eikonal equation for the crosswell model. Each equi-time contour corresponds to a wavefront. The wavefronts are quite complex and include the traveltimes in regions where raytracing suggest a shadow zone. Figure 10 depicts the shadow zones and wave guides encountered in the Langan model. To verify the accuracy of traveltimes computed from the eikonal equation, seismogram along a vertical geophone line at nx=26 and a horizontal geophone line at nz=26 were calculated by a wave equation finite difference method; the velocity model, geophone and source locations were the same as that used in the Langan model. The first arrival times of the seismograms were computer picked by identifying the first onset of a pre-determined amplitude (the value of the pre-determined amplitude will affect the picked time, though not seriously). The comparison between the two results are shown in Figure 11 (horizontal geophone line) and Figure 12 (vertical geophone line) and Figure 13 and 14 show the corresponding travel-time differences \( t_{\text{wave equation}} - t_{\text{eikonal equation}} \). The agreement is fairly good except between the depths range of 150m to 270m in Figure 14. The reason for this discrepancy may be that the source frequency used in the wave equation method peaked at 90 Hz, whereas the eikonal equation is a high frequency approximation to the arrival times. A two point raytracing method was also used
for this model. Figure 15 depicts the results accompanied by the eikonal equation results. Figure 16 is the seismogram computed by the finite difference solution to the acoustic wave equation overlaid with first arrival times computed by a raytracing method (Langan, 1988). It can be clearly seen from the two figures that the eikonal equation method is much more effective than the raytracing method for the Langan model.

2-D CALNAN MODEL

Next we applied our method to a 2-dimensional velocity model reconstructed from crosswell traveltime data by Calnan (1988). Figure 17 is the contour of the slowness distribution where the velocity varies from 3619 to 7029ft/s. The grid is 501 by 303 with a grid point spacing of 2 ft and the source is located at the grid coordinate (251,3). Figure 18 is the comparison between cross-well travel-times using the eikonal equation method and two-point raytracing method with a geophone line located at \( n_x = 303 \). In this figure, the vertical bars represents the raytraced traveltimes and the solid curve represents the traveltimes from the eikonal equation. Shadow zones in the raytracing results are characterized by a large interval between two adjacent bars. Figure 19 is the difference between the raytracing and eikonal equation traveltimes, where the maximum difference is less than \( 10^{-3} s \). It is difficult to determine which result is more accurate, because the greatest traveltime discrepancy is between the depths of 450-600 ft. where the raytracing method encounters a shadow zone.

Figure 20 is a cross well seismic profile (collected by staff members at Exxon Production and Research Company), the Calnan velocity model was reconstructed using traveltimes from seismic profiles such as this, for the same source
and geophone line locations as we used in the raytracing method. Figure 21 is the comparison between the eikonal equation traveltimes (curve) and the picked traveltimes from the real cross well data (vertical bar). The gaps between two adjacent vertical bars also correspond to regions of ambiguous traveltime picks in the real data. It is important to note that the traveltime pick error in the real crosswell data was judged to be about 1 ms; this means that the eikonal traveltime errors are less than the noise level in the real data.

CONCLUSIONS

A finite difference scheme is presented which accurately solves the eikonal equation for traveltimes in complex velocity models. It largely overcomes the causality problem in Vidale's (1988) expanding square method and can compute first arrival traveltimes in problem areas such as shadow zones and wave guides. The disadvantage with this new scheme is that it is computationally expensive requiring about $O(N^2)$ logical statements, where $N$ is the number of grid points along a square model. Careful code design, however, should significantly decrease this computation time. This scheme can also be adapted to the three-dimensional problem and perhaps to transverse anisotropy. A still unresolved problem with this new scheme is that the argument of the square root occasionally becomes negative. For the models we tested, this was not a problem.

In travel-time tomography, the eikonal equation method may offer a good alternative to the traditional raytracing method. But when the media is too complex, then both raytracing and eikonal equation methods may not be applicable. Both methods require that the seismic wavelength be less than the spatial wavelength of the velocity distribution. Therefore source frequency may be a
significant factor affecting the travel times in such regions.

ACKNOWLEDGMENTS

We are grateful to our colleagues involved with the University of Utah Seismic Tomography Group for their kindly help. We also thank the Gas Research Institute and Bruce Smith for encouraging the initiation of this project.
REFERENCES


FIGURE CAPTIONS

Figure 1. Centered finite-difference mesh, where we wish to time point D, given the travel-times at points A, B, and C.

Figure 2. Non-centered finite-difference scheme, where D is the point to be timed, and A, B and C are points whose travel-times are already known.

Figure 3. Finite-difference mesh about source point A, where Bi and Ci for \( i = 1, 2, 3, 4 \) are to be timed.

Figure 4. Figure illustrating the expanding square method (Vidale, 1988). Point A is the source point and the points shown as filled circles are about to be timed. The travel-times at hollow circles are known.

Figure 5. An example to show the invalidation of the expanding square method, where A is the source point and D is the point to be timed. \( V_2 \) and \( V_1 \) are the velocities of the two layers and \( V_2 > V_1 \).

Figure 6. A sketch to show the differences between (a) the actual wavefront and (b) the wavefront calculated by the "expanding square" method for a 2 layer model.

Figure 7. Figure illustrating the "expanding wavefront" method. (a) The solution region and the minimum traveltime point (filled circle inside a circle ). The dashed curve represents the actual wavefront. (b) The solution region is expanded to the points (hollow square points) adjacent to the minimum traveltime point. (c) New solution region and new minimum traveltime point among the new perimeter points.

Figure 8. Approximate velocity distribution of Langan model (sonic log data). Velocities vary from 1500 to 2600 m/s.
Figure 9. Contour plot of travel-times for Langan model calculated from eikonal equation. It shows the possible wavefronts in the media.

Figure 10. Raypaths of crosswell data for Langan's model. Where shadow zones, wave guides and caustics can be clearly seen. (courtesy of Bob Langan, 1988)

Figure 11. Comparison of the first arrival times calculated by the eikonal equation method and the traveltimes calculated by a finite difference solution to the wave equation. The model is the Langan model shown in Figure 8 and the horizontal geophone line is located at nz=26. The traveltme agreement between the two methods is excellent.

Figure 12. Comparison between the travel-times calculated by the eikonal equation method and the finite difference solution to the wave equation method. The location of the geophone line is along a vertical line at nx=26. There is a traveltme discrepancy between the depths of 150 and 270m. This might be expected since a high frequency result (eikonal equation) is compared to a finite frequency result (wave equation, 90 Hz).

Figure 13. Difference \( t_{wave \ equation} - t_{eikonal \ equation} \) between the traveltimes computed from the wave equation and eikonal equation at the horizontal geophone line (Figure 11).

Figure 14. Difference \( t_{wave \ equation} - t_{eikonal \ equation} \) between the traveltimes computed from the wave equation and eikonal equation at the vertical geophone line (Figure 12).

Figure 15. Comparison of traveltimes calculated by the eikonal equation method and raytracing method. "*" represents the raytracing results. Stars located along the zero traveltme line indicate that the raytracing failed
to find rays leading to these points.

Figure 16. Seismogram computed by the finite difference solution to the acoustic wave equation overlaid with first arrival times computed by a raytracing method. The velocity model is that in Figure 8 (courtesy of Bob Langan, 1988).

Figure 17. Velocity contours of Calnan model. Source is placed at node (251,3), and velocities vary from 3619 to 7029 ft/s.

Figure 18. Comparison of travel-times calculated by the eikonal equation method (solid curve) and that of the raytracing method (vertical bars). The large gaps between adjacent bars correspond to shadow zones.

Figure 19. The traveltime difference between the two results shown in Figure 18 where the largest traveltime discrepancy appears near a shadow zone at the depth of 450 ft. It is important to note that Calnan (1989) reported an average traveltime error \( t = t_{\text{observed}} - t_{\text{calculated}} \) of about 1 ms for the real crosswell.

Figure 20. Real crosswell data associated with the Calnan model for the same source and receiver locations used in Figure 18. The distance between the two adjacent traces is 10 ft.

Figure 21. Comparison between travel-times calculated by eikonal equation method (curve) and picked arrivals from real cross-well data (bar).
Figure 3

Figure 4
Figure 6
Figure 8
Contour of Traveltimes of Langan Model

Distance (m); Source at (241,231)

Figure 9
Figure 10
Traveltimes by Eik-E & W-E Results

Distance (m); Horizontal Geophone Line; "*" represents Eik-E results

Figure 11
Figure 12

Traveltimes by Eik-E & W-E Methods

Depth (m); Crosswell Data; "*" represents Eik-E results
Figure 13
Figure 14
Traveltimes by Eik-E & Raytrace Methods

![Graph showing traveltimes vs depth](image)

- Depth (m); Crosswell Data; "*" represents raytracing results

**Figure 15**
Figure 17
Comparison between Eikonal Equation and Raytracing Results

![Graph showing comparison between Eikonal Equation and Raytracing Results. The graph plots travel times (s) against cross-well data and depth (ft).]

Figure 18
Figure 19
Comparison between Eikonal Equation Results and Real Data

Figure 21
INVERTING THE EIKONAL EQUATION

Gerard T. Schuster
Abstract

A gradient optimization scheme is presented which uses traveltimes from the eikonal equation to invert for velocities. The development closely parallels that of full wave inversion.

Introduction

First arrival times can be computed by solving the two-dimensional isotropic eikonal equation

$$\left(\frac{\partial T(x)}{\partial x}\right)^2 + \left(\frac{\partial T(x)}{\partial z}\right)^2 = s(x)^2$$

(1a)

by a finite difference scheme, where $s(x)$ is the slowness of the medium, $x$ is the observation position and $T(x)$ represents the first arrival time of a seismic wave from a point source. Vidale (1988) proposed a finite difference method which solves the eikonal equation along an expanding square. This approach, however, proves to be invalid for moderate to large velocity contrasts due to causality violation; rays emerging from one face of the square can intersect another face to introduce large traveltime errors. To avoid this problem, Fuhao et al. (1990) proposed differencing along an expanding wavefront. This approach has proved to be accurate for earth models with over 200% velocity contrasts. Hence, traveltimes computed from the eikonal equation may be used by exploration geophysicists.

The main advantage in using the isotropic eikonal equation rather than ray tracing is that the finite difference solution to the eikonal equation can, in principle, accurately compute traveltimes in shadow zones, in wave guide zones, and in regions where head waves are extant. Ray tracing of traveltimes in these situations will be difficult or even fail.
Hence, eikonal equation traveltimes may be more accurate than ray traced traveltimes in transmission traveltime tomography. Therefore, we need to determine how to invert the eikonal equation.

**Inverting The Eikonal Equation**

The eikonal equation can be inverted by using a gradient optimization algorithm and Green's theorem. We will demonstrate this in the context of traveltime tomography.

In traveltime tomography, a seismic source gives rise to wavefronts where the associated traveltimes, \( d(x_g) \), are observed at geophone locations \( x_g \). The goal of traveltime tomography is to invert for velocities by adjusting the assumed slowness model, \( s(x) \), until the theoretical traveltimes, \( T(x_g) \), match the observed traveltimes. This can be accomplished by perturbing the objective function \( \epsilon \)

\[
\epsilon = \frac{1}{2} \int_{x_g} [T(x_g) - d(x_g)]^2 dx_g.
\]

(1b)

with respect to perturbations in the slowness model. The objective function is the sum of the squared errors between the observed and theoretical traveltimes. The updated slowness is then given by

\[
s(x_s)^{(i+1)} = s(x_s)^{(i)} - \alpha \frac{\delta \epsilon}{\delta s(x_s)}
\]

(2)

where \( \alpha \) is the step length which minimizes \( \epsilon \) in the search direction \( \frac{\delta \epsilon}{\delta s(x_s)} \), and the superscript \( i \) denotes the \( i \)-th iteration. This is a steepest descent procedure, but a conjugate gradient method can also be used.

The perturbation of \( \epsilon \) with respect to slowness can be explicitly derived from (1b) to give
\[
\frac{\delta e}{\delta s(x_s)} = \int_{V_s} \frac{\partial T(x_g)}{\partial s(x_s)} [T(x_g) - d(x_g)] d x_g
\]
(3)

The quantity in square brackets is the traveltime residual, and the change in the traveltime at \(x_g\) due to a perturbation in slowness at \(x_s\), \(\frac{\partial T(x_g)}{\partial s(x_s)}\), can be shown in appendix A to be \(\vec{\mathcal{G}}(x_s | x_g) s(x_s)\), where \(\vec{\mathcal{G}}(x_s | x_g)\) is the solution to

\[
\vec{\nabla} \cdot (\vec{r} \vec{\mathcal{G}}(x_s | x_g)) = -\delta(x_s - x_g)
\]
(4)

where

\[
\vec{r} = \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j}
\]

The interpretation of equation (3) is that the traveltime residuals at the geophone locations \(x_g\) are backprojected into the medium at location \(x_s\) using the Green's function \(\vec{\mathcal{G}}(x_s | x_g)\).

References


Appendix A

The analytic expression for the perturbation of traveltimes with respect to the slownesses can be derived by perturbing equation (1a) to give

\[
\left( \frac{\partial T (x)}{\partial x} \right) \frac{\partial \delta T (x)}{\partial x} + \left( \frac{\partial T (x)}{\partial z} \right) \frac{\partial \delta T (x)}{\partial z} = s (x) \delta s (x)
\]

(A.1)

\[
\vec{r} \cdot \vec{\nabla} \delta T (x) = s (x) \delta s (x).
\]

(A.2)

where \( \vec{r} = \vec{\nabla} T (x) \) is the normal to the wavefront, or the ray direction. The brute force use of this equation is to determine the finite difference solution to (A.2) for a particular \( \delta s (x_g) \). This process can be repeated for all slowness perturbations so that we have the necessary gradient information for the steepest descent method.

A more elegant approach is to use Green's theorem. That is we define the following impulse equation

\[
AG (x | x_g) = \vec{r} \cdot \vec{\nabla} G (x | x_g) = \delta (x-x_g)
\]

(A.3)

and its adjoint

\[
\widetilde{AG} (x | x_g) = - \vec{\nabla} (\vec{r} \widetilde{G} (x | x_g)) = \delta (x-x_g)
\]

(A.4)

Equation (A.3) says that the gradient of \( \widetilde{G} (x | x_g) \) is perpendicular to the gradient of \( T (x) \), or the wavefront normals. This is true everywhere except when the observation position \( x \) coincides with the geophone position \( x_g \). Therefore \( \vec{\nabla} \widetilde{G} (x | x_g) \) is interpreted as the tangent to the wavefront of the equal time contours of \( T (x) \).

Multiplying equation (A.4) by \( \delta T (x) \) and equation (A.1) by \( \widetilde{G} (x | x_g) \), and

\[
\delta T (x) \widetilde{AG} (x | x_g) - \widetilde{G} (x | x_g) \Lambda \delta T (x)
\]

(A.5)

\[
= \delta (x-x_g) \delta T (x) - \widetilde{G} (x | x_g) s (x) \delta s (x)
\]
Using the product rule of differentiation on the LHS of equation (A.5) we get

\[
LHS = - \nabla \cdot (\delta T(x) \bar{r}(x) \bar{G}(x \mid x_g)) + \bar{G}(x \mid x_g) \bar{r}(x) \cdot \nabla \delta T(x) \\
- \bar{G}(x \mid x_g) \bar{r}(x) \cdot \nabla \delta T(x) = - \nabla \cdot (\delta T(x) \bar{r}(x) \bar{G}(x \mid x_g))
\]  

\text{(A.6a)}

Inserting (A.6a) into equation (A.5), integrating the result over all space and using Gauss's theorem to reduce the 2-D volume integration to a 1-D surface integration we get

\[
- \int \nabla \cdot (\delta T(x) \bar{r}(x) \bar{G}(x \mid x_g)) d^2x = - \int_S \delta T(x) \bar{G}(x \mid x_g) \bar{r}(x) \cdot \hat{n} dx
\]

\[
= \delta T(x_g) - \int_V \bar{G}(x \mid x_g) s(x) \delta s(x) d^2x
\]

\text{(A.6b)}

The surface integral over \(S\) in equation (A.6b) is zero because we assume that the velocity along the surface is extremely large, so that the gradient of traveltime, \(\bar{r}\), is negligible at the surface.

Therefore it follows from (A.6b) that

\[
\delta T(x_g) = \int_V \bar{G}(x \mid x_g) s(x) \delta s(x) d^2x
\]

\text{(A.7)}

and if \(\delta s(x) = \delta(x - x_s) \delta s(x_s)\) then equation (A.7) becomes

\[
\frac{\delta T(x_g)}{\delta s(x_s)} = \bar{G}(x_s \mid x_g) s(x_s)
\]

\text{(A.8)}

Substituting equation (A.8) into equation (3) yields

\[
\frac{\delta \xi}{\delta s(x_s)} = s(x_s) \int_{x_k} \bar{G}(x_s \mid x_g) [T(x_g) - d(x_g)] d x_g
\]

\text{(A.9)}
The interpretation of equation (A.9) is that the traveltime residuals act as sources in equation (A.4). Solving this PDE by finite differences yields the gradient of the misfit function for all of the slowness cells. Hence, one finite difference solution to the adjoint equation (A.4) yields the perturbation of travel times with respect to all slowness perturbations. It is to be noted that the use of Gauss’s Theorem in equation (A.6) requires certain smoothness constraints on $T(x)$ as a function of $x$. This may cause difficulties in practice because traveltime wavefronts may not have continuous first derivatives.
Automatic Picking of Reflected Seismic Arrivals:
Improvement by Contour-Slice F-K Filtering

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ABSTRACT

Contour-slice F-K filtering is shown to be an efficient method for separating downgoing and upgoing wavefields in seismic crosswell data. Moreover, the reflected arrivals may be enhanced in the F-K domain by removing energy from undesirable events, such as tube waves. Reflections contain important information to be used in tomographic inversion methods, and accurate picks of the reflected travel times are necessary to obtain realistic images of the area between the wells. Due to the statistical suppression of noise by the inversion routines, the quality of tomographic reconstructions are usually dependent on the number of travel times involved. However, the amount of data available in crosswell experiments is often overwhelming, and manual extraction of the travel times may be tedious and subjective.

A computerized method for picking reflected arrivals from common shot point gathers recorded in crosswell, VSP, and surface seismic surveys is presented. The picking is performed as a correlation between a small window containing an arbitrary hand-picked reflected arrival from each record and relatively larger picking windows for the remaining traces, defined from a priori knowledge of the approximate reflection arrival times. Both the correlation operator and the picking windows are tapered at the ends in order to avoid edge effects.

Both the contour-slice F-K filtering technique and the automatic picking method has been applied to data from a high-resolution crosswell experiment. The downgoing and upgoing wavefields were successfully separated, and several reflectors were significantly enhanced. The difference between the computer-picked reflection travel times and equivalent hand-picks stayed within the time of ten samples for the unfiltered record and within four samples for the downgoing wavefield. Also, the automatically picked reflected event from the upgoing wavefield was considerably smoother than equivalent picks from the unfiltered data.
INTRODUCTION

Various tomography-type modeling methods are shown to be valuable techniques for estimating sub-surface parameter distributions (e.g., Bishop et al., 1985, Amorim, 1987; Olsen, 1989). An obvious advantage of these methods is the ability to evaluate several observations at each station so that the effect of random noise is minimized. However, a routinely extraction of large amounts of travel times from the seismic records is tedious and subjective, and an automatic picking method would be desirable. be a or less required.

Several procedures have been suggested for automatic picking of seismic arrivals. Hatherly (1982) picked the first arrivals by evaluating the seismic amplitude normalized with the standard deviation of the previous samples, and Coppens (1985) suggested a correlation of the trace with the energy in a small window normalized with the previous squared amplitudes. These algorithms rely on a seismic quiescence ahead of the seismic arrival, and should not be used to detect reflected arrivals embedded in seismic wavetrains typically containing coherent features such as first arrivals, tube waves and shear waves. Coppens (1985) and Peraldi & Clement (1972) applied correlation techniques to pick the first breaks. Correlation is expected to be unstable when the signature of the first arrival is changing abruptly between two adjacent traces (Olsen, 1987). Multi-trace operations, such as correlation procedures, have been applied successfully to extract reflected arrivals, primarily due to more pronounced coherency of these events (Paulson and Merdler, 1968).

A serious problem in recognizing reflected arrivals from unstacked seismic sections is attenuation due to transfer of energy to undesirable events (such as tube waves). By removing this energy, the reflections may be enhanced. F-K filtering techniques permit the selective separation of seismic events displaying different moveouts \( \frac{dt}{dx} \) on a seismic section. Upgoing and downgoing wave fields can be effectively separated in the
F-K domain, a property making F-K filtering attractive for improving crosswell seismic data. Standard F-K filtering techniques attenuate energy in F-K space outside of a fan- or 'pie slice' -shaped area, which corresponds to the desired signal. Contour-slice F-K filtering (after Suprajitno and Greenhalgh, 1985) nulls energy below a certain contour level, alleviating aliasing problems due to straight-edge effects encountered with standard, fan-shaped filters. By its nature, the contour-slice filter is also effective at removing low amplitude random noise.

This paper suggests a correlation algorithm for automatic picking of reflected arrivals. The picking method is applied to contour-slice F-K filtered data from a crosswell survey as well as to the unfiltered data in order to analyze the enhancement of reflected arrivals.

**CONTOUR-SLICE F-K FILTERING**

The most important process in contour-slice filtering is the determination of a spectral amplitude contour that best separates desired and undesired energy. The filter algorithm sets all amplitudes below the selected contour to zero while all amplitudes above are left unchanged. Quadrants containing data from wave fields traveling in opposite directions to the desired signal are also nulled.

The contour-slice filtering technique was applied to field data from a crosswell experiment in Texas. The well spacing is 600 ft and the shot and receiver spacing is 10 ft. A total of 98 shots located in well A and 23 shots on the surface of the earth were recorded by 96 channels in well B. Also, 24 surface records of each of 98 shots located in well A were available. The seismic data were sampled for a total time of 1 sec at a rate of .25 ms. However, the data were resampled to .50 ms for the F-K filtering technique.

Figure 1a shows the recorded seismograms from a shot point located at a depth of 500 ft in well A, and Figure 1b shows the corresponding F-K spectra. In this data, it is apparent that much more energy resides in events such as tube waves from the free
surface-borehole interface and borehole bottom than in the buried reflector events. The tube waves would be the dominant feature both in the upgoing (U.G.W.) and downgoing (D.G.W.) wave fields at any given contour level if a single contour filter was applied. But by defining a 'bracket' of reflection energy between two contour levels, applying the contour slice filter at each level, and subtracting the two results; a large portion of the tube wave energy and low amplitude noise was removed, leaving the desired reflection energy.

Aliasing was a major problem encountered with the real data. Figure 2 shows an F-K amplitude spectrum with aliasing. In general, the aliased energy interfered with reflection energy and had to be attenuated. A high-cut filter using a cosine squared ramp was therefore applied to filter all data above 230 Hz. While this alleviated the aliased energy, it also removed the high frequency information from the reflection, causing the reflected wavelet to appear broader in the filtered sections.

Figure 3 shows an example of a F-K filtered section, recorded in well B from a shot point located at a depth of 500 ft in well A. The unfiltered section is shown in (3a) (same as Figure 1a), while the corresponding downgoing and upgoing fields are shown in (3b) and (3c), respectively. Tube wave arrivals emanating from the top and bottom of the well are seen to be among the most prominent features of the unfiltered section. Also, a significant amount of energy is located in coherent events parallel to the primary tube wave arrivals. Three reflection events are clearly recognized in the unfiltered section (Figure 4): the downgoing event from the free surface/bottom of near-surface LVL (R1), the downgoing event initiated at the first arrival of the 75th trace (R2) and the upgoing event appearing from the 15th trace (R3). Although the near-surface reflections constitutes a distinct event, the reflected arrivals are rapidly attenuated with increasing distance from the interface.

The separated wavefields (Figure 3b and 3c) show a significant enhancement of the reflection events mentioned above. Moreover, the energy of the tube wave and first arrivals has been significantly reduced for both the upgoing and downgoing fields.
PICKING ALGORITHM

We suggest a picking algorithm which computes the cross correlation \( S_t \) between a user-specified operator \( C_t \) of \( n_c \) samples, referred to as the correlation operator, and a series of picking windows \( P_i \) of length \( n_p \).

\[
S_t = \sum_{i=1}^{n_c} C_i \cdot P_{i+k} \quad k = 1, 2, ..., n_p - n_c + 1
\]

The samples of the operator window are located symmetrically around the hand-picked reflected travel time of some arbitrary trace from the record to be picked. The correlation algorithm computes the relative travel times automatically, but the hand-picked arrival is needed for an absolute time frame reference of the procedure. Of course, the performance of the algorithm is dependent on the quality of the hand-picked trace, and the picking operator should be selected to be as coherent as possible with the picking windows. For example, the picking accuracy is usually decreased if the picking operator contains parts of nearby events, such as first arrivals or tube waves.

In order to avoid edge effects introduced by truncation, a linear damping has been applied at the ends of both the correlation operator and the picking windows. Also, tapering may be used to depress effects from additional coherent features during the picking procedure. The presence of such correlated noise in the picking windows is expected to increase the number of mispicks from the picking algorithm. However, if extensive tapering is applied, the windows must be carefully selected to avoid a subjective biasing of the picking result. Figure 5 illustrates the effect of tapering the picking windows and correlation operator.

The stability of the algorithm is obviously dependent on the relative and absolute length of the selected correlation operator \( (n_c) \) and picking windows \( (n_p) \). When \( \frac{n_c}{n_p} \) approaches 1, the result of the picking is strongly constrained, which may be desirable if
correlated noise is present. Desirable values of $\frac{n_r}{n_p}$ appear to be between 1/2 and 4/5, and most stable results are obtained with absolute window lengths of 3 – 7 times the predominant period of the reflected arrival.

**MISPICK DETECTION**

The picking algorithm described is developed assuming random noise and a S/N ratio greater than one at the first arrival. When these assumptions are violated, however, a number of mispicks will be delivered by the procedure. To the extreme, the main part of the picked values will be in error, and no information is extracted from the seismograms. Generally, a limited number of erroneously picked travel times are obtained due to correlated noise in a number of traces.

In order to reduce the number of mispicks delivered by the picking routine, the outliers are assumed to be limited in number (< 20 %) and located randomly in the travel time sequence. A running median filter is selected for the operator of mispick detection. This filter is closely related to the $L_1$-norm, that is known to be fairly insensitive to outliers (Menke 1984). An odd numbered filter length (NF) is chosen for consideration by the filter, and a threshold value $\Delta t$ of the difference between the predicted and picked arrival at position (NF+1)/2 is estimated.

A limited amount of randomly distributed mispicks in error of several times the shifts between the arrivals between two adjacent traces are expected to be detected by the median filter. However, these assumptions are often not obeyed, and the number of retained mispicks and rejected true arrivals is determined by the selection of NF and $\Delta t$.

Sensible values of NF and $\Delta t$ are estimated from the amount and quality of data and the characteristics of the desired method of interpretation. A sequence containing a small percentage of travel times in error is expected to be insensitive to the choice of the filter length. In order to retain stability, however, a small value of NF should be selected for a
time sequence containing a large amount of mispicks. An $L_1$-norm inversion or a manual interpretation of the data is fairly robust to a limited number of outliers (evaluated with a large value of $\Delta t$) in the data compared to a least-squares estimate. A sensible choice of $\Delta t$ seems to be on the order of the estimated average time shift of the arrival time between two adjacent traces.

**PICKING OF REAL DATA**

The reflection picking procedure has been tested on the field record shown in Figure 3 from the crosswell data set discussed above. The three significant reflectors ($R1$, $R2$ and $R3$) were picked from the unfiltered section, and the downgoing ($R1$ and $R2$) and upgoing ($R3$) events were picked from the respective F-K filtered records. Figure 4 shows the records with the picking windows superimposed. The ratio $\frac{n_c}{n_p}$ was chosen to be 4/5.

Figures 6 and 7 summarize the picking of $R1$ from the unfiltered section using small and large amount of tapering, respectively. The correlation operator (a) reflects the degree of tapering; (b) and (c) show the computer-picks compared to the equivalent hand-picks before and after mispick filtering, respectively. Finally, (d) shows the differences between the manually and automatically picked arrivals in (c). The efficiency of the mispick detection filter is reduced in areas of mispick clustering. The picking accuracy seems to be within 10 samples (5 ms), which is unacceptable for tomographic applications.

Figures 8 and 9 show the results of picking the $R1$ event from the downgoing wavefield, with a tapering similar to the amounts applied in Figure 6 and 7. Except for a single outlier, the accuracy of both travel time sequences are seen to be within 3 samples (1.5 ms), compared to the manually picked travel times. This accuracy is acceptable for most modelling purposes.

Figure 10 presents the raw (automatic) estimate of the $R2$ event from the unfiltered section for a low (10a and 10b) and a high (10c and 10d) degree of tapering. Due to
well defined picking windows, the extensively tapered correlation operator delivers the smoothest estimate of the reflection event. The same pattern is recognized for the down-going wavefield (Figure 11). Similar to the results from picking $R_1$ event, the extraction of $R_2$ is significantly improved by the F-K filtering.

Finally, the results of picking $R_3$ are shown in Figure 12 (unfiltered section) and 13 (upgoing field). The scattering of the picks from the unfiltered data is unacceptable, although improved by increased tapering. The $R_3$ picks from the F-K filtered section, however, are applicable for travel time inversion if mispick filtering or a relatively high amount of tapering is involved.

CONCLUSION

The application of a contour-slice F-K filter to real data from a high-resolution cross-well experiment proved to be effective in separating upgoing from downgoing wavefields. Additionally, the filter adequately nulled low amplitude, spurious noise. However, the reflection signal and tube wave energy of the same wave field could not be separated completely, and spatial aliasing forced the attenuation of high frequency information, thereby decreasing the data resolution. The upgoing reflections were generally less coherent after filtering than were the downgoing reflections.

A computerized correlation algorithm proved to be an efficient tool for accurate picking of reflection events from the crosswell data set. An unacceptable picking accuracy from the unfiltered record was improved to a level applicable for travel time inversion in the F-K filtered records. Tapering the ends of both the hand-picked correlation operator and the picking windows was seen to minimize the effects from edges and correlated noise. However, the windows must be carefully defined when extensive tapering is applied.

A median rejection filter of seven samples and a threshold value on the order of the
average time shift between the arrivals of two adjacent traces was able to eliminate isolated mispicks.
REFERENCES


LIST OF CAPTIONS

Figure 1  (a) Common shot point gather recorded in well B for a dynamite source, located at a depth of 500 ft in well A, (b) the corresponding F-K spectra to the CSG shown in (a). (D.G.W. = down going wave field, U.G.W. = up going wave field).

Figure 2  Example of aliasing in the F-K amplitude spectrum of a CSG from the cross-well data set.

Figure 3  CSG from the crosswell data set, recorded from a shot point located at a depth of 500 ft. (a) Unfiltered section, (b) downgoing wavefield from F-K filtering and (c) upgoing wavefield from F-K filtering.

Figure 4  The definition of the picking windows for the three events identified in the CSG shown in Figure 3. (a) Unfiltered section, (b) downgoing wavefield from F-K filtering and (c) upgoing wavefield from F-K filtering.

Figure 5  Effect of tapering the picking windows. Four sets of correlation operators and picking windows with different amount of tapering are shown.

Figure 6  Summary of the $R_1$ - extraction from the unfiltered section using a small amount of tapering. (a) correlation operator; the computer-picks compared to the equivalent hand-picks before (b) and after mispick filtering (c) and the differences between the manually and automatically picked arrivals (d).

Figure 7  Summary of the $R_1$ - extraction from the unfiltered section using a high degree of tapering. (a) correlation operator; the computer-picks compared to the equivalent hand-picks before (b) and after mispick filtering (c) and the differences between the manually and automatically picked arrivals (d).

Figure 8  Summary of the $R_1$ - extraction from the downgoing wavefield using a small
amount of tapering. (a) correlation operator; the computer-picks compared to the equivalent hand-picks before (b) and after mispick filtering (c) and the differences between the manually and automatically picked arrivals (d).

Figure 9 Summary of the $R1$ - extraction from the downgoing wavefield using a high degree of tapering. (a) correlation operator; the computer-picks compared to the equivalent hand-picks before (b) and after mispick filtering (c) and the differences between the manually and automatically picked arrivals (d).

Figure 10 Correlation operator and automatically picked travel times for the $R2$ - event from the unfiltered section using a small amount (a, b) and large amount (c, d) of tapering.

Figure 11 Correlation operator and automatically picked travel times for the $R2$ - event from the downgoing wavefield using a small amount (a, b) and large amount (c, d) of tapering.

Figure 12 Correlation operator and automatically picked travel times for the $R3$ - event from the unfiltered section using a small amount (a, b) and large amount (c, d) of tapering.

Figure 13 Correlation operator and automatically picked travel times for the $R3$ - event from the upgoing wavefield using a small amount (a, b) and large amount (c, d) of tapering.
Figure 1 (b)
Figure 2
Figure 3 (a)

Figure 3 (b)
Figure 6

Figure 7
Figure 8

Figure 9
Crosswell Attenuation Tomography

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ABSTRACT

This paper analyzes the possibilities of estimating intrinsic attenuation from seismic travel time and wavelet broadening information. Information about the attenuation factor may be used to delineate features such as steam injection zones and hydrocarbon reservoirs. A 2D full waveform modeling shows that period data, like pulse width or rise time estimates, are less influenced by focusing and radiation pattern than amplitude estimates. The forward problem is solved by a two-point ray tracing algorithm using bending rays in a tomography approach. The inverse problem is formulated in terms of the inverse of the discretized quality factor ($Q^{-1}$) and is carried out as a linear estimate by damped least squares technique. The velocity distribution is considered known from tomographic travel time inversion.

Three different computerized methods for extraction of attenuation information from the records of multiple coverage seismic surveys are presented. The location of the wavelet associated with the first arrival is performed by an automatic travel time picking routine. The automatic picking procedure is efficient and shown to be stable for realistic noise levels.

Synthetic experiments show that it is possible to extract the intrinsic attenuation factor by inverting data picks. When the noise reaches realistic levels, however, filtering of the data and constraints in the inversion must be involved. It is shown that a median rejection filter and a 2D flatness matrix are efficient tools for the extraction of information from picked field data.

Finally, the methods discussed are applied to field data from a crosswell experiment in Texas, USA. Wavelet widths were picked from the seismic records and inverted for a distribution of $Q_i$. Tomograms of the quality factor from the target area were extracted by inversion of three different data estimates.
INTRODUCTION

The seismic method has proved to be a superior tool for the detection of hydrocarbon accumulations. It has been shown that the presence of volatiles in rocks have substantial influence on the velocity ($V_p$) of elastic P-waves (Mavko & Nur, 1978). Velocity is an ambiguous source of attenuation information, however, when lithological variations such as lenses of sandy material in shaley sequences are introduced.

It is a fact that the dissipation (quality) factor ($Q$) of a rock is decreased, when volatiles are introduced (Clark & Tittmann, 1980; Mochizuki, 1982; Tittmann et al., 1980). Expressions for the quality factor in terms of different characteristics of the seismic wavelet have been established. Albright et al. (1988) determined a mean value of $Q_i$ ($Q$) from estimates of the slope and intercept of the linearized expression for the amplitudes (e.g., Telford et al., 1976). Ho-lio et al. (1988) formulated the problem using amplitudes as a tomography approach that has proved to be a valuable technique for estimating subsurface parameter distributions. The parameter vector was discretized into a model composed of rectangular cells, each of a constant value of $Q_i$. Using the ratio of the observed amplitudes for the P- and S-waves the unknown values of the initial amplitudes, radiation pattern and receiver response were eliminated from the equations. In dealing with partially saturated rocks, however, the attenuation coefficients of the P- and S-waves are not expected to be related in a straight-forward manner. Kjartansson (1979) derived an approximate linear relation between the wavelet width and traveltime, with support from field data. Gladwin and Stacey (1974) recommend the use of the pulse rise time as a source of attenuation information. Hauksson et al. (1987) and Ohtake (1987) used the pulse width and rise time characteristics of earthquake recordings to estimate $Q_i$. Badri and Mooney (1987) showed from a shallow seismic field experiment that more reliable $Q_i$ estimates are expected from the wavelet width than by using the vertical characteristics of the impulse (amplitude, rise time).
An obvious advantage of the tomographic methods is the ability to evaluate several observations at each station so that the effect of random noise is minimized. However, these methods more or less require a computerized method to perform a routinely extraction of a large amount of observations from the seismic records. Several procedures have been suggested for an automatic picking of first arrivals. This paper uses a method for the seismic wavelet location by the energy in a small running window without normalization (see appendix).

The objective of this paper is to develop a reliable method for inverting $Q$ from travel times and wavelet widths using a linear expression between wavelet broadening data travel time. Three different characteristics of the seismic wavelet as the source of information about attenuation have been evaluated. The sensitivity of the inversion result to realistic noise levels of the data is determined by synthetic tests. Finally, the methods are applied to real data from a crosswell experiment.

**SELECTION OF DATA PARAMETER**

A 2D finite difference solution of the elastic wave equation was used to calculate synthetic P-SV seismograms. The finite difference solution contains no information about the intrinsic $Q$-factor and will therefore show the accumulated effects of radiation pattern, focusing and scattering. Figure 1a shows an example of the recorded crosswell seismograms from a dynamite charge located a depth of 500 ft down the well, and the same CSG simulated by finite difference solution of the elastic wave equation for an explosion source are shown in Figure 1b. The split-operator scheme suggested by Bayliss et al. (1986) was applied with a spatial discretization of 2 ft and a peak frequency of 100 hz. The velocity model used in the finite difference modeling was obtained from travel time tomography on the same data set. In order to simulate the pressure field recorded by hydrophones for the real data the synthetic traces are calculated by the average of the
normal stresses.

\[ P \approx \frac{\sigma_{xx} + \sigma_{rr}}{2} \]

(1)

The effect of numerical dispersion was minimized by comparing simulations from finer and finer discretizations of the model.

The most widely recognized feature of the seismic wavelet related to the intrinsic attenuation factor is the amplitude. Two estimates of the amplitude from the transmitted pulse are considered here. First, the maximum value within half a period after the first arrival pick is estimated from the common shot point gather shown in Figure 1. The stars represent picks from the real data CSG (Figure 1a), and the circles are picks from the synthetic seismograms shown in Figure 1b. It is remarkable how erratic the real data picks are compared to the picks from the synthetic seismograms. Another estimate of the amplitude that is expected to be more robust to noise is the spectral amplitude, obtained by integrating the power spectrum in a fixed interval after the first pick. Clearly, by comparison with the synthetic modeling this estimate is less erratic than the time domain estimate. However, both of the amplitude estimates are seen to be effected by both focusing, scattering and intrinsic attenuation in an apparently unseparable way.

Another effect of the attenuating medium is the pulse broadening phenomena. Three different period-estimates of the transmitted pulse are considered: The time between the inflection point and the pulse maximum (Figure 3a), the rise time estimate between the zero crossing and pulse maximum (Figure 3b) and the pulse width between zero crossings (Figure 3c). As for the amplitudes, the real data picks are very erratic, especially in the pulse width estimates, which is expected to be due to superposition from slightly delayed wavelets. Obvious outliers are due to failure of the computer picking routine. However, the picks from the synthetic seismograms indicate that the period data are less effected by focusing or scattering than the amplitude estimates. From this analysis the three different period estimates of the transmitted pulse were selected for further study.
MODELING TECHNIQUE

Ray Tracing The forward solution in seismic tomography experiments is determined by tracing rays between the sources and receivers. Numerous methods have been proposed for accurate tracing of rays in laterally varying media (e.g., Langan et al., 1985; Cassel, 1982; Whittall and Clowes, 1979). For these experiments, the tracing of rays are performed in a grid of triangles, each of a constant velocity gradient (e.g., Weber, 1988). Due to a unique definition of the velocity gradients from the velocities specified at the edges of the triangles, no undesirable discontinuities occur in the model during the ray tracing. A shooting and two-point iteration technique was included in the modeling procedure.

Wavelet broadening A measure of the width of the wavelet representing the first arrival ($\tau_i$) can be expressed as

$$\tau_i = \tau_i^0 + C \cdot \int \frac{dt}{Q}$$  \hspace{1cm} (2)

where $\tau_i^0$ is the initial width at $t = 0$, $t$ is the travel time and $C$ is a constant (Kjartansson, 1979). This relation is claimed to be valid for $Q > 20$, where the proportionality factor $C$ is only weakly dependent on frequency.

For a model discretized into blocks, each of constant $Q_j$, equation (1) becomes

$$\tau_i = \tau_i^0 + C \cdot \sum_{j=1}^{NC} \frac{T_{ij}}{Q_j}$$  \hspace{1cm} (3)

where $T_{ij}$ and $Q_j$ is the travel time and quality factor of the j’th block, respectively.

If the model response ($g_i$) is considered a linear function of the distribution of $Q_j^{-1}$, the expression for the inverse problem is given by
\[ g_i = g_i^0 + \sum_{j=1}^{NC} \frac{\partial g_i}{\partial Q_j^{-1}} \Delta Q_j^{-1}, \quad i = 1, \ldots, N \]  

(4)

where \( g_i^0 \) is the response of the initial model and \( Q_j \) is the quality factor in the j’th cell. If the coefficients of attenuation are constant in each cell, the value of \( \frac{\partial g_i}{\partial Q_j^{-1}} \) is simply the travel time in each cell, \( T_{ij} \). For \( g_i^0 = 0 \), a cost function to be minimized is now selected as

\[ \sum_{i=1}^{N} \sum_{j=1}^{NC} d_i - \frac{\partial g_i}{\partial Q_j^{-1}} \Delta Q_j^{-1} \]  

(5)

where \( d_i \) is the i’th data value.

For N observations equation (5) can be cast in matrix notation as

\[ G \cdot x = d \]  

(6)

where

\[ G = \begin{bmatrix} T_{11} & T_{21} & T_{31} & \ldots & T_{NC1} \\ T_{12} & T_{22} & T_{32} & \ldots & T_{NC2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ T_{1N} & T_{2N} & T_{3N} & \ldots & T_{NCN} \end{bmatrix} \]

is the sensitivity matrix,

\[ x = (Q_1^{-1}, Q_2^{-1}, \ldots, Q_{NC}^{-1})^T \]

is the parameter vector and

\[ d_j = \tau_j^{\text{obr}} - \tau_j^{\text{cal}} \]  

(7)

are the data expressed as the difference between the real data picks and the period data calculated from an initial guess of the model parameters. The prime in equation (7) denotes the correction for the proportionality constant C and the source parameter \( \tau_i^0 \).
\[ \tau'_i = \frac{\tau_i - \tau_i^0}{C} \]

From the linear relation with the period data the reciprocal of the intrinsic \( Q \) is estimated in a least squares sense. However, because of the relative high noise level the problem must be stabilized in order to avoid oscillations in the solution. In addition to the usually damping of the parameter change vector we suggest a set of constraints on the flatness of the model parameters.

\[ \tilde{Q}_{REL}^{-1} = [G^T G + \varepsilon^2 D^T D + \varepsilon^2 I]^{-1} G^T d \]  \hspace{1cm} (8)

These constraints are expressed in the matrix \( D \) in equation (8) containing the numerical derivatives in both vertical and horizontal directions.

**SYNTHETIC EXPERIMENTS**

In order to analyze the effect from the error of the estimated source parameter as well as the noise level of the data, a set of synthetic period data was calculated by tracing bending rays in a steam injection model, and a realistic level of Gaussian noise was added. The injection model was simulated by a 15\% low in both \( Q \) and and velocity (similar to Macrides et al., 1988) compared to a constant background value (Figure 4a). A transitional rim was applied around the anomaly. Both crosswell and VSP data were used with a density comparable to the real data set (Figure 4b shows the ray density).

In the first experiment the initial source parameter was assumed to be known exactly. Without constraining the model parameters the shape of the anomaly was only weakly recovered (Figure 4c). Turning the flatness constraints on during the inversion, however, is seen to improve the result considerably (Figure 4d and 4e). Only the relative distribution of the \( Q \)-factor is seen to be recoverable.

In the second (more realistic) experiment the initial parameter was mis-estimated by 50\%. The model (Figure 5a) and ray density is the same as for the first experiment. In the
unconstrained case the anomaly is not recoverable at all (Figure 5c). However, applying the 'soft constraints' with increasing weight the reconstructed anomaly is improving considerably (Figure 5d and 5e). Again, only the relative distribution of $Q$ can be recovered.

**REAL DATA INVERSION**

The $Q$-inversion technique using wavelet broadening data is applied to field data from a crosswell experiment in Texas. The well spacing is 600 ft and the shot and receiver spacing are 10 ft. A total of 98 shots located in well A and 23 shots on the surface of the earth were recorded by 96 channels in well B. Also, 24 surface records of each of 98 shots located in well A were available. The uppermost shot and receiver in the wells were located at a depth of 30 ft and 10 ft, respectively. A sampling interval and time of .25 ms and 1 sec, respectively, were applied in the survey. No information about hydrophone calibration was available.

The cell area is estimated from the dominant wavelet frequency and the source-receiver spacing of the experiment. From the average period of the first arrival observed in the records and a FFT-analysis, a mean value of the seismic frequency is found to be 250 hz, implying a mean wavelength of 25 ft. Hence a cell width of 40 ft was selected leading to a $15 \cdot 25 = 375$ cell model.

Figure 5a shows the 94 seismograms representing the zero-offset traces in the survey. Figure 5b shows the seismic data scaled by a constant value for all the samples involved.

A mean velocity gradient of 1.5 sec$^{-1}$ is indicated from the data. Velocity anomalies of approximately 500 ft/s seem to be located at depths 500-600 ft and 800 ft. Figure 5b indicates that attenuation could be associated with the two low velocity zones. The analysis of the sonic log of the survey gives no positive indication of lithologic variation associated with the expected anomalous zones (Calnan, 1989). Figure 6a shows the rays traced from the intermediate of the 98 crosswell shots to the receivers in the other well.
Caustics and shadow zones are encountered in parts of the model but, in general, the ray paths are seen to be fairly straight. Figure 6b shows the data fit for the CDP-gather in Figure 6a. Even though the outliers have been eliminated from the CDP-gather the real picks are very erratic.

In order to stabilize the inversion it was necessary to assume a considerably smooth distribution of $Q_i^{-1}$ ($\sigma(\Delta Q^{-1}) \approx 0.01$). The velocity model obtained from iterative travel time inversion was used to calculate the travel times for the transmitted arrivals in the real data inversion (Figure 7a). The ray density shows an expected low resolution in the bottom part of the model due to lack of ray coverage (Figure 7b). The tomograms of the $Q$-factor from inversion of the three different period estimates, the inflection point to pulse maximum estimate (Figure 7c), the rise time estimate (Figure 7d) and the pulse width estimate (Figure 7e) show the same overall features: A central high $Q$-anomaly, low at the top and along the wells with a slightly higher $Q$ in a zone at the top of the right well. Unfortunately, no information about the geology was available to evaluate the inversion results.
DISCUSSION AND CONCLUSIONS

Wavelet width and travel time have been evaluated in a linear tomographic inversion for $Q$-values. The tomographic inversion using linear expressions for the wavelet broadening represents a new and unexplored technique for crosswell seismology.

The full waveform finite difference modeling using the estimated real velocity distribution and the period data such as the pulse width and rise time, seem to be less affected by focusing, internal scattering and radiation pattern than the amplitude estimates. The synthetic experiments using a crosswell model and realistic noise levels in the data suggested that the major attenuating features could be recovered in the inversion, if considerable smoothing such as flatness constraints were involved. Care should be taken in estimating the period of the source to avoid a biased inversion result.

The real data analysis suggested that it is possible to extract period data automatically on the computer, although rejection of outliers was necessary. Finally, the $Q$-tomograms from inverting three different period estimates of the transmitted wavelet did agree on the main features, though no geological information was available to evaluate the result.

The inversion results obtained from this work is based on a linear relation between the wavelet width and the quality factor. Future work will have to support this assumption.
REFERENCES


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APPENDIX

The applied wavelet location algorithm evaluates the energy in a running window of $m$ samples, $E(n,n+m)$

$$E(n,n+m) = \sum_{l=n}^{n+m} A(l)A(l) \tag{A1}$$

where $A(l)$ is the amplitude at the $l$th position in the trace.

The picking criterion is defined from a threshold value of (A1). In order to improve the stability of the algorithm when the noise level varies spatially, the threshold value for the picking of the first arrival in the $i$th trace ($T_i^p$) is selected proportional to the standard deviation of the samples preceding the window expected to contain the first arrival

$$T_i^p = C_p \cdot \sigma_i [1, p_1] \tag{A2}$$

where $\sigma_i [1, p_1]$ is the standard deviation of the samples preceding sample no. $p_1$ and $C_p$ is a constant. The value of $C_p$ is conveniently selected from a few trial-and-error pickings of a limited number of shots in the survey.

The length of the picking operator ($m$) is essential for the stability of the algorithm. Fig. 8 shows the response of a specific trace for evaluation of three different values of $m$ ($1/10 \cdot \lambda$, $1/2 \cdot \lambda$, $1.5 \cdot \lambda$). The small value of $m$ defines a steep and oscillating response at the arrival compared to a slower raise of energy and smooth response for the large value. Obviously, there is a trade-off between the accuracy and stability in the choice of operator length. However, a value of $m$ as one half of the wavelength is found to satisfy the demands of precision and stability in most cases.

When high-frequency noise is present previous to the picking window, the threshold value from (A2) is not a stable criterion for the selection of first arrivals. In order to reject the erroneous candidates of arrivals, a test on the energy of the subsequent $nt$ samples after the candidate of arrival at position $nc$, $ET(nc,nc+nt–1)$
\[ ET(nc, nc + nt - 1) = \sum_{l=nc}^{nc+nt-1} A(l) \cdot A(l) \]  
(A3)

is performed. The criterion for passing the test is defined similarly to the approach of the picking algorithm. A threshold value for the pick in the \( i \)-th trace (\( T_i^l \)) is selected by

\[ T_i^l = C_i \cdot \sigma_i [1, p_i] \]  
(A4)

An optimal value of \( nt \) involved in the test is selected by considering the frequency of the noise and the expected minimum distance between adjacent arrivals in each trace. Generally, a sensible choice of \( nt \) seems to be three times the number of samples/period.

The location algorithm described is developed assuming random noise and a S/N ratio greater than one at the first arrival. When these assumptions are violated, however, a number of mispicks will be delivered by the procedure. To the extreme, the main part of the picked values will be in error, and no information is extracted from the seismograms. Generally, a limited number of erroneously picked travel times are obtained due to correlated noise in a number of traces.

In order to reduce the number of mispicks, delivered by the picking routine, the outliers are assumed to be limited in amount (<20%) and located randomly in the travel time sequence. A running median filter is selected for the operator of mispick detection. This filter is closely related to the \( L_1 \)-norm, that is known to be fairly insensitive to outliers (Menke 1984). An odd number of samples (\( NF \)) is chosen to be considered by the filter, and a threshold value \( \Delta t \) of the difference between the predicted and picked arrival at position \( (NF + 1)/2 \) is estimated.

A limited amount of randomly distributed mispicks in error of several times the shifts between the arrivals between two adjacent traces are expected to be detected by the median filter. However, these assumptions are often not obeyed, and the number of retained mispicks and rejected true arrivals is determined by the selection of \( NF \) and \( \Delta t \).

Sensible values of \( NF \) and \( \Delta t \) are estimated from the amount and quality of data and the characteristics of the desired method of interpretation. A sequence containing a small
percentage of travel times in error is expected to be insensitive to the choice of the filter length. In order to retain stability, however, a small value of $NF$ should be selected for a time sequence containing a large amount of mispicks. A $L_1$-norm inversion or a manual interpretation of the data would be fairly robust to a limited number of outliers (evaluated with a large value of $\Delta t$) in the data compared to a least-squares estimate. A sensible choice of $\Delta t$ seems to be at the order of the estimated average time shift of the arrival time between two adjacent traces.
LIST OF CAPTIONS

Figure 1  (a) Common shot point gather recorded in the second well for a dynamite charge located at a depth of 500 ft in the first well. (b) Synthetic seismograms simulated by a finite difference solution of the elastic wave equation for the CSG shown in (a).

Figure 2  (a) Time domain estimate of the first arrival amplitude. (b) Spectral estimate of the first arrival amplitude. The stars represent picks from the real data CSG (Figure 1a), and the circles are picks from the synthetic seismograms shown in Figure 1b.

Figure 3  different period-estimates of the transmitted pulse. (a) The time between the inflection point and the pulse maximum, (b) the rise time estimate between the zero crossing and pulse maximum and (c), the pulse width between zero crossings.

Figure 4  Synthetic modeling of a steam injection model. (a) The model, (b) ray density, (c) Q-tomogram inverted without flatness constraints, and (d) and (e) show Q-tomograms inverted including flatness constraints with increasing weight. The source parameter was assumed to be known exactly.

Figure 5  Synthetic modeling of a steam injection model. (a) The model, (b) ray density, (c) Q-tomogram inverted without flatness constraints, and (d) and (e) show Q-tomograms inverted including flatness constraints with increasing weight. The source parameter was assumed mis-estimated by 50 %.

Figure 6  (a) Ray paths traced from the shot located at a depth of 500 ft in well A, (b) data fit for the CDP-gather in Figure 6a.

Figure 7  Real data inversion. (a) The velocity model used to calculate the travel times for the transmitted arrivals, (b) ray density, (c) Q-tomogram from inversion of the inflection point to pulse maximum estimate, (d) Q-tomogram from inversion of the rise time estimate and (d) Q-tomogram from inversion of the pulse width estimate.
Figure 8  The response of varying length ($m$) of the picking operator for a field seismogram.
FULL WAVEFORM ANALYSIS

SHOT # 51 REAL DATA

(a)

SHOT # 51 FINITE DIFFERENCE MODEL

(b)

Figure 1
\( A_t^{MAX}, S_{PICK} < t < S_{PICK} + T/2 \)

\[
\sum_{f=N_1}^{f=N_2} \text{POWER}(f)
\]
RAY INVERSION

(a)

(b) Figure 7

* REAL DATA PICK
o RAY CALCULATED
Static Corrections Using

Surface Wave Inversion

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Static Corrections Using
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ABSTRACT

This paper examines the feasibility of using surface wave information to provide independent constraints on S- and P-wave velocities. Several approaches to surface wave inversion are examined; 1). separate Love and Rayleigh wave inversions for S- and P-wave velocities. 2). Love wave inversion for S-wave velocities which are then applied and held constant in Rayleigh wave inversion for P-wave velocities only, 3). Love wave inversion for S-wave velocities which is then followed by Rayleigh wave inversion for S- and P-wave velocities, and 4). approach 2 followed by Rayleigh wave inversion for S- and P-wave velocities. Results suggest that S-wave velocities from Love wave inversion are much more reliable than those from Rayleigh wave inversion. Density recovery appears impractical using surface wave inversion. Inversion of synthetic data suggests that a combination of Love and Rayleigh wave inversion will provide the best results. This is especially true if the Love waves are used to reconstruct the S-wave velocities and the Rayleigh waves are then used to recover the P-wave velocities.

To verify the practicality of surface wave inversion, a nine-component surface wave experiment was performed in north-east Texas (courtesy of Arco research labs). A combination of tau-p and Fourier transforms are applied to the YY CSP gather to extract Love wave dispersion curves. These curves clearly
show the fundamental and first harmonic modes from 1.5 Hz to about 6-7 Hz. Inversion using these curves suggests shear velocities of 200 m/s at the near-surface and an abrupt change from 500 to 800 m/s at a depth of 70 meters. This is in very good agreement with the shear velocities from well log data. Results suggest that Love waves might be used to provide long wavelength shear wave statics.
INTRODUCTION

In many geophysical data sets, poor delineation of near-surface velocities contributes to a variety of processing problems including loss of data coherency, cycle skips, and false structures. Too often, residual statics, refraction statics, and/or hand statics fail to remedy the velocity problem. This difficulty is especially severe in shear wave surveys where the shear wave static corrections can be three to four times as large as the compressional static corrections.

It is therefore desirable to develop a method which could provide independent constraints on the velocity structure above the first good reflector. It would also be desirable if this method could be applied to existing seismic data sets. Such a method may be velocity inversion using surface wave data. This is a compelling choice because surface wave amplitudes are an order of magnitude larger than those of other events. Moreover, they are primarily influenced by the near-surface shear velocities, which are the primary culprits in shear wave static problems.

For years, earthquake seismologists have used inversion of earthquake generated Love and Rayleigh waves to investigate S-wave velocity distributions on a crustal scale (Schwab and Knopoff, 1972; Braille and Keller, 1975). More recently, several researchers have begun using these methods in the investigation of engineering scale problems < 3-10 meters (Cherry et al., 1980; Nazarian and Stokoe, 1986; and Barrows and Gahr, 1987). However, there are only a few papers which investigate the usefulness of surface waves in the oil industry (Wattrus, 1989). In particular, the exploitation of combined Love and Rayleigh
wave information is virtually unexplored. These data may have an important bearing on near-surface anisotropy, S-wave and P-wave statics, and the attenuative properties of the weathering zone.

There are several advantages inherent to the surface wave inversion technique. First, surface waves penetrate to a depth of several wavelengths, therefore, velocities can be reconstructed at depths much greater than those evaluated with uphole timings. Second, since the input information comes from geophone stations, the subsurface can be much more densely sampled than upholes would allow. Thirdly, the resulting velocity model may reveal detail in areas of complex layered geology such as seen in areas with lava flows, clinker beds, or near-surface limestone formations. And finally, the results are completely independent of other statics methods such as uphole or refraction, thus providing an additional source of information about the critical shallow velocities. The main disadvantage is the assumption of homogeneous lateral velocities over the length of the shot record. Hence, it is a pseudo 1-D inversion method.

The first section of this paper presents the theory of surface wave inversion along with some details of practical implementation. The next section examines the inversion of synthetic Love and Rayleigh wave data, and is followed by tests on a nine-component set of CSP seismic data.

THEORY OF SURFACE WAVE INVERSION

This section presents the methodology for inverting Rayleigh and Love wave data. The optimization technique is Gauss-Newton and the earth model is assumed to be isotropic and layered. The seismic data will be in the form of
common shot point gathers. The two gathers of particular interest are the YY
gather (Love waves generated by a cross-line component source and recorded by
cross-line component geophones) and the ZZ gather (Rayleigh waves generated
by a z-component source and recorded on a z-component geophone). From these
seismograms the dispersion curves are extracted by a combined tau-p and Fourier
transform (McMechan and Yedlin, 1981). The velocity model is adjusted until
the synthetic dispersion curves match the observed dispersion curves.

The S- or P-wave velocities in the \(i^{th}\) layer are denoted by \(V_i\), and the
data is in the form of common shot records. The seismograms in \((x,t)\) space are
slant stacked and Fourier transformed to yield dispersion curves, or phase velocity
with respect to frequency \(C(\omega, V)\). There is a non-linear relationship (Aki and
Richards, 1980) between the actual velocity model \(V(z)\) and the dispersion
curves \(C(\omega)\). The velocities can be recovered by using a Gauss-Newton method,
consisting of four steps;

Step 1: The S- and P-wave phase velocities \(C(\omega)\), where \(\omega\) is the frequency,
are expanded in a Taylor Series about an initial \(N/2\) layer velocity model
\(V^0 = (V_1^0, V_2^0, \ldots, V_N^0)^T\) :

\[
C(\omega, V) = C(\omega, V^0) + \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} (V^0 - V_i) + \text{higher orders in } (V^0 - V_i).
\]

Rearranging terms we get

\[
\Delta C(\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \Delta V_i + \cdots \tag{1}
\]
where $\Delta C(\omega, V) = C(\omega, V) - C(\omega, V^0)$. The equation is linearized by assuming that the initial guess is close enough to the actual model so that the higher orders of $\Delta V_i$ can be neglected. Therefore:

$$
\Delta C(\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \Delta V_i \quad (2)
$$

Step 2: In equation (2) there are $N$ unknown values of velocity $(V_1, V_2, V_3, \cdot \cdot \cdot V_N)^T = V$. $V$ is related to the $M$ data samples measured from the dispersion curves by

$$
\Delta C(\omega_1, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_1, V)}{\partial V_i} \Delta V_i ,
$$

$$
\Delta C(\omega_2, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_2, V)}{\partial V_i} \Delta V_i ,
$$

$$
\Delta C(\omega_3, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_3, V)}{\partial V_i} \Delta V_i ,
$$

\[ \vdots \]

$$
\Delta C(\omega_m, V) = \sum_{i=1}^{N} \frac{\partial C(\omega_m, V)}{\partial V_i} \Delta V_i ,
$$

or

$$
\Delta C = G \Delta V \quad (3)
$$
where \( \Delta C \) is the column vector of the data, \( \Delta V \) is the column vector of the model parameters, and \( G \) is the MxN sensitivity matrix where:

\[
[G]_{ik} = \frac{\partial C(\omega_i, V)}{\partial V_k}.
\]

To determine \( G_{ik} \), the \( k^{th} \) element of \( V \) is perturbed, the dispersion curve is recalculated, the difference is taken between this new curve and the original dispersion curve, and the result is divided by the velocity perturbation.

Step 3: Find \( \Delta V = (G^T G)^{-1} G^T \Delta C \).

Step 4: Update the velocity model \( V \) by \( \Delta V \) and iterate. It is advisable to normalize the columns of \( G \) prior to the inversion to minimize computational round off errors.

**INVERSION OF SYNTHETIC DATA**

The objective of this section is to determine the optimal method or methods for inverting layer velocities from synthetic surface wave data. Separate inversions of Love and Rayleigh wave data for S- and P-wave velocities will first be examined, followed by combinations of these techniques. The velocity model used for the synthetic tests has 6 layers with a roughly linear velocity gradient (Figure 1). This velocity model was chosen because it is representative of conservative ranges found in the Williston Basin of North Dakota with which the author has personal experience. Forward modeling of this velocity distribution results in the 80 point Love and Rayleigh wave dispersion curves in Figure 2.
The subsequent 20, 40, and 80 data point sets input to the inversion process are taken from these curves. A Singular Value Decomposition algorithm is used to solve equation (3). The starting S-wave velocity is 0.35 km/sec for all layers, while the P-wave velocity is 1.5 km/sec.

**Separate Inversion Algorithms**

Figure 3 demonstrates the convergence rate for inverting shear velocities from a 20 point sample of the Love wave dispersion curve in Figure 2. The solid and dashed lines represent the RMS error in the model velocities, while the dotted and dot-dashed lines represents the RMS data error. The 'Old' curves represent the results of a standard Gauss-Newton algorithm, while the 'New' curves represent results from an accelerated Gauss-Newton algorithm which multiplies the ΔV step length by a value proportional to the number of iterations up to a limit of 50. The Old inversion converges relatively quickly for the first 200 iterations, and thereafter, much more slowly. In the accelerated algorithm, all S-wave velocities converged to within 0.5% of the model values within 200 iterations.

Figure 4 represents the convergence in S-wave velocities using Rayleigh wave inversion; in this case the P-wave velocities are assumed to be correctly known. It can be seen that convergence is much more rapid than that in Figure 3, and is robust for a wide range of starting model velocities. As with the Love wave data, an accelerated Gauss-Newton method significantly increases the convergence rate.

The convergence rate in inverting simultaneously for S- and P-wave velocities from Rayleigh wave data is given in Figure 5. After approximately
300 iterations, the inversion process converged to incorrect S- and P-wave velocity values; apparently, small errors in the S-wave velocities can offset large errors in the P-wave velocities. To correctly recover P-wave velocities from Rayleigh wave data, it is clear that the significant influence of errors in S-wave velocities must be remedied.

Assuming a correct S-wave model, Figure 6 depicts the convergence rate for inverting P-wave velocities from Rayleigh wave data. A multiplication factor of 100 was used to significantly increase the convergence rate. It is clear that without the accelerated algorithm, significant convergence would not have been achieved. The next section will examine the feasibility of inverting for velocities from combinations of Love and Rayleigh wave data.

**Combined Love and Rayleigh Wave Inversion**

A combination of Love and Rayleigh wave inversion appears to hold promise for recovering both S- and P-wave velocities. Since Love waves are independent of P-wave velocities, the first step is to invert for shear wave velocities using Love waves. Figure 7 shows the results from a combined process in which Love waves are first inverted for S-wave velocities and then, after 100 iterations, the Rayleigh wave data are inverted for P-wave velocities. This process is performed using 20 (---) and 40 (- -) dispersion curve data points over the same frequency range. This combination of processes has resulted in recovery of reasonable P-wave velocities from surface wave inversion, though it would not have occurred using the non-accelerated algorithms.

The next test follows Love wave inversion for S-wave velocities with Rayleigh wave inversion for all 12 S- and P-wave velocity parameters (Figure 8).
While this process does not converge as quickly as the previous process, there is a long term trend which will eventually result in more accurate velocities. It is also clear that while the 20 data point process converges more slowly than those of the 40 and 80, it does not warrant the large increase in computing time required to invert the more highly sampled data.

The final test (Figure 9) incorporates thresholds and combines the previous two examples starting with Love wave inversion for the first 45 iterations after which a threshold of 1.0×10^{-6} km/sec in RMS S-wave data error is achieved. The Rayleigh wave algorithm then inverts for the P-wave velocities, ending in another 250 iterations after reaching the RMS P-wave data error threshold of 1.0×10^{-6} km/sec. The inversion process continues to 2000 iterations using Rayleigh wave inversion for S- and P-wave velocities in an attempt to achieve the slightly improved results of the previous example. Little improvement over the results in the previous two examples was achieved.

In summary, synthetic tests suggest that S-wave velocities can be optimally recovered by Love wave inversion. Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for P-wave velocities appears to be a successful strategy for noiseless synthetic data. In practice, however, recovering P-wave velocities from Rayleigh wave data may be difficult due to the significant influence of errors in the S-wave velocities. Inversion for densities appears impractical based on their minimal contribution to dispersion curve character as evidenced in Appendix A.
FIELD DATA

The field data for this report was collected by Arco at their test site several miles west of Sulfur Springs, Texas. The source is an ARIS multidirectional hydraulic impact type. A vertical source response is simulated by summing two records recorded with impacts in opposing diagonal directions so that the horizontal components vectorially cancel. Similarly, a horizontal source response is simulated by computing the difference between the same two records so that the vertical displacement components vectorially cancel. The geophones in the experiment consisted of 11 groups of 6 horizontal and 6 vertical 4.5 Hz elements spaced at 9.144 m (30 ft) intervals. The geophones were planted at one end of the line while the source, starting at a near offset of 219.5 m (720 ft), was moved down each line at 100.6 m (330 ft) intervals to a maximum offset of 1524 m (5000 ft). Layer velocities were determined from a previous VSP experiment. The traces in Figures 10 and 11 represent the YY and ZZ component shot records. Both raw data records exhibit dominant frequencies in the range of 4-5 Hz, and show clear signs of dispersive surface waves.

The YY and ZZ shot records are transformed into the tau-p domain, and then into the frequency-p domain by a Fourier transform over the tau parameter. This yields the dispersion data in frequency-phase slowness space. Further details of the process are included in Appendix B. Figure 12 contains the transformed YY (Love wave) seismic data. The peaks of strongest coherency are picked (dots) to yield the estimated fundamental Love wave dispersion curve. The fundamental mode is clearly represented between 2 and 6 Hz, but is significantly reduced at higher frequencies. It is suspected that deconvolution and filtering could improve the signal to noise ratio in this frequency range. The
lack of low frequency energy below 1.5 Hz could be due to the combination of finite receiver aperture and geophone attenuation. The first higher mode can be seen at $P \approx 1.5$ sec/km between frequencies 3 and 7 Hz.

Shear velocities were obtained from inversion using the fundamental Love wave dispersion curve in Figure 12. The convergence of the RMS data error is shown in Figure 13, and the reconstructed velocity model is shown in Figure 14. The measured VSP velocities are represented by the solid line, the inversion results by the dashed line, and the starting model by the dotted line. There is fairly good agreement between the predicted and actual S-wave velocities. The total one way delay time computed for the 0.1 km (330 ft) model from the measured velocities is 214 ms, while the total one way time for the inverted velocity model is 224 ms for an error of less than 5%. This is a very encouraging result for these preliminary tests. To further verify this result, dispersion curves (Figure 15) are calculated from the reconstructed velocity profile. The agreement between the two curves is very good for the fundamental, and first order harmonic.

The Rayleigh wave dispersion curves extracted from the ZZ seismograms (Figure 11) are shown in Figure 16. The observed fundamental mode curve is quite different than that predicted by forward modeling using the shear wave velocities from Love wave inversion. In fact, the shear velocities inverted from the Rayleigh waves are very different from the VSP velocities. Part of the reason may be in assuming incorrect P-wave velocities. These inconsistencies are currently being investigated.
CONCLUSION

Synthetic tests demonstrate the feasibility of using Rayleigh and Love wave data to reconstruct near-surface S-wave velocities. An accelerated Gauss-Newton method appears to be robust and accurate in inverting dispersion curves for realistic near-surface velocity models. The inversion of P-wave velocities from Rayleigh waves is much more difficult because Rayleigh waves are so sensitive to errors in the shear velocities. Density inversion from Love or Rayleigh wave data appears to be impractical.

Preliminary results from inverting CSP seismic data show much promise. The S-wave velocities reconstructed from the Love waves compare well with the velocities measured from a VSP experiment. In addition, there is very good agreement between the observed and predicted integrated traveltime to the deepest reconstructed layer. That is, the predicted S-wave static correction agrees within 5% of the actual static correction. In addition, the predicted dispersion curve for the first-order harmonic agrees well with the observed data. The S-wave velocities from the Rayleigh wave data did not agree with the VSP velocities. Present work is directed toward improving the signal to noise ratio above 6 Hz, and in resolving shear velocity inconsistencies between the Love wave and the Rayleigh wave data.

ACKNOWLEDGEMENTS

The author thanks Arco Oil and Gas Company for allowing us to participate in their 9-component surface wave experiment. I am extremely indebted to Nigel Wattrus and Jim DiSiena for their invaluable assistance in this project. Discussions with Dr. Wattrus were very helpful in this research.
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the Society of Exploration Geophysicists, Houston, Texas.


APPENDIX A

TWO LAYER MODELS

This section examines the relative importance of S-wave velocity, P-wave velocity, and density on the behavior of dispersion curves. Figure A1a represents the phase velocity of a Love wave as a function of frequency. These curves were generated using a simple 2 layer model of crustal scale using an algorithm described in Schwab and Knopoff (1972). The shallow layer has an S-wave velocity of 3.5 km/sec, a P-wave velocity of 6.1 km/sec, and a density of 2.7 gm/cc. The bottom layer has an S-wave velocity of 4.5 km/sec, a P-wave velocity of 8.1 km/sec, and a density of 3.36 gm/cc. The depth of the interface is at 35 km. The longest curve is that of the fundamental mode, while subsequent curves to the upper right represent the higher modes.

As one would expect, when the frequency approaches 0 Hz and the wavelength increases, the Love wave is influenced by deeper and deeper formations resulting in a phase velocity approaching that of the deeper layer. As the frequency increases, the deeper layer becomes invisible, and the phase velocity approaches that of the shallow layer S-waves.

Figure A1b uses an identical model calculated for Rayleigh waves. The most significant difference is that the dispersion curves asymptotically approaches 90% of the value of the S-wave velocity of the shallow layer. The Love wave computation is a much simpler and straightforward procedure resulting in more logical and predictable behavior.
The next sets of figures show what happens to the dispersion curves when the defining parameters are varied. They clearly demonstrate which geologic variables are significant and in general, for which frequencies they are most significant. Each parameter in the two layer demonstration model above has been varied by plus and minus 20%.

Figure A2 shows the effect of variation in the S-wave velocity for the upper layer. The substantial separation of the dispersion curves demonstrates that this is a significant variable. The lower-most curve in each mode represents a velocity of 2.8 km/sec, while the upper-most curve of each mode represents a velocity of 4.2 km/sec, with six intermediate values between. The Love wave graph shows that at the lowest frequencies, a variation in the shallow layer can not be distinguished, but the Rayleigh wave dispersion curves do diverge and can give some additional information. At the higher frequencies, or shorter wavelengths, variations in shallow velocities are clearly distinguished.

The model in Figure A3 incorporates the +/- 20% shear wave velocity variation in the deeper layer. It is apparent that nearly all resulting divergence appears in the lower frequencies of each mode as one would expect. S-wave velocity again demonstrates a significant contribution.

Figure A4 is associated with variation in the shallow layer's P-wave velocity. Since the Love wave is a shear wave, one would not expect any variation in its dispersion curve, and none is exhibited. The variation seen in the fundamental mode of the Rayleigh curve implies that P-wave velocities can be modeled and measured to some extent with surface waves.
Figure A5 is associated with a +/- 20% variation in the P-wave velocity of the deeper layer while the upper layer remains constant. Once again, the Love wave dispersion curves show no effect. The Rayleigh wave curves show only a slight deviation mostly at the low end of the fundamental. The last two examples demonstrate that most of the P-wave information will be contained in the fundamental mode of the Rayleigh wave, and that this is the place to look if one is interested in inverting for P-wave as well as for S-wave velocities.

The next two figures demonstrate the minimal significance of even a +/- 20% variation in density. Figure A6 has a shallow density range of 2.16 to 3.24 gm/cc, and Figure A7 has a deep density range of 2.69 to 4.03 gm/cc. Since variations of this magnitude would not be observed in our work, and since there is such a small contribution even for these excessive values, modeling for densities is considered impractical.
APPENDIX B

SURFACE WAVE EXTRACTION

The extraction of dispersion curves from seismic shot records is one of the keys to the success of the process. By performing a tau-p transformation of a shot record, followed by Fourier transform over the tau parameter, one will recover a frequency-phase slowness record representing the dispersion curves contained in the original data. The major benefit in using this method is that errors introduced by reflections and random noise will be minimized due to the inclusion of a large number of traces. This section will show the results of using the software packages based on this algorithm applied to synthetic seismic data.

The first example uses the forward modeling program discurv.f to generate frequency-phase velocity data pairs from which fundamental mode values are picked at 1/8 Hz intervals and input into the MATLAB script dispgen.m. Dispgen.m then generates a synthetic seismic section (Figure B1) containing only the dispersive curve surface wave which can then be transformed into SEG-Y format using mat2segy.f. The SEG-Y data is input into taup.f to calculate the tau-p data which is then input into tauplt.f for plotting and formatting, then into zconjF.f for the Fourier transform over tau, and finally into zdecF.m for dispersion curve plotting. The extracted dispersion curve is shown in Figure B2. Since only fundamental mode energy was used in the modeling, all other curves represent aliasing.

The second example generates synthetic seismic records using the finite
difference programs mod.f, sh.f, and psvr2.f. The velocity model is from a preliminary velocity model for the Arco test site. Figures B3 and B4 are the synthetic SH and PSV records plotted using the 'rsx' set of programs. In the finite difference set of programs, mod.f is used to generate the models which are then used by both sh.f and psvr2.f. The output is then converted to ASCII files c.dat and balmax.dat with sh2mat.f or psvr2mat.f. These data files can then be plotted in MATLAB with fdpot.m or converted to SEG-Y format with mat2segy.f for further processing described in the first example.

The extracted dispersion curve results (Figures B5 and B7) can be compared to the forward model results (Figures B6 and B8) generated with discurv.f for the same velocity model. Both the Love and Rayleigh wave dispersion curves can be seen to match the forward models exactly. The fundamental mode values can easily be picked down to about 2 Hz. The first higher modes can also be matched to the forward model values though their amplitudes with respect to the fundamental mode are greatly diminished.

Several aspects of the results should be addressed. The first is the problem of aliasing. Both synthetic examples have maximum offsets of 1.5 km (5000 ft). Example 1 has 24 traces spaced at 64 m (210 ft) while the second has 48 traces spaced at 32 m (105 ft). The aliasing manifests itself as additional curves in the lower right hand corner of the plots in Figures B2, B5, and B7. It can be seen that the finer sampling in the second and third figures forces these curves away from the fundamental mode curve, helping to expose the higher modes. It is also apparent that the aliasing in
the more coarsely sampled record does not interfere with the ability to pick data values from the fundamental mode curve.

When extractable, higher order modes would no doubt aid a great deal in the inversion process, though writing a program that could automatically identify, separate, and pick higher mode values would be more difficult. The higher order modes are also much weaker than the fundamental as evidenced in Figures B5 and B7. In the presence of real world noise, these higher order modes may be difficult to pick out reliably.

It has been seen that the width, and subsequently the ease of picking frequency-slowness data, is greatly affected by the amount of offset contained in the shot record. Figure B5 shows a significant widening in the fundamental curve below 2 Hz. A 48 trace record with a maximum offset of 2.25 km (1.4 miles) results in a constriction of the dispersion curve in Figure B9 down to 1.5 Hz, while a 48 trace 4.8 km (3.0 miles) record will afford easy picking down to 0.5 Hz as evidenced in Figure B10.

The deeper that one wishes to measure velocities, the lower the frequencies one must measure. Limitations at these frequencies arise quickly in industry seismographs and geophones, as well as with attenuation at larger offsets. It has also been seen that the width of the entire dispersion curve widens as the near offset is placed farther from the shot.
FIGURES

Figure 1. Synthetic test velocity model. S-wave velocities on the left, P-wave velocities on the right.

Figure 2. Dispersion curve data points used in the inversion process.

Figure 3. Comparison of Love wave convergence for S-wave velocities using standard and accelerated algorithms.

Figure 4. Comparison of Rayleigh wave convergence for S-wave velocities given correct P-wave velocities using standard and accelerated algorithms.

Figure 5. Comparison of Rayleigh wave convergence for S- and P-wave velocities using standard and accelerated algorithms.

Figure 6. Comparison of Rayleigh wave convergence for P-wave velocities given correct S-wave velocities using standard and accelerated algorithms.

Figure 7. Combined process: 100 iterations solving for S-wave velocities using Love wave inversion followed by Rayleigh wave inversion for P-wave velocities only.

Figure 8. Combined process: Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for S- and P-wave velocities simultaneously.
Figure 9. Triple process: Love wave inversion for S-wave velocities followed by Rayleigh wave inversion for P-wave velocities followed by Rayleigh wave inversion for S- and P-wave velocities simultaneously.

Figure 10. Arco test site YY Love wave shot record. Station spacing is 9.14 m (30 ft).

Figure 11. Arco test site ZZ Rayleigh wave shot record. Station spacing is 9.14 m (30 ft).

Figure 12. Extracted Love wave dispersion curves.

Figure 13. Convergence rate diagram of Love wave inversion for S-wave velocities.

Figure 14. Final S-wave velocity model from Love wave inversion.

Figure 15. Extracted Rayleigh wave dispersion curves.

Figures A1 - A7 based on two layer model with interface depth at 35 km.

Figure A1. Love and Rayleigh wave Dispersion Curves. Layer 1 S-wave velocity: 3.5 km/sec, P-wave velocity: 8.1 km/sec, density: 2.70 g/cc. Layer 2 S-wave velocity: 4.5 km/sec, P-wave velocity: 8.1 km/sec, density: 3.36 g/cc.
Figure A2. Same model as in Figure A1 calculated through a variation in the shallow layer S-wave velocity of + and - 20%.

Figure A3. Variation in this case is + and - 20% of the Layer 2 S-wave velocity.

Figure A4. Same model calculated with a + and - 20% variation in the shallow layer P-wave velocity.

Figure A5. Variation in this case is + and - 20% of the Layer 2 P-wave velocity.

Figure A6. Density variation of + and - 20% in the shallow layer density.

Figure A7. Density variation of + and - 20% in the deeper layer's density.

Figure B1. Synthetic seismic section created using fundamental mode dispersive wave energy only.

Figure B2. Extracted dispersion curve from synthetic in Figure B1.

Figure B3. Synthetic Arco test site SH wave seismic section generated with the finite difference program sh.f.

Figure B4. Synthetic Arco test site PSV wave seismic section generated with the finite difference program psvr2.f.
Figure B5. Extracted SH dispersion curve from synthetic in Figure B3.

Figure B6. Forward modeled SH dispersion curves.

Figure B7. Extracted PSV dispersion curve from synthetic in Figure B4.

Figure B8. Forward modeled PSV dispersion curves.

Figure B9. Dispersion curve extracted from 48 trace 1.4 mile wide synthetic.

Figure B10. Dispersion curve extracted from 48 trace 3.0 mile wide synthetic.
Figure 1

Figure 2
ACCELERATED CONVERGENCE RATE FOR SIMULTANEOUS RAYLEIGH WAVE $V_s$ & $V_p$

![Graph showing convergence rates for old and new models and data for $V_s$ and $V_p$.](image)

Figure 5

ACCELERATED CONVERGENCE RATE FOR RAYLEIGH WAVE $V_p$

![Graph showing convergence rates for old and new models and data for $V_p$.](image)

Figure 6
Figure 7

Figure 8
CONVERGENCE RATE, LOVE Vs, RAYLEIGH Vp, LOVE/SIMULTANEOUS RAYLEIGH

Model Error

20 Sample Points

40 Sample Points

Data Error

RMS

Error

10^0

10^-1

10^-2

10^-3

10^-4

10^-5

0 200 400 600 800 1000 1200 1400 1600 1800 2000

Iterations

Figure 9
Figure 10

Courtesy of Arco Oil and Gas Company
Figure 11

Courtesy of Arco Oil and Gas Company
Figure 12
Figure 13

Figure 14
Figure 15
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model

+/- 20% Variation in Vs2

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure A3
LOVE WAVE

Omega (radians/sec)

2 Layer Model

+/- 20% Variation in Vp1

RAYLEIGH WAVE

Omega (radians/sec)

Figure A4
LOVE WAVE

2 Layer Model

+/- 20% Variation in Vp2

RAYLEIGH WAVE

Figure A5
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model

+/- 20% Variation in rho1

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure A6
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model
+- 20% Variation in rho2

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure A7
Figure B2
Figure B3
Figure B5
Figure B6
Figure B7
Figure B8
Figure B9
Figure B10
Parsimonious Staggered Grid Finite-Differencing

of the Wave Equation

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ABSTRACT

A parsimonius staggered grid differencing scheme is presented which requires much less storage than the conventional staggered grid method. For three dimensional elastic wave propagation, this scheme only stores displacement components, not stress, and so requires 66% of the memory needed by the standard staggered grid method. The storage requirement is the same as the 2-2 differencing scheme used by Kelly et al. (1976) for the second-order wave equation. Its advantage is that it is stable and accurate for media with fluid-elastic contacts and for a wide range of Poisson ratios. The parsimonius staggered grid scheme can accommodate much larger models than the standard staggered grid scheme. A disadvantage is that its computer programming is slightly more involved.
INTRODUCTION

A major problem with some finite-difference solutions of the wave equation is that they become unstable or inaccurate for a range of Poisson’s ratios, typically larger than .25 (Levander, 1988; Marfurt, 1984). Thus, these finite-difference schemes are not applicable to problems involving weathering zones with large Poisson ratios or across fluid-solid contacts such as the sea bottom. An example is the 2-2 scheme of Kelly et al. (1976) which will be denoted as the standard 2-2 finite-difference scheme. To overcome this problem, staggered grid finite-difference schemes have been proposed for the coupled first-order equations of stress-strain and conservation of momentum (Madariaga, 1976; Virieux, 1984; and Levander, 1988). Other workers, notably Yee (1966) used this method for the equivalent first-order equations in electromagnetic theory, namely Maxwell’s equations. The staggered grid method can accommodate large gradients in physical parameters because, in part, 1). central differencing of 1st-order field derivatives are naturally centered around the grid point of the physical parameter, and 2). there are no explicit spatial derivatives of physical parameters; coupled first-order stress-strain and momentum equations do not contain derivatives of physical parameters. Hence, differencing errors are negligible in regions with large parameter gradients.

The major drawback with staggered grid methods is that they require more memory than the conventional differencing schemes (e.g., Kelly et al., 1976). For a three-dimensional problem, the conventional 2-2 differencing of the second-order elastic wave equation computes only $3N^3$ displacement values at each time step, where $N$ is the number of grid points along an edge of a gridded cube. This means that two cubes of grid points ($6N^3$) need to be stored at each time step. In contrast, the 2-2 staggered grid method computes 6 stress components and 3 velocity components every two time steps. This means that a cube of stress components (6 stresses per grid point) and a cube of velocity components (3 components per grid point) need to be stored at
any one time step, for a storage demand of $9N^3$ field values per time step. Therefore, a 2-2 staggered grid scheme for three-dimensional elastic wave propagation requires 1.5 times more storage than that using the conventional 2-2 differencing of the second-order wave equation. This storage demand can severely limit the model size for staggered grid methods.

**PARSIMONIOUS STAGGERED GRID METHOD**

To overcome the storage limitations of the staggered grid method, we propose a parsimonious staggered grid method. Parsimonious staggered grid differencing only requires, for a three-dimensional problem, the storage of two cubes of displacement components per time step, where there are three displacement components at each grid point. Hence, the storage requirements for the parsimonious staggered grid scheme are the same as for the standard 2-2 finite-differencing scheme. It also enjoys the advantage of being stable for fluid-solid interfaces and for a wide range of Poisson ratios. It can be shown by induction that the parsimonious staggered grid method is mathematically equivalent to the staggered grid method (see Appendix).

**Theory.**

For two-dimensions, the conservation of momentum equations are

\[
\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \tag{1a}
\]

\[
\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}
\]

and the stress-strain relations are

\[
\tau_{xx} = (\lambda+2\mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} \tag{1b}
\]
\[ \tau_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} \]

\[ \tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \]

where \( \lambda \) and \( \mu \) are Lamé's parameters, \( \rho \) is density, and \( u \) and \( v \) are displacement components in the \( x \) and \( y \) directions respectively. The staggered grid scheme reparameterizes \( \dot{u} = \frac{\partial u}{\partial t} \), differentiates equation (1b) with respect to time, so that equations (1) become

\[ \rho \frac{\partial \dot{u}}{\partial t} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \]  

(2a)

\[ \rho \frac{\partial \dot{v}}{\partial t} = \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \]

and

\[ \frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial \dot{u}}{\partial x} + \lambda \frac{\partial \dot{v}}{\partial y} \]  

(2b)

\[ \frac{\partial \tau_{yy}}{\partial t} = (\lambda + 2\mu) \frac{\partial \dot{v}}{\partial y} + \lambda \frac{\partial \dot{u}}{\partial x} \]

\[ \frac{\partial \tau_{xy}}{\partial t} = \mu \left( \frac{\partial \dot{v}}{\partial x} + \frac{\partial \dot{u}}{\partial y} \right) \]

The standard 2-2 staggered grid scheme used by, say Virieux (1986), assigns the velocity and stress components to the grid points shown in Figure (1). The 1st-order spatial derivatives of stress and velocity in equation (2) are approximated by 2nd order correct centered difference approximations, e.g.,
\[
\frac{\partial u(i+1/2\Delta x, j, \Delta y, t)}{\partial x} = \frac{\dot{u}_{i+1/2j} - \dot{u}_{ij}}{\Delta x}
\]

or
\[
\frac{\partial \tau_{xx}(i \Delta x, j \Delta y, t)}{\partial x} = \frac{(\tau_{xx})_{i+1/2j} - (\tau_{xx})_{i-1/2j}}{\Delta x}
\]

where \( \Delta x \) and \( \Delta y \) are the distances between grid point in the x and y direction respectively. The i and j indices correspond to the grid point numbering in the x and y directions respectively.

The time derivatives in equation (2) are replaced by centered differences in time, except the stress time derivatives are centered at time \( t+1/2 \) and the velocity time derivatives are centered at time \( t \), e.g.,
\[
\frac{\partial \dot{u}(i \Delta x, j \Delta y, t+1/2)}{\partial t} = \frac{\dot{u}_{ij}^{t+1} - \dot{u}_{ij}^{t}}{\Delta t}
\]
\[
\frac{\partial \tau_{xx}(i \Delta x, j \Delta y, t)}{\partial t} = \frac{(\tau_{xx})_{ij}^{t+1/2} - (\tau_{xx})_{ij}^{t-1/2}}{\Delta t}
\]

The differencing scheme for the x component in equation (2a) is shown in Figure (2) where the stress components are confined to the \( t+n/2 \) time panels and the velocity components are confined to the \( t+m \) time panels, where \( m \) (n) can take on all (odd) integer values. Stress and velocity panels are staggered in this way for all time steps. Thus, for any one time step one panel of stress components and one panel of velocity components need to be stored. For three dimensional problems, this means that a staggered grid scheme requires 1.5 times more storage than the conventional 2-2 differencing scheme for the second-order elastic wave equation.

To overcome this storage limitation, we propose a parsimonious staggered grid fo-
mulation. The parsimonious staggered grid scheme solves for displacement components using equation (1). The following is a 2-2 parsimonious staggered grid algorithm;

1). Replace the time derivatives in equation (1a) by second-order correct centered differencing, shown in Figure (3). Assume the pivot is at time t.

2). The stress values at the pivot time t on the right hand side of equation (1) are computed by using the displacement components at time t to central difference the stress-strain equations in equation (1b). The grid point locations for the stresses and displacements are the same as the standard staggered grid method in Figure (1).

3). The stress values computed in step 2 are then used to compute stress gradients by centered differencing formulae.

4). Using the stress gradients at time t in step 3 and the displacements at time t-1, the displacements are computed at the t+1 panel. Steps 1-4 are repeated after incrementing the time step.

To minimize storage, steps 2 and 3 are combined so that only one differencing formula is used to compute

\[ u_{ij}^{t+1} = 2u_{ij}^{t} - u_{ij}^{t-1} + \frac{\Delta t^2}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) \]  \hspace{1cm} (3a)

and

\[ v_{ij}^{t+1} = 2v_{ij}^{t} - v_{ij}^{t-1} + \frac{\Delta t^2}{\rho} \left( \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) \]  \hspace{1cm} (3b)

where the stresses are computed from centered differences of equation (1b), e.g.,
\[ \tau_{xx}(i+1/2\Delta x, j \Delta y, t) = (\lambda + 2\mu) \frac{u^t_{i+1/2,j} - u^t_{i,j}}{\Delta x} + \lambda \frac{v^t_{i+1/2,j+1/2} - v^t_{i+1/2,j-1/2}}{\Delta y} \quad (4a) \]

\[ \tau_{xy}(i \Delta x, j + 1/2 \Delta y, t) = \mu \frac{u^t_{i,j+1} - u^t_{i,j}}{\Delta y} + \frac{v^t_{i+1/2,j+1/2} - v^t_{i-1/2,j+1/2}}{\Delta x} \quad (4b) \]

The spatial derivative operators in equation (3a) or (3b) are also computed by centered differences, e.g.,

\[ \frac{\partial \tau_{xx}(i \Delta x, j \Delta y, t)}{\partial x} = \frac{(\tau^t_{xx})_{i+1/2,j} - (\tau^t_{xx})_{i-1/2,j}}{\Delta x} \quad (5a) \]

\[ \frac{\partial \tau_{xy}(i \Delta x, j \Delta y, t)}{\partial y} = \frac{(\tau^t_{xy})_{i,j+1/2} - (\tau^t_{xy})_{i,j-1/2}}{\Delta y} \quad (5b) \]

Equations (4) are substituted into (5), and these in turn are substituted into (3) to yield the differencing formula in terms of displacement only (see Figure 3). This combination of steps 2 and 3 is the reason for parsimony, stresses do not have to be stored. Only two panels of displacement need to be stored at any one time. In addition it retains the beneficial property of centering all space derivatives in the same manner as the standard staggered grid formulation. In fact, the two schemes are equivalent.

**Numerical Examples.**

The line source elastic response of a water layer over a half space model in Figure (4a) is computed by the standard staggered grid method and compared to the parsimonius staggered grid solution in Figure (4b). The two solutions are identical to five significant figures (Figure 4c). The standard 2-2 scheme of Kelly et al. (1976) would fail in this circumstance.

The line source elastic response to the complicated model in Figure (5a) is com-
puted by the two schemes and compared in Figures (5b) and (5c). Again the solutions are identical to five significant figures.

CONCLUSIONS

A 2-2 parsimonious staggered grid scheme is presented which only requires storage of two displacement panels at any one time step. This is the same storage requirement as the standard 2-2 differencing of the second-order wave equation, and, for a three-dimensional elastic problem, 66% that of a standard staggered grid method. Similar to the staggered grid method, it is accurate and stable for fluid-elastic interfaces and high Poisson ratios. The advantage of parsimonious staggered grid is that it can accommodate much larger models than the standard staggered grid scheme. Its disadvantage is that its computer programming is not as easy. Extensions to higher order differencing schemes are straightforward.

REFERENCES


APPENDIX A

PROOF OF THE EQUIVALENCE BETWEEN
STANDARD AND PARSIMONIOUS STAGGERED SCHEMES

We use mathematical induction to implement this proof. We will assume that the stress and velocity values are the same at time step \( t=n \), and prove that they are the same at the \( t=n+1 \). We then will show that they are equivalent at the initial conditions \( t=0 \).

First denote standard staggered scheme variables by primes and no primes denote the parsimonious variables. The parsimonious velocities, \( u_{ij}^{n+1/2} \) and \( v_{ij}^{n+1/2} \), are given by:

\[
\dot{u}_{ij}^{n+1/2} = \frac{u_{ij}^{n+1} - u_{ij}^{n}}{\Delta t}, \quad \dot{v}_{ij}^{n+1/2} = \frac{v_{ij}^{n+1} - v_{ij}^{n}}{\Delta t} \quad (A-1)
\]

At time step \( t=n \), we assume equality between parsimonious and standard variables

\[
\dot{u}'_{ij}^{n+1/2} = \frac{u_{ij}^{n} - u_{ij}^{n-1}}{\Delta t} = \dot{u}_{ij}^{n+1/2}, \quad \dot{v}'_{ij}^{n+1/2} = \frac{v_{ij}^{n} - v_{ij}^{n-1}}{\Delta t} = \dot{v}_{ij}^{n+1/2} \quad (A-2)
\]

\[
(\tau'_{xx})_{ij}^{n} = (\tau_{xx})_{ij}^{n}, \quad (\tau'_{yy})_{ij}^{n} = (\tau_{yy})_{ij}^{n}, \quad (\tau'_{xy})_{ij}^{n} = (\tau_{xy})_{ij}^{n}
\]

where the velocities and stresses with prime " \( ' \) " denote the staggered scheme variables. At time step \( t=n+1 \), from equation (A-1), we have,

\[
\frac{\dot{u}_{ij}^{n+1/2} - \dot{u}_{ij}^{n-1/2}}{\Delta t} = \frac{u_{ij}^{n+1} - 2u_{ij}^{n} + u_{ij}^{n-1}}{\Delta t^2} = \frac{1}{\rho} \left[ \left( \frac{\partial (\tau'_{xx})_{ij}^{n}}{\partial x} \right) + \frac{\partial (\tau'_{xy})_{ij}^{n}}{\partial y} \right]. \quad (A-3)
\]

By assumption \( \dot{u}'_{ij}^{n+1} = \dot{u}_{ij}^{n+1} \), so equation (A-3) implies that future time values of parsimonious \( \dot{u} \) and standard \( \dot{u}' \) are equal
Similarly, it can be proved that \( \dot{\psi}^{n+\frac{1}{2}} = \psi_i^{n+\frac{1}{2}} \). Hence, the standard and conventional velocities are equal at future values of time \( n+1/2 \).

From the main body of this paper, we know that,

\[
(\tau_{xx})^{n+1}_{ij} = (\lambda + 2\mu) \frac{\partial u_i^{n+1}}{\partial x} + \lambda \frac{\partial v_i^{n+1}}{\partial y},
\]

Substituting (A-1) into the above equation, we get

\[
(\tau_{xx})^{n+1}_{ij} = (\lambda + 2\mu) \frac{\partial (\dot{u}_i^{n+\frac{1}{2}} \Delta t + u_i^{n})}{\partial x} + \lambda \frac{\partial (\dot{v}_i^{n+\frac{1}{2}} \Delta t + v_i^{n})}{\partial y}
\]

\[
= (\lambda + 2\mu) \frac{\partial u_i^{n+\frac{1}{2}} \Delta t}{\partial x} + \lambda \frac{\partial v_i^{n+\frac{1}{2}} \Delta t}{\partial y} + (\tau_{xx})^{n}_{ij}
\]

But the unprimed velocities at \( t=n+1/2 \) on the RHS of equation (A-5) can be equated to primed velocities by equation (A-4), and the RHS unprimed stress can be equated to the primed stress by equation (A-2). Hence,

\[
(\tau'_{xx})^{n+1}_{ij} = (\tau_{xx})^{n+1}_{ij}
\]

In the same way, we can prove \( (\tau'_{yy})^{n+1}_{ij} = (\tau_{yy})^{n+1}_{ij} \) and \( (\tau'_{xy})^{n+1}_{ij} = (\tau_{xy})^{n+1}_{ij} \).

The last part of this proof is to show that standard and parsimonious variables are equal at a past time. This is done by setting the parsimonious and standard staggered grid initial conditions at \( t=1/2 \) and \( t=1 \) equal to one another. Figure (A.1) illustrates this proof.
FIGURES

Figure 1. The grid points in which the stress and velocity components are staggered. The $u$ components are restricted to the $(i \Delta x, j \Delta y)$ grid points, the $v$ components are restricted to the $(i+1/2 \Delta x, j \Delta y)$ grid points, the $\tau_{xy}$ components are restricted to the $(i \Delta x, j+1/2 \Delta y)$ grid points, and the $\tau_{yy}$ and $\tau_{yy}$ are restricted to the $(i+1/2 \Delta x, j \Delta y)$ grid points. The parsimonious staggered grid scheme is the same as above except that the displacements take the place of velocity components.

Figure 2. Standard staggered grid method. The first-order equations in equation (2) are stepped forward in time by using the stresses at time $T+1/2$ and velocities at time $T$ to compute the velocities at time $T+1$. The stresses at time $T+1.5$ are computed using the velocities at time $T+1$ and stresses at time $T+1/2$. Note that at any one time step a panel of stresses and velocities need to be stored. The stencil in this figure is for the $u$ velocity component.

Figure 3. Parsimonious staggered grid method. Equations (1) are stepped forward in time by using the displacements at time $T$ and $T-1$ to compute the displacements at time $T+1$. The stress gradient at time $T$ on the right hand side of equation (1a) is computed using the differenced stress strain equations in equation (1b). The stencil in this figure is for the $u$ component.

Figure 4. (a) Water layer over an elastic layer. (b) $x$-component and $z$-component seismograms for the line source and hydrophones shown in (a). The seismograms computed by the standard staggered grid scheme were identical to those computed by the parsimonious staggered grid method. (c). Comparison between the standard (*) and parsimonious (-) seismograms at trace 25 and trace 85.

Figure 5. (a) Complicated model underlying a water layer. (b) $x$-component and $z$-component seismograms for the line source and hydrophones shown in (a). The seismograms computed by the standard staggered grid scheme were identical to those computed by the parsimonious staggered grid method. (c). Comparison between the standard (*) and parsimonious (-) seismograms at trace 20 and trace 80.
Figure (1).
Figure (2).
Figure 3.
Figure (4a).
Figure (4b).
Figure (4c).
Figure (4c cont.)
Figure (5a).
Figure (5b).
Figure (5b cont.)
Figure (5c).
Figure (5c cont.)
Figure A.1
2-D AND 2.5-D FINITE-DIFFERENCE ELECTROMAGNETIC FIELD MODELING

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ABSTRACT

This paper presents a new time domain finite difference method for modeling 2.5-D electromagnetic fields. A new system of equations is derived from Maxwell's equations which in a conductive earth reduces to diffusion equations. Based on these new equations, this method only needs to solve for two fundamental magnetic components $h_x$ and $h_z$ which are decoupled from the three components of electric field $e$ and one of the components of the magnetic field $h$. These four components can be calculated from the two fundamental magnetic components if they are needed. The finite difference scheme is based on a combination of the staggered grid, Dufort-Frankel, and parsimonious schemes. This hybrid scheme is stable for large conductivity contrasts because the staggered grid scheme does not explicitly involve the derivatives of conductivity with respect to the space coordinates. By introducing a transformation in the time variable, the number of time steps is reduced from $N$ to about $10 \cdot \sqrt{N}$. This reduces the computational time by 1 or 2 orders of magnitude. The computer code is explicit in time, uses only real arithmetic, and is easy to code and apply to various conductive models.
INTRODUCTION

Electromagnetic exploration for minerals is frequently performed with transient ("time-domain") EM systems. Physical and numerical modeling are important for interpreting real data. In some situations, the medium is approximately 2-D but the source is a point source and the receivers are distributed along a plane. In this case a 2.5-D modeling method can be used to model the electromagnetic response. In 2.5-D problems, the medium is assumed to be 2-D (invariant in y direction), the source can be a point source, and the receivers can be distributed along a plane. In the time domain, 2.5-D numerical modeling method is not published in the literature. In this paper, we present a finite difference method for solving Maxwell’s equations for a 2.5-D model.

THEORY

Decoupling of $h_x, h_z$. In a conductive earth, Maxwell’s equation reduces to the diffusion equations,

$$ e_x = \rho \left( \frac{\partial h_x}{\partial y} - \frac{\partial h_y}{\partial z} \right) $$  \hspace{1cm} (1.1)

$$ e_y = \rho \left( \frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} \right) $$  \hspace{1cm} (1.2)

$$ e_z = \rho \left( \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} \right) $$  \hspace{1cm} (1.3)
\[-\mu \frac{\partial h_x}{\partial t} = \frac{\partial e_x}{\partial y} - \frac{\partial e_y}{\partial z}\] (1.4)

\[-\mu \frac{\partial h_y}{\partial t} = \frac{\partial e_x}{\partial z} - \frac{\partial e_z}{\partial x}\] (1.5)

\[-\mu \frac{\partial h_z}{\partial t} = \frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y}\] (1.6)

where, \(\rho\) is the resistivity, and \(\mu\) is the magnetic permeability which is assumed to be constant in the earth.\((e_x, e_y, e_z)\) and \((h_x, h_y, h_z)\) are the three components of electric field and magnetic field respectively. Replacing \(e_z\) in equation (1.4) by equation (1.3), noting that \(\rho\) is independent of \(y\), we get

\[-\mu \frac{\partial h_x}{\partial t} = \rho \left( \frac{\partial^2 h_y}{\partial x \partial y} - \frac{\partial^2 h_x}{\partial y^2} \right) - \frac{\partial e_y}{\partial z}.\] (2.1)

Using \(\nabla \cdot h = 0\), we have

\[\frac{\partial h_y}{\partial y} = - \frac{\partial h_x}{\partial x} - \frac{\partial h_z}{\partial z}.\]

Plugging this relation into equation (2.1), we get

\[\mu \frac{\partial h_x}{\partial t} = \rho \left[ \frac{\partial^2 h_x}{\partial x^2} + \frac{\partial^2 h_x}{\partial y^2} + \frac{\partial^2 h_x}{\partial x \partial z} \right] + \frac{\partial e_y}{\partial z}.\] (2.2)

By the same derivation, we can derive

\[\mu \frac{\partial h_z}{\partial t} = \rho \left[ \frac{\partial^2 h_z}{\partial y^2} + \frac{\partial^2 h_z}{\partial z^2} + \frac{\partial^2 h_x}{\partial x \partial z} \right] - \frac{\partial e_y}{\partial x}.\] (2.3)

The combination of equation (2.2), (2.3) and (1.2) leads to
\[
\frac{\partial h_x}{\partial t} = \rho \left[ \frac{\partial^2 h_x}{\partial x^2} + \frac{\partial^2 h_x}{\partial y^2} + \frac{\partial^2 h_z}{\partial x \partial z} \right] \frac{\partial e_y}{\partial z} + 
\]

\[
\frac{\partial h_z}{\partial t} = \rho \left[ \frac{\partial^2 h_z}{\partial y^2} + \frac{\partial^2 h_z}{\partial z^2} + \frac{\partial^2 h_x}{\partial x \partial z} \right] \frac{\partial e_y}{\partial x} - 
\]

\[
e_y = \rho \left[ \frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} \right] 
\]

Plugging equation (3.3) into (3.1) and (3.2), we get

\[
\frac{\partial h_x}{\partial t} = \rho \left[ \frac{\partial^2 h_x}{\partial x^2} + \frac{\partial^2 h_x}{\partial y^2} + \frac{\partial^2 h_z}{\partial x \partial z} \right] \frac{\partial}{\partial z} \left[ \frac{\partial h_x}{\partial z} \rho - \frac{\partial h_z}{\partial x} \right] 
\]

\[
\frac{\partial h_z}{\partial t} = \rho \left[ \frac{\partial^2 h_x}{\partial y^2} + \frac{\partial^2 h_z}{\partial z^2} + \frac{\partial^2 h_x}{\partial x \partial z} \right] \frac{\partial}{\partial x} \left[ \frac{\partial h_x}{\partial z} \rho - \frac{\partial h_z}{\partial x} \right] 
\]

In this equation, \( h_x \) and \( h_z \) are decoupled from the six components in equation (1).

\( \tau - t \) transformation. In this section, we discuss the transformation of equation (4) from the \( t \)-domain to the \( \tau \)-domain to save computer time. Oristaglio and Hohmann (1984) discussed an expanding time step \( \Delta t \) using a Dufort-Frankel finite difference scheme,

\[
\Delta t = \sqrt{t} dx. 
\]

The variable \( \Delta t \) is based on the physical phenomenon that the induced electric field diffuses through the conductor with velocity,
\[ v = \frac{\partial X_{\text{max}}}{\partial t} = \sqrt{\frac{1}{2\mu \sigma t}}. \]

As time increases, the velocity decreases so that the finite difference time step can be enlarged (finite difference stability criteria is proportional to the product of velocity and time increment). To make use of this physical characteristic in the finite difference calculation, we introduce a \( \tau - \tau \) transformation,

\[ t = \tau + \frac{a}{2} \tau^2 \quad (6.1) \]

where, \( a \) is a constant which will be investigated soon. Differencing equation (6.1) leads to

\[ \Delta t = \Delta \tau (1+a \tau). \quad (6.2) \]

From this equation we can see that for a fixed \( \Delta \tau \), \( \Delta t \) becomes larger as \( t \) or \( \tau \) become larger. Since \( \Delta t = \Delta \tau \) as \( \Delta \tau \rightarrow 0 \), \( \Delta \tau \) should at least honor the same requirement as that of \( \Delta t \) (Oristaglio and Hohmann, 1984)

\[ \Delta t = \frac{\mu \sigma \Delta x^2}{4}. \quad (6.3) \]

For safety, we take

\[ \Delta \tau = \frac{\mu \sigma \Delta x^2}{8}. \quad (6.4) \]

We will now determine the coefficient \( a \). According to equation (5.2) and (6.2), the expanding \( \Delta t \) satisfies,

\[ v \cdot \Delta t = \Delta \tau \cdot (1+a \tau) \cdot \sqrt{\frac{1}{2\mu \sigma \tau}} \leq \frac{\Delta x}{c_1} \quad (7) \]
where $c_1$ is a constant ($c_1 > 1$). Let $\tau \to \infty$ so that equation (7) can be solved to get

$$a = \frac{8}{c_1^2 \Delta \tau}$$  \hspace{1cm} (8)

To verify the validity of coefficient $a$, we calculate the ratio of $\frac{v \cdot \Delta t}{\Delta x}$, which should (by physical considerations) be less than 1. Using equation (7), we can derive,

$$\frac{v \cdot \Delta t}{\Delta x} = \frac{1 + \frac{8}{c_1^2} k}{4 \sqrt{k \cdot (1 + \frac{4}{c_1^2} k)}}$$

where, $k$ is the index of time step, $k = \frac{\tau}{\Delta \tau}$. Figure 1 shows the curves for $c_1 = 1, 2, 10,$ and $20$. From Figure 1, we can see the ratio is always less than 1 for $c_1 \geq 2$. Plugging equation (8) into (6.1),

$$t = \tau + \frac{4}{c_1^2 \Delta \tau} \tau^2$$ \hspace{1cm} (9.1)

Dividing $\Delta \tau$ on both sides of equation (9.1), we get

$$N = N' + \frac{4}{c_1^2} N'^2$$ \hspace{1cm} (9.2)

where, $N = \frac{t}{\Delta \tau}$ is the number of time steps in the $t$-domain, and $N' = \frac{\tau}{\Delta \tau}$ is the number of time steps in the $\tau$ - domain. When $N$ is large, we have
\[ N' = \frac{c_1}{2} \sqrt{N} \] (9.3)

For safety, we take \( c_1 = 20 \), so \( N' = 10\sqrt{N} \). A more careful choice of \( c_1 \) may save more computer time.

By a \( \tau-\tau \) transformation described in equation (6.1), we can transfer equation (4) from the t-domain into the \( \tau \)-domain,

\[
\mu \frac{\partial h_x}{\partial \tau} (1 + a \tau) = \rho \left[ \frac{\partial^2 h_x}{\partial x^2} - k_y^2 + \frac{\partial^2 h_z}{\partial z \partial x} \right] \frac{\partial}{\partial z} \left[ \left( \frac{\partial h_z}{\partial z} - \rho \frac{\partial h_x}{\partial x} \right) \right] \] (10.1)

\[
\mu \frac{\partial h_z}{\partial \tau} (1 + a \tau) = \rho \left[ k_y^2 h_z + \frac{\partial^2 h_z}{\partial z^2} + \frac{\partial^2 h_x}{\partial x \partial z} \right] \frac{\partial}{\partial x} \left[ \left( \frac{\partial h_x}{\partial z} - \rho \frac{\partial h_z}{\partial x} \right) \right] \] (10.2),

where, we have taken the Fourier transformation of \( y \). Letting \( k_y = 0 \), this equation reduces to two equations (2-D medium and line source). After \( h_x, h_z \) are known, we can calculate \( h_y \) by solving the following equation which is obtained by substituting equation (1.1) and (1.3) into (1.5) and taking a \( \tau-\tau \) transformation,

\[
-\mu \frac{\partial h_y}{\partial \tau} (1 + \tau) = \frac{\partial}{\partial y} \left[ \left( \frac{\partial h_z}{\partial y} - \rho \frac{\partial h_y}{\partial z} \right) \right] \frac{\partial}{\partial z} \left[ \left( \frac{\partial h_x}{\partial x} - \rho \frac{\partial h_y}{\partial y} \right) \right] \] (11)

In this equation, \( \frac{\partial h_z}{\partial y} \) and \( \frac{\partial h_x}{\partial y} \) are known. We can solve this equation in a manner similar to solving equation (10) and can calculate \( (e_x, e_y, e_z) \) from equations (1.1) and (1.3) after \( (h_x, h_y, h_z) \) are known. This procedure will be described in the next section.

**NUMERICAL IMPLEMENTATION**
Except the last term in equations (10.1) or (10.2), all terms are replaced by a standard central finite difference scheme. For example,

\[
\frac{\partial h^{\tau}_{(i,j)}}{\partial \tau} = \frac{h^{(\tau+1)}_{(i,j)} - h^{\tau}_{(i,j)}}{2\Delta \tau}
\]  

(12.1)

and

\[
\frac{\partial^2 h^{\tau}_{(i,j)}}{\partial x^2} = \frac{h^{\tau}_{(j+1,k)} - 2h^{\tau}_{(j,k)} + h^{\tau}_{(j-1,k)}}{\Delta x^2}
\]

where \(\Delta \tau\) is given by equation (6.3), and \(h^{\tau}_{(i,j)}\) represents the magnetic field, either \(h_x\) or \(h_z\) at time \(\tau\) and space point \(x = j\Delta x, z = k\Delta z\).

To avoid taking a derivative of \(\rho\) with respect to \(z\) or \(x\) in the last term of equation (10.1) or (10.2), we use a parsimonious staggered grid scheme (Luo and Schuster, 1989). We also adapt the standard staggered scheme of Yee (1966). Figure 2 illustrates this scheme, in which, \(h_x\) and \(h_z\) are staggered in space and the derivatives of \(\frac{\partial h_x}{\partial z}\) and \(\frac{\partial h_z}{\partial x}\) are calculated at the middle points of the mesh sides. These first derivative values are used to calculate the second derivative \(\frac{\partial}{\partial z}\) and \(\frac{\partial}{\partial x}\) in equation (10).

To incorporate the \(\tau-t\) transformation and retain stability, we need to use the Dufort - Frankle scheme (Dufort, 1953). That is

\[
h^{\tau}_{(i,j)} = \frac{h^{(\tau+1)}_{(i,j)} + h^{(\tau-1)}_{(i,j)}}{2}.
\]

NUMERICAL EXAMPLES
Figure 3 shows the numerical solution in the t-domain compared to the analytical point source solution in a homogeneous medium.

Figure 4 shows the comparison of the numerical solution in the t-domain (stars) and in the \( \tau \) - domain (solid line). In this case, \( c_1=20 \).

Figure (5a) shows a homogeneous medium with a conductive body. Both the ground surface and the conductor have conductivity 1 \( \Omega^{-M} \). The background conductivity is 300 \( \Omega^{-M} \). Figure 3b show the response at receiver locations A and B and Figure 3b shows the response at receiver locations A and C. The calculations required only 30 minutes the time \( \tau \) domain compared to over 40 hours in the time domain.

CONCLUSION

A time domain finite difference method is presented for solving Maxwell's diffusion equations in the 2.5-D situation. This scheme is accurate for models with high conductivity contrasts and can require one or two orders of magnitude less computer time than conventional methods in the time domain. More tests will be performed in the future and we hope to provide the sponsors with a fortran code sometime in 1990.

Acknowledgements

I am grateful to Jerry Hohmann for encouraging discussions. I also thank Jerry Schuster for suggesting the use of a staggered grid method to solve Maxwell's equations.
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Oristaglio, M.L., and Hohmann G.W., 1984, Diffusion of electromagnetic fields into a two-dimensional earth: A finite difference approach; Geophysics, 49, 870-894.


Luo Y. and Schuster G. T., 1989, Parsimonious staggered grid finite differencing of the elastic wave equation; U of Utah Tomography mid-year report.
Figure 1.
Fig. 4

Fig. 3
Figure 5a.

hz component for impulse point magnetic dipole

\( (x,y)=(800,0) \) real line
\( (x,y)=(800,220) \) dash line

Figure 5b.
4th-Order Finite-Difference Solution To The 2-D Acoustic Wave Equation By A Staggered Grid Method

Jerry Schuster

This is a short description of the 4th-order finite difference code that is contained in the floppy disk "pp4.f". It was successfully executed on three UNIX based computers: a SPARC workstation, a MicroVAX, and a Stellar GS1000 computer. The simplicity in code design allows for easy adaptation to reverse time migration and full wave inversion codes.

"pp4.f" is a fortran program which solves the 2-D acoustic wave equation for crosswell source-receiver geometries via a 2-4 staggered grid finite difference scheme. The accuracy is fourth-order in space and second-order in time, and can be used for modeling acoustic waves in arbitrary 2-D earth models. If the input variable "ifre" is set equal to zero then a free-surface boundary condition is imposed on the top surface; otherwise there will be absorbing boundary conditions on all four sides.

To generate synthetic seismograms, two steps are necessary:

1). Generate earth model. This is done by specifying P-velocities, densities, and finite difference parameters in the file "indat", b). specifying interface coordinates in the file "inmod", and c). executing the program "mod.f". The program "mod.f" will then generate the data file "vel" and the summary file "mod.dat". "vel" is the file which assigns a layer number to each nodal point; these numbers will be read by "pp4.f" and used to assign velocity values to each nodal point.
2). Generate synthetic seismograms. Once "mod.f" has been executed then "pp4.f" can be executed to generate the files "pre1" and "pre2". "pre" contains the pressure seismograms along a horizontal line of geophones at node depth "nzg"; and "pre1" contains pressure seismograms along a vertical line of nodes at geophones at offset "nxg1". An example of the input files and the graphical output is given in Figure 1. Figure 1 depicts the line source response of an acoustic half-space having a velocity of 0.313 m/s. The source is a Ricker wavelet with peak frequency of 3 Hz and the source receiver parameters are listed in the figure. The solid line is the finite difference solution and the * depicts the analytic solution using a method of images. In this case the source is 76 m below the free-surface while the receiver is 20 m directly below the source location.

**Theory**

For 2-D acoustic media, the stress-strain relations and Newton's law reduce to

\[
\frac{\partial \ddot{u}}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} \tag{1a}
\]

\[
\frac{\partial \dot{w}}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial z} \tag{1b}
\]

\[
\frac{\partial P}{\partial t} = -\kappa \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \tag{1c}
\]

where \(\kappa, \rho, \ddot{u}, \dot{w},\) and \(P\) denote, respectively, bulk modulus, density,
Figure 1. The 2-4 finite difference solution (-) is compared to the analytic solution (*) for a line source in a half-space. The receiver is located 20 m directly below the source and the free-surface is 76 m above the source. The input files "indat" and "inmod" for this model are shown below the figure.
Figure 2. Stencil for 4th-order staggered grid finite difference scheme.
horizontal particle velocity, vertical particle velocity and pressure.

A staggered grid finite difference scheme will be used with the spatial staggering stencil given in Figure 2. This staggering yields a 4th-order finite difference approximation given as

\[ u_{i+1/2,j+1/2}^{t+1/2} = u_{i+1/2,j+1/2}^{t-1/2} - \frac{r}{\rho_{i+1/2,j+1/2}} \]

\[ c1 \cdot (P_{i+1,j+1/2}^{t} - P_{i,j+1/2}^{t}) + c2 \cdot (P_{i+2,j+1/2}^{t} - P_{i-1,j+1/2}^{t}) \]  

\[ \dot{w}_{i,j}^{t+1/2} = \dot{w}_{i,j}^{t-1/2} - \frac{r}{\rho_{i,j}} \]

\[ c1 \cdot (P_{i,j+1/2}^{t} - P_{i,j-1/2}^{t}) + c2 \cdot (P_{i,j+3/2}^{t} - P_{i,j-3/2}^{t}) \]

\[ p_{i,j+1/2}^{t+1} = p_{i,j+1/2}^{t} - r \chi_{i,j+1/2} \]

\[ c1 \cdot (\ddot{u}_{i+1/2,j+1/2}^{t+1/2} - \ddot{u}_{i-1/2,j+1/2}^{t+1/2}) + c2 \cdot (\ddot{u}_{i+3/2,j+1/2}^{t+1/2} - \ddot{u}_{i-3/2,j+1/2}^{t+1/2}) \]

\[ + c1 \cdot (\dot{w}_{i,j+1/2}^{t+1/2} - \dot{w}_{i,j}^{t+1/2}) + c2 \cdot (\dot{w}_{i,j+2}^{t+1/2} - \dot{w}_{i,j-1}^{t+1/2}) \]

where \( c1 = 9/8 \) and \( c2 = -1/24 \). and \( r = \Delta t / \Delta x \). The numerical dispersion and stability criteria should be similar to that in Levander (1988), albeit adjusted for the zero shear modulus case.

**Free-Surface Boundary Conditions And Stability.** If "ifre" = 0 in the "indat" file then a free-surface boundary condition is implemented near
the topmost line of nodes. This condition was implemented by imposing asymmetry of pressure across the free-surface marked in Figure 2. The asymmetry condition follows from an image solution analysis. That is, the free-surface boundary is replaced by an image plane with a source of opposite polarity above the image plane at \((x, -z)\), where \((x, + z)\) is the source location below the image plane. This strategy seems to be successful since the response of a line source in a half space calculated by the finite difference scheme matched that of the analytical solution (Figure 1). I have also tested this code for large models \((1000 \times 1000)\) with over 10,000 time steps and the solution appears to be stable.

In principle, stability of the finite difference scheme with absorbing boundaries should be analytically analyzed for arbitrary models. Because of the difficulty in this endeavor, I chose instead to analyze free-surface stability by a numerical eigenvalue analysis; this is applied to a 1-D 4th-order staggered grid code with a free-surface boundary condition. Eigenvalue analysis is performed by first setting the horizontal particle velocities in equation (2) equal to zero and casting the resulting equations into matrix vector notation as

\[
\mathbf{v}^{t+1} = A \mathbf{v}^t
\]

where the \((2N-1) \times 1\) vector \(\mathbf{v}^{t+1} = (u^{t+1/2}, P^{t+1})\) contains the \(N\) pressures, and the \(N-1\) vertical particle velocities at the node points of the model. The elements in the \(2N-1 \times 2N-1\) matrix \(A\) are the constants dictated by the finite difference equations in equation (2) and the boundary conditions, such as absorbing boundaries and free-surface conditions. The finite
difference scheme in equation (3) will be stable if the eigenvalues of $A$ have magnitude less than or equal to 1. Hence stability of free-surface boundary conditions can be assessed by numerically determining if the eigenvalue magnitudes are less than or equal to one.

Figure 3 depicts the plot of $r = c_{max} \Delta t / \Delta x$ versus maximum eigenvalue of $A$ for a 1-D acoustic model. The eigenvalues were computed by a double precision matrix solver in "MATLAB". In addition a free-surface boundary condition is at the top of the model using asymmetry of pressure across the free-surface. This is exactly the same free-surface boundary condition in the 2-D code "pp4.f". Figure 3 suggests that stability is achieved for the 1-D 4th-order staggered grid when $r = c_{max} \Delta t / \Delta x$ is less than .85. Note that a 2-D scheme will demand a more restrictive stability condition, but the point of this exercise was to convince myself that the imposition of pressure asymmetry at the free-surface did not lead to an inherently unstable finite difference formulation.

A recent paper by Kamel (1989) suggested that the eigenvalues of $A$ can be cheaply calculated by an iterative "power method". I attempted to compute eigenvalues of $A$ using the power method for the 2-D scheme in "pp4.f"; I found that the associated iterative series did not converge. The reason for this convergence failure was revealed by the numerically exact eigenvalue analysis (using double precision arithmetic in MATLAB) in Figure 3; i.e., I discovered that the eigenvalues come in complex conjugate pairs. In this case a power method analysis is questionable since "the eigenvalue predictions will not converge (Jennings, p. 293, 1977)".

References
Figure 3. Eigenvalue analysis of 1-D 4th-order finite-difference code with a free-surface boundary condition. The eigenvalue magnitudes of the matrix A in equation (3) are numerically determined for vel*dt/dx between .5 and 1.0. Instability of the finite difference scheme occurs when vel*dt/dx is greater than .85. For this example the number of pressure nodes is 15. The same result was achieved for NZ=30.


pp4.f  Mon Jan 22 16:14:47 1990  1

C ---------------------------------------
C pp4.f computes acoustic pressure seismograms by a 2-4 staggered grid method
C (2nd order correct in time, 4th order correct in space). A line source is at
C (nx,ns) and a horizontal line of receivers is located at depth "nzg" and a
C vertical line of receivers is located at offset "nxpl". The Ricker wavelet has
C peak frequency of "vm" Hz, time and space steps are "dt" and "dz", and the
C pressure seismograms are written into the file "pre". Eseokal seismogram
C storage is achieved by skipping every "nxskip" traces and skipping "ntiskp"
C values in a trace when writing to "pre". First create earth model by
C executing "mod.f" program.
C Written by Jerry Schuster (12/11/89), partly adapted from YL
C Luo's 2-2 sh program.
C
C c prel -output file- pressure seismograms at level nzg
C c pres2 -output file- pressure seismograms at level nxpl
C c indat - input file- parameters for FD scheme and earth model
C c imod - input file- layer geometry parameters for earth model
C c vel - input file- velocity pointers for each grid point of earth model,
C c generated by "mod.f" program
C c data - subroutine- open files, read input parameters (e.g. velocities and
C c densities generated by mod.f, or parameters for FD
C c scheme from indat)
C c wrdata - subroutine- write seismograms into file pre & prel
C c abscb - subroutine- apply 1st order Clayton-Enquist BC's to boundaries
C c pdiff2 - subroutine- apply 2-2 scheme to pressure for points that are 1 nod
C c from boundaries
C c pdiff2 - subroutine- apply 2-2 scheme to particle velocities for points that
C c are 1 node from boundaries
C c atten - subroutine- apply exponential damping to pressures that occupy nodes
C c within 50 points of a boundary
C
C Hookes and Newton's Laws solved by staggered grid FD scheme
C
C dp/dt = -kappa* ( du/dx + dw/dz ) + source
C
du/dt = -1/density * dp/dx
C
dw/dt = -1/density * dp/dz
C
C staggered grid numbering scheme
C
C WARNING: NEVER INTERROGATE FIELDS ABOVE Free Surface
C
C +-----------------------*
C |                     |
C |  * = W (1:nx-1, 1:nz-1)  |
C | [ ] = U (1:nx-1, 1:nz)  |
C | [ ] X                  |
C | X = pressure (1:nx, 1:nz)  |
C |                     |
C +-----------------------*
C
C Below: Upper Left Hand corner of model
C
C +-----------------------*
C |                     |
C | 1,1 X                |
C | X 2,1               |
C | 1,1                 |
C |                     |
C +-----------------------*
C
C parameter(nx0=853,nx0=853,nz0=12000,kw=3,cl=9./8.,
1 c2=-1./24.)
C integer 2 pointr
C common/bush/pointr(nx0,nx0)
C common/earth/velo(250), xkappa(250), dden(250)
C common/field/ u0(nx0,nx0), w0(nx0,nz0), pre(nx0,nz0)
C dimension sou(nx0)
C
c call data(nx,nz,nt,dx,dt,vm,nx,s,ns,nzg,ntiskp,
1 nlay,dtx,sou,nxsklp,nx,nsnap,nxgl,ifre)
C ku0 = 1
C if(ifre.eq.0) ku0 = 2
C
c Loop over Time Steps
C
do 100 ntst=1,nt
C
C c Apply 4th-order differencing to interior particle velocities,
C c free-surface BC's and 2nd order differencing 1 node from boundary
C
do 10 k=ku0,nx
C
do 20 j=2,nx-2
C
c = pointr(j,k)
C d = dden(ii2)
C w0(j,k) = w0(j,k) -
1 d*( c1* ( pre(j+1,k) - pre(j,k) ) +
2 c2* ( pre(j+2,k) - pre(j+1,k) ) )
continue
C
do 20 k=2,nx-2
C
do 20 j=2,nx-1
C
c = pointr(j,k)
C d = dden(ii2)
C w0(j,k) = w0(j,k) -
1 d*( c1* ( pre(j+1,k) - pre(j+1,k+1) ) +
2 c2* ( pre(j+2,k-1) - pre(j+2,k) ) )
continue
C
c call pdiff2(nx,nz,ifre)
C if(ifre.eq.0) then
C do 21 j=1,nx-1
C
c = pointr(j,1)
C d = dden(ii2)
C w0(j,1) = w0(j,1) -
1 d*( c1* ( pre(j+1,1) ) +
2 c2* ( -pre(j+1,4) - pre(j+1,3) ) )
21 continue
Cendif
C
C 4th-order differencing of pressure & Free-Surface BC
c do 50 k=k0,nz-2
d0 50 j=3,nx-2
liz = pointr(j,k)
d = xxkappa(liz)
p = pre(j,k) -
1 d*x (2, cl(j,liz, j), w(j-1,k) ) +
2 c2*x ( w0(j+1,k) - w(j-2,k) ) +
1 c1*x ( w0(j-1,k-1) - w0(j-1,k) ) +
2 c2*x ( w0(j-1,k+1) - w(j-1,k) )
50 continue

50 c Apply absorbing BC's, 2nd order differencing 1 point from boundaries,
c and exponential damping along 50 node swath adjacent to boundaries.
c
   call abscbc(nx,nz,dtm,ifre)
c   call pdiff2(nx,nz,ifre)
c
   if (ifre.eq.0) then
do 51 j=1,nx
   pre(j,2) = 0.
   pre(j,1) = -pre(j,3)
51 continue
   endif
   call atten(nx,nz,dtm,dx,ifre)

51 c Add source and write seismograms to "pre & sesap"
   if (ntst.lt.ntnt) pre(nx,nz,nz)*pre(nx,nz,nz)*sou(ntst)*dt
c   call wrdata (ntst,ntiskp,nz,nxg1,nx,nxskip,nsnap,nsnap)
c
100 c continue stop end

50 c subroutine wrdata (ntst,ntiskp,nz,nxg1,nx,nxskip,nsnap,nsnap)

   c Store seismograms and snapshots
   c
   c Parameter vector
   c
   c inl Horton/brass.
   c
   parameter (nx0=853,nz0=853,nt0=12000)
   common/fld/ u0(nx0,nx0),w(nx0,nx0),pre(nx0,nx0),litt(ntst/ntiskp)
   if (litt.eq.0) then
      write(9) (pre(j,nz),j=1,nx,nxskip)
      write(21) (pre(nxg1,j),j=1,nx,nxskip)
   endif
   litt = nsnap*(ntst/nsnap)
   if (litt.eq.ntst) then
      write(19) (pre(j,k),j=1,nx,nxskip,k=1,nz,nxskip)
   endif
   write(6,*),'Time step 1 = ',ntst
   call flush(6)
c
   return
end

50 c subroutine data (nx,nz,nx,nt,dx,dt,vm,nxs,nz,ng)
c
   1
   c Set source function (Ricker wavelet)
c
   c b = (3.1415926*vm)**2
   a = 2.*b
do 50 i=1,nts+1
  t2=((1-t)*dt)+2
  sou(i+1)=(1.-a*t2)*exp(-b*t2)
  sou(i)=sou(i+1)
nts = 2*nts + 10
end

subroutine udiff2(nx,ny,nz,ifre)

parameter(nx0=853,nz0=853,nt0=12000)
integer*2 pointr
common/bush/pointr(nx0,nz0)
common/earth/velo(250),xkappa(250),dden(250)
common/field/u0(nx0,nz0),w0(nx0,nz0),pre(nx0,nz0)

2nd Order Differentiating at Bottom Boundary and Top Boundary
k=nz-1
do 10 j=2,nx-1
  ilz = pointr(j,k)
d = xkappa(ilz)
pree(j,k) = pree(j,k) - d*( ( u0(j,k) - u0(j-1,k) ) +
     ( w0(j-1,k-1) - w0(j-1,k) ) )
10 continue
if(ifre.ne.0) then
  k = 2
do 11 j=2,nx-1
  ilz = pointr(j,k)
d = xkappa(ilz)
pree(j,k) = pree(j,k) - d*( ( u0(j,k) - u0(j-1,k) ) +
     ( w0(j-1,k-1) - w0(j-1,k) ) )
11 continue
endif

if(nx=0) then
  k = nz-1
do 13 j=2,nx-1
  ilz = pointr(j,k)
d = dden(ilz)
w0(j,k) = w0(j,k) - d*( pre(j+1,k) - pre(j+1,k+1) )
13 continue
endif

2nd Order Differencing at left and right boundary
j=1
do 20 k=1,nx
  ilz = pointr(j,k)
d = dden(ilz)
u0(j,k) = u0(j,k) - d*( pre(j+1,k) - pre(j,k) )
20 continue
j=nz-1
do 21 k=1,nx
  ilz = pointr(j,k)
d = dden(ilz)
u0(j,k) = u0(j,k) - d*( pre(j+1,k) - pre(j,k) )
21 continue

subroutine absbc(nx,ny,dtx,ifre)

absbc applies 1st-order Clayton-Enquist absorbing boundary conditions at left, right and bottom grid boundaries. If ifre .ne. 0 then apply also at top.

parameter(nx0=853,nz0=853,nt0=12000)
integer*2 pointr
common/bush/pointr(nx0,nz0)
common/earth/velo(250),xkappa(250),dden(250)
common/field/u0(nx0,nz0),w0(nx0,nz0),pre(nx0,nz0)

absorbing BC's at top and bottom boundaries
k=nz

do 211 j=1,nx
  ilz = pointr(j,k)
d = velo(ilz)
pree(j,k)=pre(j,k)-dtx*d*(pre(j,k)-pre(j,k-1))
211 continue
if(ifre.ne.0) then
k=1
   do 221 j=1,nx
      ilz = pointr(j,k)
      d = velo(ilz)
   pre(j,k)=pre(j,k) + dtx*d*(pre(j,k) - pre(j-1,k))
   endif
   c absorbing BC's at left and right boundaries
   c
   j=nx
   do 231 k=2,nz-1
      ilz = pointr(j,k)
      d = velo(ilz)
   pre(j,k)=pre(j,k) - dtx*d*(pre(j,k) - pre(j-1,k))
   j=1
   do 241 k=2,nz-1
      ilz = pointr(j,k)
      d = velo(ilz)
   pre(j,k)=pre(j,k) + dtx*d*(pre(j+1,k) - pre(j,k))
   c return
   end
   c
   subroutine atten(nx,nz,dtx,dx,abre)
   c apply exponential damping to pressures at side and bottom boundaries
   c and out to 50 nodes from boundaries. If abre not equal to 0 then
   c apply at top boundary also.
   c******************************************************
   parameter(nx=853,nz=853,nt0=12000)
   integer*2 pointr
   common/bush/pointr(nx0,nz0)
   common/earth/velo(250),xkappa(250),dden(250)
   common/field/u0(nx0,nz0),w0(nx0,nz0),pre(nx0,nz0)
   as = .95
   na = 50
   nm = na - 1
   nb = na+1/(na*dx)*log10(as)/log10(2.73)
   c damp pressure na node region along left and right boundaries.
   c
   do 10 lx=1,na
      aa = exp(-alpha*lx*dx)
   do 10 li=1,nl
      pre(nz-na+lx,li)=pre(nz-na+lx,li)*aa
   pre(nb-lx,li) = pre(nb-lx,li)*aa
   10 continue
   c damp pressure na nodes from bottom boundary to the bottom.
   c
   if(ifre.ne.0) then
      do 20 li=1,nb
         aa = exp(-alpha*lx*dx)
      do 20 li=1,nl
         pre(li,na-li)=pre(li,na-li)*aa
      20 continue
   endif
   c
   do 21 li=1,nb
      aa = exp(-alpha*lx*dx)
   do 21 li=1,nl

mod.f generates the file "vel". "vel" contains numbers which assign
c a layer number to each node point.
c Coordinates of line segments describing interface boundaries are
given in "inmod". The coordinates (x, z) associated with the
c interface geometry are read into mod.f and the lines below
c this interface are given a specified velocity, density value.
c Make sure line segments start from left to right. Start from
c top and work way down.
c
- input file - input parameter file for finite difference
c program pp4.f

- input file - parameter file for finite difference

- input file - model parameter file (contains node pt
c info for defining interface geometry).

- output file - information file for interface geometry,
c and stability, dispersion info for FD's.

- output file - layer # assigned to each grid pt of model.

c Input format (file="inmod") for interface node pts is the
value of (x(z),layer #) of each node pt of an interface.
- The free-surface interface pts are given first, and the next
- shallowest interface pts are given next, and this continues
- until you get to deepest interface. Velocity info for each
- layer is given in file "indat". There is a restriction on the
- type of model "mod.f" can generate, namely, only "downward convex"
- interfaces. "downward convex" are interfaces defined by points in
- which a vertical line beginning at any interface point and ending at
- the z=inf plane does not intersect any other interface.

EXAMPLE: "inmod" file for a 2-layer fault model:

\[
\begin{align*}
\text{FREE SURFACE} & \quad (0,0) \quad \cdots \quad (700,0) \\
\text{Layer 1} & \quad (700,0) \quad \cdots \quad (700,45) \\
\text{Layer 2} & \quad (700,45) \quad \cdots \quad (700,90) \\
\end{align*}
\]

EXAMPLE OF A MODEL WITH A DOWNWARD NON-CONVEX INTERFACE

\[
\begin{align*}
\text{FREE SURFACE} & \quad (0,0) \quad \cdots \quad (700,0) \\
\text{Layer 1} & \quad (700,0) \quad \cdots \quad (700,45) \\
\text{Layer 2} & \quad (700,45) \quad \cdots \quad (700,90) \\
\end{align*}
\]

Compute Lamé's parameters from rho, vs

\[
vmax=2, \quad vmin=0.005. \\
\text{write(16,*)'Finite Difference Parameters'} \\
\text{write(16,9)'

\text{Layer Parameters'} \\
\text{write(16,9)'

\text{do 4 i=1,nlay} \\
\text{read(14,*) vs, rho} \\
\text{xini(1) = vs} \\
\text{write(16,9)'

\text{Parameters for Layer 'i, i} \\
\text{write(16,9)'

\text{if(vmax<vs) vmax=vs} \\
\text{if(vmin<vs) vmin=vs} \\
\text{write(16,9)'

\text{continue} \\
\text{write(16,9)'

\text{Stability and Dispersion Information'} \\
\text{write(16,9)'

\text{write(16,9)'

\text{dx = dx5.} \\
\text{if(dx<dx5) then} \\
\text{write(16,9)'

\text{else

\text{\texttt{parameter(nnx=301, nny=700)}}} \\
\text{\texttt{character(8, name, name) common/velo/xs5(30,30),}} \\
\text{\texttt{lxxe(30,30), zss(30,30), x(1000), z(1000)}}
mod.f

write(16,*) '******************************************
write(16,*) '******************************************
write(16,*) 'Spatial Sample Too coarse; min wavelength = w, da5
write(16,*) '******************************************
write(16,*) '******************************************
endif

sta = vmax*dt/dx
if(sta.lt..55) then
write(16,*) 'Stability Criteria: .55 > cmax*dt/dx = ', sta
else
write(16,*) '******************************************
write(16,*) '******************************************
write(16,*) 'Stability Violated; desired sta= .55, actual sta=', sta
write(16,*) '******************************************
write(16,*) '******************************************
endif

C Assign 1st layer parameters throughout model.

C do 6 j=1,nz
C do 6 i=1,nx
C cl(i,j)=1
6 continue

C read node pts of segments

C io=0
write(16,*) '--------------------------------------------
write(16,*) ' Interface Node Information '
write(16,*) '--------------------------------------------

C do 12 l=1,10000
read(15,*,end=33) x(l),z(l),nll(l)
if(nll(l).eq.nlay) go to 33
write(16,*) '
write(16,*) ' (x, z) = (',x(l),',',z(l),'): layer =',nll(l)
write(16,*) '(nx,nz) = (',x(l)/da5,','),z(l)/da5, ')
io = io + 1
12 continue
33 continue
io = io + 1
ns = 0

C assign layer # & coordinates of segments to xss, zss and xeexsee

C do 13 l=1,io
if(nll(l).eq.nll(l+1)) then
ns = ns+1
nll(l) = nll(l+1)
xss(ns,nl) = x(l)
xeex(ns,nl) = x(l+1)
zss(ns,nl) = z(l)
zeeex(ns,nl) = z(l+1)
nsseg(nn) = ns
else
ns = 0
endif
13 continue

C Loop over layers
C do 100 nl=1,nlay
C Loop over segments which make up an interface

C nseg = nsseg(nl)
do 90 ns = 1,nseg
xs = xss(ns,nl)
zss = zss(ns,nl)
xeex = xeex(ns,nl)
zeeex = zeeex(ns,nl)
nx = nx/(dx+1)
nz = nz/(dx+1)
nz = nz/(dx+1)
slope = (zeex-zs)/(xeex-xs)
dz = (zeex-zs)/(nx-nx+1)
90 continue

C Loop over pts which make up a segment
C do 50 i=xss,nx
li = i-xss
iz = i+(xs+dz*li)/dx
45 continue
50 continue
90 continue
100 continue

C Write the layer number for each node into "vel"
C do 200 j=1,nz
write(13) cl(i,j),j=1,nx
200 continue
C end

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INVERSION OF 3-D GRAVITY DATA USING RIGHT
RECTANGULAR PRISMS AND DAMPED LEAST SQUARES

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ABSTRACT

A modified Levenberg-Marquardt method is used to invert 3-D gravity data. Right rectangular prisms are used to model the anomalous bodies. Soft constraints are used to reduce the step size, and are also applied with a flatness matrix to reduce model complexity. Synthetic inversions show rapid convergence with excellent resolution when a geologically sound initial guess is made. Tests also indicate that gridding of randomly spaced data can lead to poor data fit and difficulty in choosing an appropriate stopping criteria.

The routine is also used on residual Bouguer gravity data from the Salt Lake Valley, Utah, in an attempt to model the low density basin sediments above the consolidated basement. However, convergence is never achieved and models resolved are geologically unsound. From tests with this real data it is concluded that the data set is possibly too noisy or that the regional gravity field removed may be inadequate. Also, the inversion as attempted may be ill-posed and a different inversion method needed to achieve success.
INTRODUCTION

The gravity data in the Salt Lake Valley (SLV) is inverted to find the depth to the 'basement' surface. This 'basement' surface, for the purpose of this gravity inversion is the boundary between unconsolidated and semiconsolidated sediments younger than Cretaceous, and the consolidated rocks below (Arnow and Mattick, 1968; Mattick, 1970). This interface is coincident with a very large impedance contrast which is the primary cause of focusing and diffraction of seismic energy entering the SLV (Hill, 1988).

The Salt Lake Valley is the largest metropolitan area along the Intermountain Seismic Belt. Unfortunately, the SLV segment of the Wasatch Fault is one of only two segments along the entire length of the Wasatch Fault zone (~370 km in length) which has not had a large earthquake in historic time (Arabasz et al., 1980). The estimated earthquake recurrence time, and the large displacement possible on the SLV segment, make the SLV segment the largest contributor to the earthquake hazard in the Salt Lake City region (Youngs et al., 1987).

Past investigations of resonance and focusing of seismic energy entering the SLV have been limited. Strong-ground-motion seismometers are not abundant in the SLV and therefore the real event records are few. Previous investigators have recorded artificially generated events but were unable to study the effects of source properties, energy travel path, and directivity since the energy source was not within their control (King et al., 1983; Hays and King, 1982; Rogers et al., 1984). Studies using synthetic models have been limited to 2-D assumptions or very simplistic 3-D models (Hill, 1988; Schuster, unpublished).

Since there are numerous factors affecting the seismic response at a point on the surface it is extremely important to minimize the assumptions when studying earthquake response. Hill (1988) showed that the 'basement' interface
is the key to seismic energy focusing in the valley, but assumed the valley to be two-dimensional. If we assume this boundary exists throughout the entire valley, we can remove the 2-D assumption and generate a 3-D valley model by inverting the SLV gravity data.

Gravity data coverage is generally good in SLV with only a few exceptions (Figure 1). Well data do exist within SLV, but basement inferred penetrations are limited to locations close to the edges of the valley, and are typically drillers' logs from water wells. In the northern portion of SLV there are three seismic reflection lines. Two of these are dip lines, and the third is a strike-tie line. As can be seen from Figure 1, the gravity data is the most abundant geophysical data set in SLV.

Two refraction lines have also been recorded in SLV, but these were unreversed and generally sampled section deeper than the basin sediments (Bashore, 1982). Shallow shear waves have been recorded at sites in the Salt Lake City area but are too shallow to assist in locating the basement surface (Wong, 1979). Other gravity studies have been undertaken in SLV but were only 2 or 2 1/2 D studies, or did not use the available seismic data (Wong, 1979; Bashore, 1982; Fox, 1983). This study of the SLV uses the gravity data in a 3-D inversion applying the seismic data and well depths for constraints. The basement topography resulting from this study will be used for studying the SLV's resonance and focusing of incoming seismic energy.

The 3-D inversion code is run on the Stellar computer and the algorithm used is a modified version of the Levenberg-Marquardt least-squares minimization algorithm presented by Press et al. (1986). The gravity signal calculated in the forward process is a numerical adaptation of the vertical attraction of gravity due to a right vertical prism discussed by Nagy (1966) and Goodacre (1973).
Figure 1. Geophysical data in the Salt Lake Valley area.
The original code for this algorithm was obtained from C. Schlinger of the University of Utah.

**THEORY**

The theories for inversion of geophysical data have been well covered by other researchers and detailing this would be redundant and unnecessary. Suffice it to say that the inversion used for the results presented is a standard $L_2$ norm minimization using Gauss-Jordan elimination with full pivoting. However, additions to the standard least squares inversion will be discussed.

The function used to generate a gravity field is presented by Nagy (1966) and discussed by Goodacre (1973). This expression uses a right-vertical prism to generate a synthetic gravity signal (see appendix for this function). Inverting for the bottom coordinate of each prism in the model presents us with a non-linear inversion requiring an iterative solution. This study determines the Jacobian using an analytical expression rather than a more standard finite difference approach. This reduces calculations required and possible instability caused by approximate Jacobian values. (See appendix for the Jacobian formulation). Other non-standard least squares methods applied are diagonal damping to reduce step size and flatness damping to reduce excessive complexity in the solution. The cost function thus defined is:

$$\chi^2 = J^T \Delta d - (J^T J + \psi D^T D + \mu I) \Delta m,$$

(1)

where $J$ is the Jacobian matrix, $D$ is the flatness matrix, $I$ is the identity matrix, $\Delta d$ is the data residual vector, $\Delta m$ is the change in parameters vector, and $\psi$ and $\mu$ are weighting values.

essential to resolving the model realistically while fitting the data. Imposing flatness constraints in this 3-D study requires two flatness matrices: \( D_v \) to 'link' the north-south adjacent parameters, and \( D_h \) to 'link' east-west adjacent parameters. Equation 1 thus becomes:

\[
\chi^2 = J^T \Delta d - (J^T J + \psi \Gamma_{D_v}^T D_v + \psi \Gamma_{D_h}^T D_h + \mu I) \Delta m.
\]  

(2)

Finally, to allow for constraining of particular parameters due to independent control data, 'soft' constraints are applied. Soft constraints are used for the well control since this data is not of exploration quality. Additionally, 'hard' constraints make the use of the flatness constraints dubious. These constraints are applied by adding the term \( IS \) where \( S \) is a column vector and will weigh those parameters with independent control more to reduce their step size at any iteration. So, the cost function is finally:

\[
\chi^2 = J^T \Delta d - (J^T J + \psi \Gamma_{D_v}^T D_v + \psi \Gamma_{D_h}^T D_h + \mu I + IS) \Delta m.
\]  

(3)

With this equation an iterative inversion scheme is established for generating a new model which improves the data fit at each inversion.

RESULTS

Synthetic Tests

To test the procedure for use in determining the depth to basement in the Salt Lake Valley, inversions were carried out on a full scale synthetic model of the SLV using the observed data station locations. Tests performed were designed to show the importance of the initial guess, data distribution, and damping. These tests also show the usefulness of the parameter variance generated by the inversion process. A plausible model of the SLV with 192 prisms is used to generate gravity values for the tests. The 192 prisms are 2500 meters on a side and vary from 100 to 2000 meters in thickness. A density contrast of -0.26 g/cc
is used for all 192 prisms, hereafter model 3. The synthetic data set is contoured in Figure 2. Stopping criteria used is the normalized chi squared:

\[
\chi^2 = \frac{\sum_{i=1}^{N} (d_{\text{obs}_i} - d_{\text{calc}_i})^2}{(N-M)\sigma^2},
\]

where \(d_{\text{obs}_i}\) and \(d_{\text{calc}_i}\) are the observed and calculated gravity values at the \(i^{th}\) station, \(N\) is the number of observations, \(M\) is the number of inverted parameters and \(\sigma\) is the assumed data standard deviation.

Two initial guesses were tested with this model. Guess 1 used a value of 2500 meters below the surface for the base of each prism. Requiring \(\chi^2 < 1\), guess 1 converged in five iterations; however, there were numerous errors between the true and resolved model. The larger error correlated with areas of shallow depth, but even more strongly with edges of the valley. Greater station coverage reduced the error observed in shallow depth areas. Figure 3 displays one west-east cross-section of this inversion and some statistics about the data and the inversion results.

The second initial guess tested was a better model of a valley than guess 1. All prisms adjacent to basement surface outcrops were given a depth of 100 meters and all other prisms started with a 1500 meters depth. The inversion starting with guess 2 achieved \(\chi^2 < 1\) in 4 iterations. The model found was also much closer to the true model than when using initial guess 1. Resolution was essentially perfect at all but 5 prisms. These 5 prisms correspond to shallower depth and either low station density or model edge areas. Figure 4 shows these inversion results for the same cross-section as that of Figure 3.

To test if areas of low resolution could be improved by using an evenly distributed data set, the synthetic data generated at the true data station locations were gridded using a commercial minimum curvature routine. This gridded data
Test3 Model observed 33x41 1250 m grid (mgals)

Figure 2. Gravity field contour map generated by test model3
Figure 3. a) West-east profile of Salt Lake Valley from inversion using initial guess 1 with non-gridded data. b) Errors in depth, standard deviations, station density coverage, and width-depth ratios for parameters in a).
Figure 4. a) West-east profile of Salt Lake Valley from inversion using initial guess 2 with non-gridded data. b) Errors in depth, standard deviations, station density coverage, and width-depth ratios for parameters in a).
set was spaced at 1250 meters in both north-south and east-west directions (half the model prism size), giving one station in the center of each prism and 3 along each prism edge. The inversion was run using initial guess 2 for ten iterations, although little improvement in data fit was observed after the second iteration. Convergence was never achieved in the $\chi^2$ sense with $\chi^2 > 7852$ at the tenth iteration. Model resolution was surprisingly good despite this high $\chi^2$ value, however it was much worse than that achieved with the non-gridded data set. Areas with poor station coverage and large error in resolution using the non-gridded data still showed these large errors. Error was reduced for the two prisms on the edge of the model but these were still not perfectly resolved. Finally, the inversion with this gridded data resulted in errors in prisms which were perfectly resolved with the non-gridded data. Figure 5 shows the same cross-section as Figures 3 and 4 for this inversion. Many of the errors for this inversion exceeded 10 % and several exceeded 40%.

The results from the inversion using the gridded data illustrate the good correlation between the width-depth ratio and the resolved model error. This correlation is not as evident in the non-gridded inversion results. The results also indicate that the parameter standard deviation is a poor indicator of model error. This standard deviation shows better correlation with the width-depth ratio. With the introduction of errors not present in the non-gridded data inversions, it was deemed that the gridding of the observed data was not beneficial.

**Real Data Inversions**

Before inverting the real data, it was plotted in 2500 meter wide profiles and checked for erroneous data points. Several spikes were found and removed as well as numerous duplications. Next, data beyond 2500 meters outside of the valley model were removed. Finally, a planar regional was removed identical to
Figure 5. a) West-east profile of Salt Lake Valley from inversion using initial guess 2 with gridded data. b) Errors in depth, standard deviations, station density coverage, and width-depth ratios for parameters in a).
Figure 6. Residual Bouguer gravity contour map of the Salt Lake Valley data used in the real data inversion.
that used by Fox (1983). Figure 6 is a contour map of this residual gravity field.

Despite the encouraging indications of the synthetic tests, the inversions attempted with real data were disappointing. Results of 20 inversions with a variety of initial guesses, assumed data noise levels, constraint parameters, and damping methods provide the same observations: Damping had to be increased after almost every iteration to keep the $\chi^2$ value decreasing. Little improvement was realized in the $\chi^2$ value after the first few iterations ($< 10$), and additional iterations degraded the plausibility of the resulting model. Convergence in the $\chi^2$ sense would only be achieved if the assumed data standard deviation is near 18 mgals. This noise level is on the order of the observed data and these inversion results are more a function of the initial guess rather than the data due to the damping required to maintain stability.

CONCLUSIONS

The objective of this study, to find the basement surface of the Salt Lake Valley, has not yet been achieved. However, the inversion results from synthetic tests have been encouraging in the model resolution achieved and the time required for convergence to occur. Synthetic tests also show the benefit of a more sensible initial guess and the risk of gridding randomly spaced gravity data for inversion purposes. Also, the poor correlation in the test results between the computed parameter standard deviations and the model error suggests that the standard deviations may not be a good indicator of model resolution.

The poor inversion results using the real gravity data observed in the SLV provide us with several conclusions: This data set may be corrupt even though contour maps of the data appear reasonable. This data, collected over a period of 20 years, may need to be reprocessed. The linear regional removed may be inadequate for the problem posed here. One possible cause for the lack of
convergence in the real data inversion that still needs to be studied is the data's spatial frequency and its depth resolving power. In general, the depths in the valley are believed to be less than 2000 meters. Much of the data were collected in profiles with fairly small station spacings, but the profiles are generally spaced quite far apart. These wide spacings of the data set may limit resolution of the shallow depths in the valley. So, errors introduced by gridding may have to be tolerated to get sufficient coverage to achieve desired depth resolution. The possibility that the problem is ill-posed for the inversion method tested should also be considered and other inversion procedures investigated.
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APPENDIX

The vertical component of gravitational attraction due to a right vertical prism is given by:

$$
g_z(x, y, z) = -\gamma \frac{w}{\sqrt{(u^2+y^2+z^2)^3}} \int_{w_1}^{w_2} \int_{y_1}^{y_2} \int_{x_1}^{x_2} du dv dw,
$$

(A1)

where \(x, y, z\) are the cartesian coordinates of the observing station, and \(u_2, u_1, v_2, v_1, w_2, w_1\) and \(w_1\) are the east, west, north, south, top, and bottom coordinates of the prism faces. The prism faces are perpendicular to the coordinate axes. Nagy (1966) shows the integrated result of (A1) as:

$$
g_z(x, y, z) = -\gamma \left[ u \ln \left( u + \sqrt{u^2 + v^2 + w^2} \right) + v \ln \left( v + \sqrt{u^2 + v^2 + w^2} \right) - w \tan^{-1} \left( \frac{uv}{w \sqrt{u^2 + v^2 + w^2}} \right) \right]
$$

(A2)

From equations A1 and A2, it can be seen that the sensitivity matrix when inverting for the bottom coordinate of a right vertical prism the sensitivity matrix is not a constant, instead it is a Jacobian matrix of partial derivatives \(J\), and the inversion must therefore proceed iteratively.

Many iterative inversion schemes use a numerical calculation to develop the Jacobian matrix \(J\). This requires at least one and possibly two evaluations of the data functional (A2) for every observed data value. To avoid unnecessary calculations and to reduce errors from a numerical approximation, the partial derivative of equation A2 wrt \(w_1\) was evaluated. The form of this analytical Jacobian was found to be:

$$
\frac{\partial G_z(x, y, z)}{\partial w_1} = \gamma \left[ \frac{\partial f_{11}}{\partial w_1} + \frac{\partial f_{12}}{\partial w_1} \frac{\partial f_{21}}{\partial w_1} \right] - \left[ \frac{\partial f_{15}}{\partial w_1} + \frac{\partial f_{16}}{\partial w_1} \frac{\partial f_{25}}{\partial w_1} \right] + \frac{\partial f_{17}}{\partial w_1} \frac{\partial f_{18}}{\partial w_1} \frac{\partial f_{27}}{\partial w_1}
$$

(A3)
where \( \frac{\partial f_{11}}{\partial w_1} = \frac{-4a_1w_1c}{b_1\sqrt{r_1^2+r_i^2}} \), and \( \frac{\partial f_{21}}{\partial w_1} = a_ib_ir_1^{1/2}c \left\{ \frac{1+c^2r_i^{-2}}{r_i^2+(a_ib_i)^2} \tan^{-1} \left( \frac{a_ib_i}{c\sqrt{r_i}} \right) \right\} \)

Substitutions required to make the equation complete are

\[ c = z - w_1, \quad r_i = a_1^2+b_1^2+c_i^2, \quad a_1 = b_2 = a_3 = b_5 = x - u_2, \quad b_1 = a_2 = a_3 = a_4 = y - v_2, \text{ and} \]

\[ a_5 = b_4 = a_7 = b_8 = x - u_1, \quad b_2 = a_5 = b_7 = a_8 = y - v_1. \]

**Algorithm Testing**

To ensure the forward calculation is coded correctly on the computer, the values calculated are compared to values resulting from an analytical equation. The vertical component of gravity due to an upright cylinder is given by Telford *et al.*, (1976) as:

\[ g_z = 0.01277\rho \left[ L + \sqrt{z^2+R^2} - \sqrt{(z-L)^2+R^2} \right] \]

resulting in mgals when spatial units are in feet. \( L \) is cylinder height, \( R \) is cylinder radius, \( \rho \) is cylinder density, and \( z \) is the height above the cylinder where the observation is made. 5 prisms are used to simulate the cylinder. The error between the analytic form and the prism approximation is 2.7%. This error is attributed to the coarse approximation of the cylinder using only 5 prisms.

The analytical Jacobian routine is tested for validity by comparing results with a finite difference Jacobian using the forward code already tested. Using a \( \Delta h \) of 0.1 meters the difference is 8.1%. This shows the algorithm is correct and the error that would be introduced if a finite difference approximation were used. Small scale inversions of simple models with a variety of initial guesses show the procedure is able to make accurate steps to quickly converge on the solution. Other tests show areas where the process has singularities and adjustments are coded to provide for these situations.
Preliminary Results For Wave Equation Inversion Of

Real VSP Data

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The wave equation travel time inversion (WT) method (Luo and Schuster, 1990) is applied to the Bridenstein VSP data set (Salo and Schuster, 1989). This project was started less than a week before the January 26 annual meeting. Thus, these are preliminary results and should be judged as indicating the lowest bound for the capabilities of the WT method applied to real data.

D–Day Minus 4. The Bridenstein VSP data (courtesy of Arco, Doug Foster and Jim DiSiena) is given in Figure 1. The source was a Vibroseis truck laterally offset 1000 feet from the receiver well. Three component receivers are used but only the Z-component seismograms are depicted in Figure 1. There is 50 feet between adjacent receivers and the peak frequency of the source wavelet is about 80 Hz. Salo and Schuster (1989) applied ray trace tomography to the transmission and reflection traveltimes to invert for the velocity profile shown in Figure 2. The light line is the P-velocity profile from the sonic log and the dark line represents the reconstructed velocity model. The arrows represent the layer interface locations where large amplitude reflections were observed to emanate.
**D-Day Minus 3.** The direct waves in Figure 1 were windowed to give the data in Figure 3. Note a time shift of 300 ms is applied to the data to economize on the computational time of subsequent finite difference modeling. The source wavelet (to be used in the WT inversion) was extracted by averaging the direct wave over several adjacent traces in Figure 3. With only 3 days before the meeting we had to economize computational time by neglecting the low velocity zone in the first 600 feet of the Figure 2 model; this was necessary in order to minimize finite difference computation time. This forced the removal of the shallowest 11 traces from the seismograms in Figure 1.

**D-Day Minus 2.** The wave equation traveltime method (WT) is applied to the Figure 3 data. Figure 4 shows the reconstructed model after 10 iterations, where the dotted line is the WT reconstruction, the dark line is the velocity derived from ray trace traveltime tomography (Salo and Schuster, 1989) and the light line corresponds to the sonic log velocity. The WT reconstruction is a smoothed approximation to the sonic velocity. This velocity reconstruction is judged to be less accurate than that given by ray trace tomography. We believe this discrepancy is largely caused by neglecting the first 600 feet in the model for the WT inversion.

**D-Day Minus 1.** Future studies will remedy this last problem by taking into account the entire section of the earth model.

**REFERENCES**

Figure 1. Bridenstein VSP data (courtesy of ARCO). The receivers are offset every 50 feet in depth, the source is a Vibroseis offset by 1000 feet from the source well, and the Z-component seismograms are depicted above. The vertical axis numbering corresponds to the geophone number.
Figure 2. Velocity vs depth profile for the Bridenstein well. Dark line represents the reconstructed velocity from the VSP data (Reflection+transmission traveltime tomography) and the light line represents the sonic log velocities. Arrows indicate interface locations which give rise to strong amplitude reflections.
Figure 3. Edited VSP data from Figure 1 to be used in WT inversion. Traces 121 to 130 were removed (depth less than 600') and the traces were shifted by 300 ms. In addition, all but the direct waves were muted out.
Figure 4. Comparison between velocity profile reconstructed by WT method from Figure 3 data and sonic/VSP velocities. Dotted line is the WT velocity profile, light line is sonic log velocity profile, and dark line is the VSP derived velocity profile. Starting model is assumed to be homogeneous with velocity=4.0 km/s.
Reflection+Transmission Crosswell Tomography

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ABSTRACT

The merits of transmission+reflection crosswell tomography are compared to transmission crosswell tomography. Theoretical analysis shows that both data sets will be characterized by a model null space dimension \( \geq N-1 \), where \( N \) is the number of pixels between the source and receiver wells. This assumes straight rays, horizontal reflectors, and vertical boreholes. \( N-1 \) of the associated null eigenvectors represent strictly horizontal velocity variations in the model. Computer tests suggest that the minimum null space dimension of \( N-1 \) is achieved for square models if there are at least two sources and two receivers per well pixel. Coarser source and receiver spacing may enlarge the null space to be greater than \( N-1 \) for transmission data; however, the inclusion of reflection data may reduce the null space dimension to its theoretical minimum of \( N-1 \). This is a primary benefit of reflection+transmission crosswell tomography, namely it allows for a finer velocity resolution than allowed by transmission crosswell tomography.

The null space of crosswell data can be eliminated entirely if 1). offset VSP data is included, or 2). a long row of pixels is reparameterized as a single long pixel, or 3). deviated boreholes are used, or 4). reflections from dipping layers are incorporated, or 5). severe ray bending is extant. Of these remedies, the inclusion of offset VSP data with the crosswell data will give the most robust results and is highly recommended. These remedies are verified by tomographic reconstructions of both real and synthetic crosswell data.
INTRODUCTION

Geotomography is perhaps the most systematic and useful methodology for extracting earth information from a seismic crosswell data set. In traveltime tomography the earth model is divided into MxN cells (Figure 1) of unknown slowness, and each source gives rise to an observed transmitted traveltime at each of the receivers. Additional information in the form of reflection, refraction and diffraction traveltimes can also be used to enhance the inversion. L traveltime picks contained in the vector $t$ are related by the traveltime integral to MxN unknown slownesses in $s$. The L by 1 data residual vector $\Delta t = t_{predicted} - t_{observed}$ is related to the MxN by 1 slowness perturbation vector $\Delta s = s_{predicted} - s_{actual}$ by

$$ L \Delta s = \Delta t $$

(1a)

where the elements of the L by MxN matrix $L$ consist of partial derivatives of the traveltime with respect to slowness changes. At each step of a Gauss-Newton method, the slowness residual is given by

$$ \Delta s = [L^T L + \varepsilon I]^{-1} L^T \Delta t $$

(1b)

where $\varepsilon$ is a damping parameter to stabilize the solution. In this paper we will restrict ourselves to traveltime tomography applied to the crosswell geometry (e.g., Figure 1).

Previous Crosswell Tomography Research

In 1979, Dines and Lytle described a straight-ray reconstruction algorithm
appropriate for use with electromagnetic and seismic data in weakly refractive media. They later extended the method to strongly refracted rays (Lytle and Dines, 1980). Since then researchers have applied the traveltime tomographic method to a variety of problems, including the use of transmitted crosswell arrivals to monitor changes in elastic wave velocities and amplitudes in a hot dry rock geothermal system (Pearson et al., 1983), crosswell surveys to delineate fractures in crystalline rock (Wong et al., 1983 and 1984; Bregman, 1986) and elastic property changes during heating of granitic rock masses (Paulsson et al., 1985), VSP tomography (Pujol et al., 1985) and CDP tomography to delineate subsurface velocities (Bishop et al., 1985; Bording et al., 1987; and Stork, 1988). A number of researchers have performed synthetic tests to empirically evaluate the effectiveness of crosswell tomography in the presence of low velocity zones (Ivansson, 1985) or with the inclusion of reflection data (McMechan et al., 1987). Although numerical simulation studies provide valuable insights into the tomography algorithm, there is still a need to establish rigorous theory which can explain and predict the resolution properties of tomographic experiments (e.g., Fawcett and Clayton, 1984).

The objective of this paper is to establish the theoretical benefits of reflection+transmission traveltime tomography in crosswell data. It is found that straight ray crosswell transmission data will always be characterized by a null space dimension of at least N-1, where the two adjacent wells are separated by N pixels. This is also true for reflection+transmission data, when the reflections emanate from horizontal layers. However, the null space can be eliminated entirely if either the reflector is dipping or offset VSP data is included. If the reflection traveltime errors are more than twice that of transmission traveltimes, then reflection data may not improve the reconstruction. These theoretical predic-
tions are confirmed by inversion of both synthetic and real crosswell data.

THEORY OF REFLECTION+TRANSMISSION CROSSWELL TOMOGRAPHY

This section establishes the benefits of including reflections with transmission data in crosswell tomography. Crosswell data will refer to traveltime data picked from seismic energy originating from sources in one well and recorded by receivers in an adjacent well; crosswell data combined with offset VSP traveltime data will be referred to as VSP+crosswell data. For vertical wells, horizontal reflectors, straight rays, sufficiently fine source and receiver coverage and a crosswell geometry with M pixels deep and N pixels across, it is shown that:

1). the $[L^T L]$ matrix associated with transmission crosswell data will be characterized by a null space dimension of at least $N-1$. $N-1$ of these null space vectors represent strictly horizontal velocity variations.

2). the $[L^T L]$ matrix associated with transmission+reflection crosswell data will also be characterized by a null space of at least dimension $N-1$. $N-1$ of these null space vectors also represent strictly horizontal velocity variations. However, coarse source or receiver intervals may enlarge the null space dimension of the transmission data to be greater than $N-1$, while the null space of reflection+transmission data may still be characterized by a dimension of $N-1$. This is a primary benefit of including reflection data, namely a finer resolution of the velocity model compared to that from transmission data.

3). If the reflector is dipping, then the null space of $[L^T L]$ is empty for transmission+reflection crosswell data. This contrasts with the minimum
null space dimension of N-1 associated with transmission crosswell data. The null space can also be eliminated by including VSP data or by reparameterizing a row of pixels into one long pixel.

To prove these assertions we will most often use specific crosswell models but the proofs for the general MxN parameterizations are straightforward and given in Calnan (1989).

**Null Space Dimension ≥ N−1**

We will first prove that the minimum null space dimension for the 1-D parameterization in Figure 2 is N-1 (where the model is N cells wide), and then show that this is also true for a 2-D parameterization.

**Vertical Layers And Transmission Data.** Figure 2 depicts a 1-D layered model in which the slowness variation is strictly in the horizontal direction. The following argument reveals that transmission traveltime data (for negligible ray bending) are insufficient for resolving this type of slowness variation. The traveltime equation for rays 1 and 2 of the figure gives the traveltime for each:

\[ t_1 = l_1s_1 + l_1s_2 + l_1s_3 + l_1s_4 \]  \hspace{1cm} (2a)

and

\[ t_2 = l_2s_1 + l_2s_2 + l_2s_3 + l_2s_4, \]  \hspace{1cm} (2b)

where \( l_i \) is the length of the \( i \)th ray in a layer and \( s_i \) is the slowness of the \( i \)th layer. Since the rays do not bend then \( l_2 \) differs from \( l_1 = l_2 \cos \theta \) by a multiplicative constant; hence, the two equations are linearly dependent.

\[ t_1 = (l_2s_1 + l_2s_2 + l_2s_3 + l_2s_4) \cos \theta = t_2 \cos \theta. \]  \hspace{1cm} (3)
All other possible transmission traveltime equations are also linearly dependent by the same argument. Unique determination of the four velocities requires a rank 4 system of equations, and such a system of equations cannot be constructed with straight rays. In fact, the system of equations will have a rank of 1 regardless of the number of rays or the number of vertical layers. It is easy to show that the model null space of the Figure 2 model is spanned by the vectors $(-1 \ 1 \ 0 \ 0)^T$, $(-1 \ 0 \ 1 \ 0)^T$, and $(0 \ 0 \ 1 \ -1)^T$. That is the null space vectors represent strictly horizontal velocity variations.

More generally, if $N$ equals the number of unknown slownesses, then $[\mathbf{L}^T \mathbf{L}]$ of Equation (1) will have $N-1$ zero eigenvalues. This means that we can only hope to calculate an average slowness for the medium as a whole between the two boreholes; the four individual velocities cannot be correctly determined.

**Vertical Layers And Reflection Data.** Including reflection data (rays 3 and 4 in Figure 2) from horizontal interfaces does not improve this situation. Travel-time equations for rays 3 and 4 can be expressed as a constant multiplied by any of the other traveltime equations. Therefore, these equations are also linearly dependent similar to equation (3).

**2-D Model Parameterization.** The 1-D result can be extended to the case of a 2-D pixel parameterization (Figure 1) with the following argument. Elements of the partial derivatives matrix for the 2-D model in Figure 1, $\mathbf{L}_{2D}$, are ray path segment lengths within pixels. The $i$th column consists of segment lengths of rays which intersect the $i$th pixel. However, the column space of $\mathbf{L}_{2D}$ spans the column space of $\mathbf{L}_{1D}$. This can be shown by noting that the vectorial sum of the first 4 column vectors of $\mathbf{L}_{2D}$ (corresponding to the leftmost column of pixels in
Figure 1) will be equal to the first column vector of $L_{1D}$ (corresponding to the leftmost vertical rectangle in Figure 2). The sum of the next 4 column vectors of $L_{2D}$ (corresponding to the second leftmost column of pixels in Figure 1) will be equal to the second column vector of $L_{1D}$ (corresponding to the second leftmost vertical rectangle in Figure 2). That is, each column vector of $L_{1D}$ is a vectorial sum of 4 distinct column vectors of $L_{2D}$. In general, if an $M \times N$ grid of pixels is reparameterized as a $1 \times N$ model of $N$ vertical layers, columns of the new partial derivatives matrix ($L_{1D}$) are calculated simply by summing appropriate columns of $L_{2D}$. Since $L_{1D}$ is characterized by $N-1$ linearly dependent vectors and each of these $N-1$ column vectors is constructed by a distinct sum of the column vectors of $L_{2D}$, then $L_{2D}$ must have a null space dimension greater than or equal to $N-1$. Equivalently, $L_{2D}^T L_{2D}$ is characterized by at least $N-1$ zero eigenvalues. This result is not surprising: vertically-oriented features are at least as difficult to detect in 2-D models as they are in 1-D models.

If reflection-transmission data is used, then the null space dimension of $[L^T L]$ will also be greater than or equal to $N-1$. This is clearly seen in Figure (2) where the reflection rays give the same information as the transmission rays and travel times. Hence, the null space dimension of $[L^T L]$ for reflection-transmission data will be greater than or equal to $N-1$.

The above arguments lead us to conclude the following property:

**Property 1.** In general, for an $M \times N$ model parameterization the null space dimension of $[L^T L]$ will be at least $N-1$ for either transmission data or for reflection-transmission data. $N-1$ of these null space vectors will be associated with strictly horizontal velocity variations. This assumes that the rays are straight and the reflectors are horizontal. Coarser source or receiver spacing will enlarge the dimension of this null space.
Synthetic Tests. To illustrate Property 1, rays were traced in a model parameterized into 6x6 pixels with 2 sources and 2 receivers per well pixel. For transmitted rays only the eigenvalues associated with the $L^T L$ matrix are given in Figure 3a which confirms Property 1's prediction of N-1=6-1=5 zero (to machine precision) eigenvalues. The associated null space eigenvectors are shown in Figure 3b, which are, as predicted, associated with strictly horizontal velocity variations.

Extra traveltime equations can be incorporated by assuming that the top and bottom boundaries of the 6x6 model are reflectors. Using both transmission and reflection travel times to form $L^T L$, Figure 4a also shows that the null space dimension is N-1=6-1=5; these null space vectors are associated with strictly horizontal velocity variations (similar to Figure 3a).

It is obvious that coarser spacing between adjacent sources and adjacent receivers can increase the null space dimension beyond N-1. An example is given for a 12x8 model parameterization where each well pixel only contains one source or receiver. In this case $[L^T L]$ is characterized by 14 zero eigenvalues (Figure 5a), of which 7 of these eigenvalues are characterized by the irregular eigenvectors in Figure (5b). Including reflections from the top and bottom boundaries rectifies this problem so that we achieve (not shown) the minimum number of zero eigenvalues, N-1=7-1=6, which are associated with strictly horizontal velocity variations. This illustrates a primary benefit of transmission+reflection data relative to transmission crosswell data, that is it allows for a finer resolution of the velocity model for a coarse spacing of sources and receivers. Louis (1985) discusses the behavior of singular values for arbitrary ranges of view angles and Schuster (1989) discusses the resolution characteristics of reflection+transmission VSP tomography.
Elimination of the Null Space.

The null space of $L^T L$ can be eliminated by incorporating receivers and/or sources on the surface, or including reflections from dipping interfaces, or using deviated boreholes, or by re-parameterizing one row of pixels into a single long pixel of unknown slowness. These assertions will now be proved.

Property 2. Assume an $L^T L$ associated with crosswell transmission traveltimes and an MxN model parameterization. The null space of $L^T L$ can be eliminated by re-parametrizing a single row of pixels into a long pixel of unknown slowness.

This can be proved by noting that ray a in Figure 6a is sufficient to determine the velocity of the long pixel (pixel #1). Pixel #1 is considered to be stripped from the model. Ray b only intersects pixel #1 and #2, so it can be used to uniquely determine the velocity of pixel #2. Pixels #1 and #2 are now stripped. Ray c intersects only pixels #1, #2 and #3, so it uniquely determines the velocity of pixel #3. Pixels #1, #2 and #3 are stripped from the model. This argument is continued until all of the pixel velocities are uniquely determined. If a layer between wells is homogeneous, then the parameterization in Figure 6 may be justified. The benefit is a full rank $L^T L$ matrix. As noted before, a coarse spacing of receivers or sources will enlarge the null space dimension greater than N-1. In most cases, our numerical tests on square models demonstrated that about 2 receivers and 2 sources per pixel were sufficient to decrease the null space dimension to its theoretical minimum of N-1.

Synthetic Tests. To verify Property 2, transmitted rays were traced through the 12x8 model in Figure 7a. There were 24 sources evenly distributed on the left
hand side and 24 receivers evenly distributed on the right hand side. The reconstruction using the conventional parameterization is given in Figure 7c and the new parameterization reconstruction is given in Figure 7b. The new parameterization is the same as the old except for the long homogeneous pixel near the bottom (denoted by dotted lines in Figures 7a and 7b). Note the poor reconstruction in Figure 7c arising from the null space of dimension N-1=8-1=7. The reconstruction in Figure 7b is almost perfect because the null space is empty, as predicted by property 2. Analysis of the singular values of $L$ also confirms this observation.

**Property 3.** Assume an $L^T L$ associated with crosswell traveltimes and an MxN model parameterization. The null space of $L^T L$ can be eliminated by including reflections from a dipping interface.

In this situation, reflected ray path length is a function of source and receiver depth as well as shooting angle. As a result, the linear dependence of the system of equations is reduced. It can be shown, using arguments similar to the previous pixel stripping argument, that the null space can be eliminated with the inclusion of reflection data from dipping reflectors. When boreholes are not perfectly parallel, transmitted ray path length and reflected ray path length both become functions of depth and shooting angle. Therefore, this perturbation in well geometry also reduces linear dependence (Ivannson, 1987).

**Synthetic Tests.** Figure 8a shows a crosswell geometry of $12 \times 12$ pixels with 12 shots or receivers in each borehole. 432 straight transmitted and reflected rays were generated, with reflecting interfaces at the top and bottom of the model. As shown in Figure 8b, this geometry results in 11 eigenvalues close to
or equal to 0, consistent with property 1. The bottom reflector is then given a small amount of dip, as shown in Figure 9a, and 432 rays are again generated. The resulting singular values are plotted in Figure 9b. Inclusion of reflections emanating from a dipping reflector eliminates the null space of $L$.

**Property 4.** Offset VSP data combined with crosswell data also eliminates the null space of $L^T L$. This assumes a sufficiently dense sampling of sources on the surface.

This property can be proved using a pixel stripping argument. The surface pixels are indexed in increasing number as source offset increases from the receiver well. The surface source nearest to the geophone well shoots a ray to the shallowest geophone. This ray only penetrates the #1 pixel, and so uniquely determines its velocity. A further offset source can be found whose rays only penetrate pixels #1 and #2 and terminate in the shallowest geophone. This uniquely determines pixel #2. This argument can be continued until the velocities in the shallowest row of pixels are determined. Since the shallowest pixel velocities are determined, then property 2 can be used to justify the unique reconstruction of all other pixels.

**Property 5.** The null space of $L^T L$ can be eliminated by allowing ray bending to occur. This can eliminate the null space at the expense of the problem becoming non-linear.

Ray bending breaks the linear dependency embedded in equation 2; the rays from a single source are no longer linearly dependent on each other. However, the problem may exist that insufficient ray bending (mild velocity contrasts) may be characterized by singular values that are quite small, in fact, almost as small as the zero eigenvalues of a straight ray experiment. Hence, the velocity
reconstructions may be deficient in the correct lateral velocity gradients.

SYNTHETIC TESTS

This section presents results of inverting synthetic data that honors Snell's law. The objective is to assess whether reflection+transmission traveltime data from bending rays improves the fidelity of crosswell tomographic reconstructions. The ray tracer is a modification of the one written by Weber (1988) and the flow chart for the non-linear inversion is given in Figure 10.

High Velocity Zone Model

Two high velocity zone (HVZ) models are given in Figure 11a and 12a with a parameterization of 13 cells across and 10 nodes deep. Figure 11a is a HVZ embedded in a homogeneous medium and Figure 12a is a HVZ embedded in a velocity gradient medium. The homogeneous HVZ model should be characterized by weak ray bending (illustrating property 1) while the gradient HVZ model is characterized by strong ray bending (illustrating property 5). Twelve source-receiver positions are spaced at 50 ft intervals starting at a depth of 25 ft in both boreholes. Reflectors were taken to be the top and bottom horizontal boundaries of the model. This geometry yields two groups of synthetic data; one of 144 transmitted traveltimes and one of 144 transmitted traveltimes plus 288 reflected traveltimes.

Noise–free Tests. Noise-free data generated with the HVZ model of Figure 11a were inverted with an initial "guess" of a 6000 ft/s uniform velocity model. The first inversion used only transmitted arrivals and continued for 17 iterations
(Figure 11b) and the second included reflection+transmission data and was carried out for 7 iterations (Figure 11c). The reflections are clearly beneficial in this case; furthermore, the reconstruction with reflections (Figure 11c) exhibits vertical features similar to those predicted by property 1. This is an example in which the transmission data with coarse receiver spacing failed to achieve a minimum null space of N-1, while the transmission+reflection data diminished the dimension of the null space to N-1.

Noise-free data generated with the HVZ model of Figure 12a were inverted with an initial "guess" of a 6500 ft/s uniform velocity model. The final result after 7 iterations for transmissions-only data is given in Figure 12b, and the final result after 11 iterations for transmissions plus reflections is given in Figure 12c. The larger data set gives a better reconstruction, as seen in a plot of $E_{m_i}$ (RMS model error) versus iteration number (Figure 13). Figure 13 is a plot of $E_{t_i}$ (scaled RMS traveltime error) versus iteration number; for both data sets, the greatest decrease in $E_{t_i}$ comes in the first iteration, which was characteristic of all synthetic tests. This behaviour was characteristic of all the synthetic tests.

The reconstructions in Figures 11 and 12 suggest that the inversions in all four cases succeeded somewhat in detecting the horizontal component of the velocity gradient. Results of the previous section suggest that it is ray bending, in this case, that permits the detection of the horizontal velocity variations.

Noise Tests. In order to test the effect of noise on crosswell reconstructions, zero-mean random noise with a normal distribution and a standard deviation of .5 ms was added to the synthetic transmission data, and noise with a standard deviation of 1 ms was added to the synthetic reflection data.

Six data sets are considered for the noisy synthetic tests: two sets of
transmissions (well-to-well), two sets of transmissions plus reflections (reflections emanating from the top and bottom horizontal boundaries of the model), and two sets of transmissions plus reflections plus surface data. The initial model is a 6500 ft/s uniform velocity model. Given in Figure 14 are the final results of inversions with noisy HVZ data sets. Convergence was achieved in 7 iterations for the transmissions+reflections+surface data (Figure 14b), 8 iterations for the transmission data (Figure 14c) and 7 iterations for the transmission+reflection data (Figure 14d). Figure 14b suggests that the addition of the surface data permits the detection of the horizontal component of the velocity gradient, a result consistent with property 4. Reconstructions for low-velocity zone models yielded similar results as those for the HVZ tests.

In summary, addition of noisy reflection data to the transmission data resulted in little or no improvement in reconstruction accuracy. This contrasted with the significant improvement in reconstruction accuracy for noise free data. Apparently, the ray bending in noisy data is insufficient to move some of the eigenvalues very far from the origin so that the solution is still quite sensitive to data noise. The most accurate reconstructions with noisy data were seen when the surface data were included in the inversion.

REAL DATA TOMOGRAPHY

Tomographic reconstructions using a high-quality crosswell data set are now presented. The survey geometry is characterized by two vertical, parallel boreholes which are about 1000 ft deep and separated by 600 ft. Energy from 98 equi-spaced explosive shots in one borehole was recorded by 96 equi-spaced hydrophones in the other borehole. Additionally, 23 shots and 24 geophones
were positioned on the ground surface between the two boreholes, yielding surface data equivalent to a double VSP experiment.

Figure 15 is a representative unprocessed shot gather of the crosswell data (shot depth = 360 ft). In addition to the direct arrival, there is a strong reflection emanating from (presumably) the free surface between the two boreholes. This arrival was manually picked on most of the crosswell shot gathers. A static shift of 1.28 ms was subtracted from the manual reflection picks to adjust for the fact that the reflection peak, rather than the reflection first break, was picked on seismograms.

The model parameterization is 49 nodes in the vertical direction and 31 nodes in the horizontal direction, yielding 1519 unknown slownesses. There are approximately 2 source/receiver positions per node in the vertical direction and about .75 source/receiver positions per node in the horizontal direction.

Transmission Data Reconstruction. A total of 8090 transmitted traveltimes were inverted with an initial "guess" (starting model) of a 6000 ft/s constant velocity medium. In this case the shallowest shot was at a depth of 25 feet so that the parameterization depth started at a depth of 25 feet rather than at the surface. Six iterations were carried out and the associated reconstructions are given in Figure (16) Also in the figure is a bar plot of velocities derived from a sonic log in the source well (the well on the left). The sonic log and reconstructions do not agree. The data donor suggested that the sonic log may be unreliable because the shot well may have been damaged from the dynamite blasts. Secondly, gas may be leaking from an underlying reservoir, again causing velocities to be lower than the sonic log would indicate.
Transmissions Plus Surface Data Reconstructions A total of 4037 surface data traveltimes were added to 8104 transmission arrivals, giving a data set of 12,141 traveltimes. Using a 6000 ft/s homogeneous starting model, only three iterations (Figure 17) were needed for convergence. A 60 foot thick near-surface LVZ not apparent in the transmissions-only reconstructions (Figure 16) is the most conspicuous feature in Figure 17. Its absence in Figure 16 is explained by the fact that very few of the crosswell transmitted rays penetrate this near-surface LVZ; rays with minimum traveltimes dive into the material of higher velocity.

From properties 1 and 4 it is concluded that the transmission+surface data reconstruction (Figure 17) is more reliable than the transmission data reconstruction (Figure 16). This conclusion is verified by the following evidence: 1). Figure 17 contains a realistic low-velocity zone while it is absent in Figure 16, 2). the vertically truncated high velocities (dark blue) in the lower right-hand of Figure 17 is consistent with a fault interpreted from a CDP seismic section, 3). the homogeneous area at intermediate depth corresponds to a no reflection zone on the CDP seismic data, and 4). transmission+VSP surface data offer more data and a greater range of view angles than the transmission data.

It is also seen in the reconstructions of Figure 17 that the main features of the final reconstruction (Figure 17d) are noticeable in the first reconstruction (Figure 17). This suggests a nearly linear relationship between the model and data, i.e., the rays are nearly straight. This is verified in Figure 18 which depicts the rays from a single shot traced through the Figure 17d model. Rays traced from other shot locations reveal the same weak ray bending. In this case Property 1 suggests that the the transmission data model is damaged by a null space of dimension \( N-1 = 30-1 = 29 \), while the transmission+surface data model is characterized by an empty null space.
Transmissions Plus Surface Data Plus Reflections. Inversion for this data group was carried out with 4112 reflection traveltimes added to the data of the last section, giving a total of 16,104 traveltimes. The starting model was the uniform-velocity model.

Unfortunately, reconstructions from this data set were judged to be unreliable due to unreasonable linear artifacts in the reconstructed models. These linear features were oriented along the raypaths of the reflected rays. The cause for this problem is thought to be due to the unreliability of the reflection traveltime picks. Picking errors occurred because of (1). overlapping arrivals obscuring the first break time of the reflected arrival, and (2). the indeterminacy of the reflector location. It is believed that the picked reflections were a combination of reflections from the free-surface and the bottom of the near surface LVZ.

CONCLUSIONS

It is shown that crosswell data with negligible ray bending will be characterized by an \([L^T L]\) with null space dimension at least greater than or equal to N-1, where N is the number of pixels between the source and receiver well. N-1 of these null eigenvectors will be associated with strictly vertical layering components of the model. Computer experiments suggest that about 2 sources and 2 receivers per pixel are sufficient to achieve the minimum null space dimension of N-1 for square models. The addition of straight ray reflection data will not reduce this null space dimension to be less than N-1, but will allow for the same model resolution at a coarser source and receiver spacing.

The \([L^T L]\) null space can be eliminated entirely if 1). reflections from a dipping layer are used with the transmission data, or 2). surface+crosswell transmission data are used, or 3). deviated boreholes are used for the source and
receiver wells, or 4). transmission data is used with a model having a single long pixel that connects the source and receiver wells, or 5). severe ray bending is extant. Synthetic tests verify these conclusions. However, computer experiments suggest that reconstructions from reflection+transmission crosswell data might show little improvement over that from transmission data when realistic noise levels are added to the reflection data.

Reconstructions from a real data crosswell+surface data set verify the above conclusions. That is, transmission+surface data reconstruction proved to be more reliable than transmission or transmission+reflection data.
REFERENCES


Calnan, C., 1989, Crosswell tomography with reflection and transmission data; MS thesis, University of Utah, Salt Lake City, Utah.


Louis, A., 1986, Incomplete data problems in x-ray computerized tomography: Singular value decomposition of the limited angle transform:


FIG. 1. Crosswell parameterization of $4 \times 4$ pixels. Small boxes mark source/receiver locations.
FIG. 2. One-dimensional, vertically-layered crosswell model and its mirror image. Snell's law is neglected at all interfaces except the ground surface, causing rays to be straight.
FIG. 3. Singular value decomposition of 144 traveltime equations for straight transmitted rays traced through a crosswell model parameterized into a 6x6 grid of pixels. (a). Singular values, (b) singular vectors corresponding to five smallest singular values.
FIG. 3. (continued)
FIG. 4. Singular value decomposition of 432 traveltime equations for straight transmitted and reflected rays traced through a crosswell model parameterized into a 6x6 grid of pixels. (a). Singular values.
FIG. 5. Singular value decomposition of 144 traveltime equations for straight transmitted rays traced through a crosswell model parameterized into a 12x8 grid of pixels. (a). Singular values, (b) singular vectors corresponding to six zero eigenvalues. The other eigenvectors associated with near zero eigenvalues correspond to strictly horizontal velocity variations.
FIG. 6. Pixel stripping with transmitted rays only. (a) Parameterization and rays. (b) Determination of slowness $s_2$. (c) $s_3$. 
FIG. 7. (a) Assumed true model. (b) SVD reconstruction with parameterization of Figure 7a.
(c) SVD reconstruction with conventional parameterization.
FIG. 38. (a) Crosswell parameterization of $12 \times 12$ pixels. (b) Singular values for 432 transmitted and reflected rays.
FIG. 9. (a) Crosswell parameterization of $12 \times 12$ pixels and a dipping reflector. (b) Singular values for 432 transmitted and reflected rays.
\[ i = \text{iteration number} \]
\[ v_i = \text{velocity result of } i \text{th iteration} \]
\[ \Delta v_i = \text{velocity update for } i \text{th iteration} \]
\[ L_i = i \text{th partial derivatives matrix} \]
\[ E_i = \text{RMS travelttime residual} \]
\[ \varepsilon' = \text{damping parameter} \]
\[ \delta = \text{lower-limit threshold for } |\Delta v_i| \]

\( i=0 \)

Trace rays through \( v_0 \) to generate \( L_1 \) and \( E_\omega \).

\( i=1 \)

Compute \( \Delta v_1 \) with several different values of \( \varepsilon \); choose \( \Delta v_1 \) for which

\[ v_1 = v_0 + \Delta v_1 \]

is closest to \( v_{\text{true}} \).

Trace rays through \( v_1 \) to generate \( L_2 \) and \( E_{\text{true}} \).

Is \( E_{\text{true}} < E_\omega \)?

yes

Recompute \( \Delta v_1 \) with larger value of \( \varepsilon \) to give new \( v_1 \)

no

\( i=2 \)

Compute \( \Delta v_i \) with several different values of \( \varepsilon \); choose \( \Delta v_i \) for which

\[ v_i = v_{i-1} + \Delta v_i \]

is closest to \( v_{\text{true}} \).

Is \( |\Delta v_i| \geq \delta \) ?

yes

Recompute \( \Delta v_i \) with larger value of \( \varepsilon \) to give new \( v_i \)

no

Trace rays through \( v_i \) to generate \( L_{i+1} \) and \( E_{\text{true}} \).

Is \( E_{\text{true}} < E_{i-1} \)?

yes

(no)

next page

FIG. 10 (a) Flowchart of first phase of synthetic test procedure. (b) Flowchart of second phase of synthetic test procedure.
Recompute $\Delta v_i$ with smaller value of $\epsilon$ to give new $v_i$.

Trace rays through $v_i$ to generate $L_{i+1}$ and $E_{v_i}$.

Is $E_{v_i} < E_{v_{i-1}}$?

$\nu_i = \nu_{i-1} + \Delta v_i$

Compute $\Delta v_i$ with several different values of $\epsilon$; choose $\Delta v_i$ such that $|\Delta v_i| \geq |\Delta v_{i-1}|$.

Is $|\Delta v_i| \geq \delta$?

Recompute $\Delta v_i$ with larger value of $\epsilon$ to give new $v_i$.

$v_{final} = v_i$

quit

FIG. 10 (continued)
FIG. 11. Reconstruction of a HVZ within a uniform background medium. (a) True model. (b) Reconstruction after 17 iterations with transmitted data only. (c) Reconstruction after 7 iterations with transmitted plus reflected data.
FIG. 12 Reconstruction of a HVZ within a velocity gradient. (a) True model. (b) Reconstruction after 7 iterations with transmitted data only. (c) Reconstruction after 11 iterations with transmitted plus reflected data.
FIG. 13 Error plots for inversion with noise-free HVZ data set. (a) $E_m$ versus iteration number. (b) $E_t$ versus iteration number.
FIG. 14. Best reconstruction (with noisy data) of a HVZ within a velocity gradient. (a) True model. (b) Reconstruction after 2 iterations with transmitted plus reflected plus surface data. (c) Reconstruction after 2 iterations with transmitted data only. (d) Reconstruction after 3 iterations with transmitted plus reflected data.
FIG. 15 Real data shot gather. Shot depth = 360 ft, shallowest traces are at bottom of plot.
FIG. 16 Tomographic reconstructions of crosswell transmissions (uniform starting model). Thin vertical color bar represents sonic-log derived velocities. (a) Iteration 1. (b) Iteration 2. (c) Iteration 3. (d) Iteration 4. (e) Iteration 5. (f) Iteration 6.
FIG. 17. Tomographic reconstructions of transmissions plus surface data (gradient starting model). (a) Starting model (b) Iteration 1. (c) Iteration 2. (d) Iteration 3.
PSVR2.F PROGRAM DESCRIPTION

PSVR2.F is a fortran program which solves the 2-D elastic wave equation via a 2-2 staggered grid finite difference scheme. The accuracy is second-order in space and time, and can be used for modeling elastic waves in arbitrary 2-D earth models, including H$_2$O/elastic interfaces.

To generate synthetic seismograms, two steps are necessary:

1. Generate earth model. This is done by a). specifying P-velocities, Vs/Vp ratios, densities, and finite difference parameters in the file "indat", b). specifying interface coordinates in the file "inmod", and c). executing the program "mod.f". The program "mod.f" will then generate the appropriate files (="density", "lam", "lambmu", and "mu") for the Lame constants associated with model. An example is given below. WARNING: mod.f is a primitive model generator and will not work for non-convex interfaces. However, minor adjustments in the code or input file will remedy this problem.

```
x: z layer #
  0.0,0.1
  1000.0,0.1
  0.0,350.0,2
  1000.0,350.0,2

"inmod"

(0,0)-----------------------------*(1000,0)
Layer 1

(350,0)-----------------------------*(1000,0)
  * src
Layer 2
```

2. Generate synthetic seismograms. Once "mod.f" has been executed, then "psvr2.f" can be executed to generate the files "psvx" and "psvz". These last two files contain the x and z particle velocity synthetic seismograms recorded along a specified line of nodal points. An example of the input files and the graphical output (Figure 1) of the synthetic seismograms is given below for a line source in a two layer medium. The accuracy of this result was verified by comparison to that of a 4th order Bayliss code. Figure 2 contains a comparison between the results from a staggered grid code and the fourth order Bayliss code for a horizontal displacement line source in a homogeneous media (Vp=2km/s, Vs=1km/s, Ricker wavelet peak frequency=10 Hz, dx=5m, dt=0.015s, distance between src-rec=200m). The free-surface boundary condition was not checked but it gives reasonable results.

```
indat
201,201,1400,10.0,1,1 nx,ny,nntimestep,vmax, ntraceskip, ntimestep
5,0.001,100 dx,dt,nntimesamplesinricker
101,101,141 nxsrc,nysrc,nzgeophone
0,0,50 logplane(0=src,1=plane wave),lompv(0=xcomp,1=zcomp),nskpl=
2,1000,2000 nlay,snaphot information (snapstart= ts,snapinterval=ti)
2000.0, 2, .50 vp, rho, gamma, layer 1
3000.0, 2, .50 vp, rho, gamma, layer 2
```

Figure (1a).
Figure (1b). Trace 101 from figure 1a, X-component velocity seismogram resulting from a horizontal line source at (101,101) node point and interrogated at node point (101,141). Node points are separated by a distance of 5 meters. The model is a two layer model with the free-surface at the top node points and the layer interface between node points (0,71) and (201,71). This result matched that from a 4th-order Bayliss code. Layer parameters are given in files "inmod" and "indat" on previous page.
Figure 2. Comparison between a 2-2 staggered grid result and a 4-2 Bayliss result. The model is a line source in a homogeneous medium.
mod.f
Thu Jul 13 12:26:27 1989

******************************************************************************
c.mod.f generates density, lambda+2mu, mu, and lambda files.
c.Coordinates of line segments describing interface boundaries are
given in "inmod". The coordinates (x,z) associated with the
c.interface geometry are read into mod.f and the nodes below
c.this interface are given a specified velocity, density value.
c.Make sure line segments start from left to right and are convex.
c.Start from top and work down.
c.
c indat - input file parameter file for finite difference
          program psvr2.f

 inmod - input file model parameter file (contains node pt
          info for defining interface geometry).

c mod.dat - output file information file for interface geometry,
c and stability, dispersion info for FD's.

c mu - output file mu values at grid pts of model.
c clam - output file lambda values at grid pts of model.
c clam2mu - output file lambda+2mu values at grid pts of model.
c density - output file density values at grid pts of model.

c Input format (file="inmod") for interface node pts is the
value of (x,z,nlayer) of each node pt of an interface.
The free-surface interface pts are given first, and the next
shallower interface pts are given next, and this continues
until you get to deepest interface. Velocity info for each
layer is given in file "indat".

c EXAMPLE: "inmod" file for a 2-layer fault model:
c
x , z , layer $                      FREE SURFACE
0.0 , 0.0 , 1 jaw                       (0,0)------------------------(700,0)*
700.0 , 0.0 , 1 jaw                      Layer #1
0.0 , 20.0 , 2 jaw                      (500,20)*
500.0 , 20.0 , 2 jaw                      Layer #2
500.0 , 45.0 , 2 jaw
700.0 , 45.0 , 2 jaw

(500,45)------------------------(700,45)*

Once you execute "mod.f", you can execute "psvr2.f" to generate
synthetic seismograms.

c NUMBERING INFORMATION (WARNING: NEVER INTERROGATE FIELDS ABOVE Free Surface)
c
  *--------------=* - U (1:nx, 1:nz)
c  |              [ ] - W (1:nx-1, 1:nz-1)
c  |  V mu [ ] V <- sigmaxx , sigmazz (1:nx-1, 2:nz-1)
c  |  lam [ ] <- density
c  *--------------=* - lam+2mu

c NUMBERING INFORMATION (WARNING: NEVER INTERROGATE FIELDS ABOVE FS)
c
  *--------------=* - 1,1

c  *--------------=* - 2,1

c  V [ ] - 2,1 Free surface

******************************************************************************
c parameter(nxx=800, nxs=800)
c common/velo/ca(nxx,nxs),cm(nxx,nxs),cl(nxx,nxs),xss(30,30),
c dens(nxx,nxs),xss(30,30),zss(30,30),xse(30,30),x(1000),z(1000)
c dimension nxm(50),nzm(50),nlm(50),nx(50),nzs(50),
c nlay(30),nl(500)
c
Open Files

c open(unit=11,file='lam2mu',form='unformatted')
c rewind(11)
c open(unit=12,file='mu',form='unformatted')
c rewind(12)
c open(unit=13,file='lam',form='unformatted')
c rewind(13)
c open(unit=14,file='indat')
c rewind(14)
c open(unit=15,file='inmod')
c rewind(15)
c open(unit=16,file='mod.dat')
c rewind(16)
c open(unit=17,file='density',form='unformatted')
c rewind(17)
c
read(14,*) nx,nz,ntl,vm,ntskip,ntskip
read(14,*) dx,dy,nts,zho
read(14,*) nxz,nxz,nz3
read(14,*) ioplan,iosav,nskps
read(14,*) nlay,ismf,ismf
write(16,*)' Finite Difference Parameters '
write(16,*)
write(16,*)'nx, nz, '
write(16,*)'time step, space step = ',nx,nz
write(16,*)'Number time steps = ',ntl
write(16,*)'time step skipped, trace skip = ',ntskip,ntskip
write(16,*)'Nodes plane wave started from edge and tapered = ',nskps
write(16,*)' Source/Receiver Parameters '
write(16,*)
write(16,*)'source coordinates (nx,nz) = ',nx2,nz2
write(16,*)'Geophone Depth = ',nz3
if (iosav.eq.0) then write(16,*)'X-component source particle velocity '
else write(16,*)'Z-component source particle velocity '
endif
if (ioplan.eq.0) then write(16,*)'Source is a Line Source '''
```c
else
    write(16,'*') 'Source is a vertical traveling plane wave'
endf

c
Compute Lane's parameters from rho, vp, vs

vmax=2.
vmin=90000.
write(16,'*') '----------------------------------------'
write(16,'*') 'Layer Parameters '
write(16,'*')
do 4 i=1,nlay
    read4(16) vp,rho,gamma
    vs = vp*gamma
    xlm2m(i) = rho*vp**2
    xmu(i) = rho*vs**2
    xlm1(i) = xlm2m(i) - 2.*xmu(i)
    xdens(i) = rho
    write(16,'*') 'Parameters for Layer ',i
    write(16,'*') 'vp, vs, rho = ',vp, vs, rho
    write(16,'*') 'lam+2mu = ',xlm2m(i)
    write(16,'*') 'mu = ',xmu(i)
    write(16,'*') 'lambda = ',xlm1(i)
    if(vmax.lt.vp) vmax=vp
    if(vmin.ge.vs.and.gamma.gt.0) vmin=vs
    write(16,'*')
    continue
write(16,'*') '----------------------------------------'
write(16,'*') 'Stability and Dispersion Information '
write(16,'*')
write(16,'*') 'Maximum, Minimum velocity = ',vmax,vmin
wm = (vmin)/(vmin*2.5)
dx5 = dx*10.
    if(dx5.lt.wm) then
        write(16,'*') 'Minimum Wavelength > 10*dx; ',wm, 'dx5
    else
        write(16,'*') '----------------------------------------'
        write(16,'*') 'Spatial Sampling Too coarse: min wavelength vs
1 10*dx ',wm, 'dx5
    write(16,'*') '----------------------------------------'
write(16,'*')
write(16,'*') '----------------------------------------'
sta = vmax*dt/dx
    if(sta.lt.706) then
        write(16,'*') 'Stability Criteria: .706 > cmax*dt/dx = ',sta
    else
        write(16,'*') '----------------------------------------'
        write(16,'*') 'Stability Violated; desired sta = .706, sta = ',sta
write(16,'*') '----------------------------------------'
c
Assign 1st layer parameters throughout model.
c
do 6 j=1,nz
do 6 i=1,nx
    ca(i,j)=xlm2m(i)
    cm(i,j)=xmu(i)
    cl(i,j)=xlm1(i)
    dens(i,j)=xdens(i)
6 continue

c
read node pts of segments
c
io=0
write(16,'*') '----------------------------------------'
write(16,'*') 'Interface Node Information '
write(16,'*')
do 12 i=1,10000
    read(15,*,end=33) x(i),z(i),nll(i)
    if(nll(i).gt.nl) go to 33
write(16,'*')
write(16,'*') ' (x, z) = (',x(i),',',z(i),')
write(16,'*') '(nx,nz) = (',x(i)/dx+1,')
   io = io+1
12 continue
33 continue
io = io - 1
ns = 0
c
assign layer & coordinates of segments to xss, zss and xee, zee
c
do 13 i=1,io
    if(nll(i).eq.nll(io)) then
        ns = ns+1
        nl = nll(i)
        xss(ns,nl) = x(i)
        zee(ns,nl) = x(i+1)
        zss(ns,nl) = z(i)
        zee(ns,nl) = z(i+1)
        nseg(nl) = ns
    else
        ns = 0
    endif
13 continue

c
loop over layers
c
do 100 nl=1,nlay

c
loop over segments which make up an interface
c
    nseg = nseg(nl)
do 90 ns = 1,nseg
    xss = xss(ns,nl)
    zss = zss(ns,nl)
    xee = xee(ns,nl)
    zee = zee(ns,nl)
    nxs = xss/dx + 1
    nzs = zss/dx + 1
    nx = xss/dx + l
    nzs = zss/dx + l
    slope = (ze-zs)/(xe-xe)
dx = (ze-zs)/(nxe-nxs+1)
c
loop over pts which make up a segment
c
do 50 i=nxs,nxe
    ll = i-nxs
    iz = 1 + (zs + dz(i))/dx
    do 45 k = iz,nz
        ca(i,k) = xlm2m(nl)
        cm(i,k) = xmu(nl)
        cl(i,k) = xlm1(nl)
        dens(i,k) = xdens(nl)
45 continue
```
45      continue
50      continue
55      continue
60      continue
65      continue
70      continue
75      write model parameters
80      do 10 j=1,nz
85          write(11) (ca(i,j),i=1,nx)
90      continue
95      continue
100     do 20 j=1,nz
105     write(12) (cm(i,j),i=1,nx)
110     continue
115     do 30 j=1,nz
120     write(13) (cl(i,j),i=1,nx)
125     write(17) (dens(i,j),i=1,nx)
130     continue
135     continue
140     end
psvr2.f Thu Jul 13 12:25:21 1989 1

c ***********************************************************************
c c psvr2.f computes the 2-D P-SV response of an elastic medium
 c by a 2nd-order staggered grid method. To run psvr2 first create
 c a model and dump lambda, mu and lambda+2mu values in files listed
 c below (Virieux, 1986, April issue of Geophysics).
 c This can be done using the mod.f program.
 c
 c 1) Absorbing BC's are 1st order Clayton-Engquist at the sides,
 c and the incorporation of attenuation (exponential damping)
 c along a 35 gridpoint wide region along sides and bottom of model.
 c
 c Written by Yi Luo, University of Utah (5/22/89)
 c Bells and whistles by GT Schuster.
 c
 c Filename - Mode - Description
 c -------- ---- ---------------
 c psvx -output- X-velocity seismograms along geophone at nz3 depth.
 c pssvz -output- Z-velocity seismograms along geophones at nz3 depth.
 c snap -output- X-velocity snapshots
 c snapz -output- Z-velocity snapshots
 c outdat -output- Output parameter data file.
 c lam2nu - input- lambda+2mu at each <> node pt.
 c lam - input- lam at each <> node pt.
 c mu - input- mu at each V node pt.
 c lndat - input- Input data parameters.
 c dens - input- density at * and [] nodes.
 c
 c *------------------------*
 c | * = U (1:nx, 1:nz) |
 c | | [ ] = W (1:nx-1, 1:nz-1) |
 c | | V mu [ ] V |
 c | < sigmaxx , sigmazz (1:nx-1, 2:nz-1) |
 c | |
 c | lam |
 c |
 c | lam2nu |
 c
 c NUMBERING INFORMATION (WARNING: NEVER INTERROGATE FIELDS ABOVE PS.)
 c (Your Free-surface should always start about 1 node below top of model)
 c 1,1 *------------------------* 2,1
 c |
 c 1,1 *------------------------* 1,1
 c |
 c 1,2 *------------------------* 2,2
 c |
 c 1,2 *------------------------* 1,2
 c |
 c 2,2 *------------------------* 2,2
 c |
 c 1,3 *------------------------* 2,3
 c |
 c 1,3 *------------------------* 1,3
 c |
 c 1,3 *------------------------* 1,3
 c |
 c 2,3 *------------------------* 2,3
 c
 c c c c c c
 c c c c c c
 c c c c c c
 c c c c c c
 c c c c c c
 c 1,4 *------------------------* 2,4
 c |
 c 1,4 *------------------------* 1,4
 c |
 c 1,4 V [ ] V 2,4
 c |
 c 1,4 *------------------------* 1,4
 c |
 c 1,5 *------------------------* 2,5
 c |
 c
 c ***********************************************************************

c parameter(nxx=700, nz=700, ntt=10000)
c common/para/dx, dt, dtx
common/para/nx, nz
common/velo/sou(ntt), ca(nxx, nz), cm(nx, nz), cl(nxx, nz),
idsens(nxx, nz)
c common pxl(nxx, nz), ps1(nxx, nz), xl1(nxx, nz),
c ul(nxx, nz), vl(nxx, nz)
c integer *2 nanap, nanapa
common nnanap(nxx, nz), nanapa(nxx, nz)
c call opref c

c c Read Data

c call datain(nxx, nz, nz, t3, t1, t3, 14, 11, 12, 13, nt, ts, 1, 2, 1, lopsv, loplan, tnt, tk, nsp, ntsb)

c c Create Ricker Wavelet in sou(ntt)

c b=(3.1415926*vm)**2
a=2.*b
407 do 1=1, ntt
407 sou(1)=0.
407 continue
407 do 1=1, nts+1
407 t2=((1-1)*dt)**2
407 sou(nst)+=(1.-a*t2)*exp(-b*t2)
407 10 sou(nst+2)=sou(nst+1)

c c Loop over Time steps

c do 30 i=1, nts+1

c c Compute (u,v) velocity values

c call uvx
407 do 20 j=2, nts+1
407 u1(j,k)=u1(j-1,k)+
407 1 dtx/dens(s,j,k)* (px1(j,k)-px1(j-1,k))
407 continue
407 do 20 k=2, nts+1
407 v1(j,k)=v1(j,k)+
407 1 dtx/dens(s,j,k)* (px1(j,k+1)-px1(j,k))
407 continue
407 call addasou(loplan, lopsv, nsp, nx, nz, nts+1, tsb, dt, 1)
c call atten

338
compute stress components

do 201 k=2,nz-1
do 201 j=1,nx-1
         au1(j+1,k)=-uj(j,k)
av=vl(j,k)-v1(j,k-1)
pixl(j,k)=pixl(j,k)*dx*{( ca(j,k)*au+cl(j,k)*av)
pixl(j,k)=pixl(j,k)*dx*{( ci(j,k)*au+ca(j,k)*av)
201 continue
do 202 k=2,nz-1
202 continue

c free-surface boundary condition at the top boundary

c  k=1
do 203 j=1,nx
xyl(j,k)=0.
pixl(j,k)=pixl(j,k+1)
203 continue

c call wrtsel(nx3,1,ntskip,ntsnap,ntsbeg,nx,nz)
30 continue

c stop

end

 subroutine vsed

*******************************************************************************
c vsed applies 1st-order Clayton-Enquist absorbing
 c boundary conditions at left, right and bottom
 c grid boundaries.

*******************************************************************************

parameter(nxx=700,nzx=700,ntt=10000)
common/para/dx,dt,dtx
common/para1/nx,nz
common/velo/sou(ntt),ca(nx,nzx),cm(nx,nzx),cl(nx,nzx),
 lden(nx,nzx)
common/pixl(nx,nzx),pzi(nx,nzx),xyl(nx,nzx),
 1
 1

*******************************************************************************

 subroutine datain

*******************************************************************************

dx - input - space step

dt - input - time step

nts - input - 1/2 # of points in ricker wavelet
(x,z) source position in nodal pts
psv2.f

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3

c nz3 = input # grid points below free surface for geophones

c nt1 = input # of time steps

c nx,nz = input # of grid points in x, z directions

c ntskip = input skip every ntskip traces when writing to psv2, pvsz2

c ntskip = input skip every ntskip time samples when writing to psv2, pvsz2

c nskp1s = input impose vertical traveling plane wave at the nz2 depth

c nskip = input starting step time for snapshots

c ntsbeg = input skip every ntsbeg time samples in writing files snapu, snapw

c lopsv = input =0: source has particle velocity in x-direction

c lopsv = input =1: source has particle velocity in z-direction

c loplan = input =0: source is line source at (nx1,nz2)

c loplan = input =1: source is plane wave at depth nz2

c ca = input lamda+2mu at sigmaxx, sigmass <> nodes

c cm = input nu at W [ ] nodes

c cl = input lamda at <> sigmaxx, sigmass nodes

c dens = input density at [ ], * nodes

c vm = input center frequency associated with maximum spectral

amplitude of Ricker source wavelet (Hz)

*------------* 
* U (1:nx, 1:nz)

| V mu [ ] | V <> = sigmaxx, sigmass (1:nx-1, 2:nz-1)

| V = sigmass (2:nx-1, 1:nz-1)

| lam =

*------------* 

parameter(nx=700, nz=700, ntt=10000)
common/para/dx, dt, dx
common/para/nx, nz
common/velo/sou(ntt), ca(nx,nz), cm(nx,nz), cl(nx,nz),
ldens(nx,nz)
common pxl(nx,nz), psl(nx,nz), xyl(nx,nz),
u1(nx,nz), v1(nx,nz)

1

Read input parameters
read(iod,*) nx,nz,nt1,vm,ntskip,ntskip
read(iod,*) dx,dt,nts
read(iod,*) nx2,nz2,nz3
read(iod,*) iloplan,lospv,nskp1s
read(iod,*) nlay,ntsnap,ntsbeq
dx=dt/dx
dx

Read model parameters
do 10 j=1,nz
read(iod1) (ca(i,j),i=1,nx)
10 continue
do 20 j=1,nz
read(iod2) (cm(i,j),i=1,nx)
20 continue
do 30 j=1,nz
read(iod3) (cl(i,j),i=1,nx)
30 continue

c c Zero field arrays

do 40 ix-1,nx

do 40 ix-1,nz

c

c return

c subroutine opefi
open(unit=15, file='outdat')
close(unit=15)
open(unit=9, file='psv2')
close(unit=9)
open(unit=9, file='psv2', form='unformatted')
rewind(9)
open(unit=10, file='psv2', form='unformatted')
rewind(10)
open(unit=11, file='lam2mu', form='unformatted')
rewind(11)
open(unit=12, file='mu', form='unformatted')
rewind(12)
open(unit=13, file='lam', form='unformatted')
rewind(13)
open(unit=14, file='indat')
rewind(14)
open(unit=15, file='outdat')
rewind(15)
open(unit=16, file='snapw', form='unformatted')
rewind(16)
open(unit=17, file='snapu', form='unformatted')
rewind(17)
open(unit=21, file='density', form='unformatted')
rewind(21)
return

end

subroutine addsoi(ioplan, ilopsv, nskp1s, nx, nz, nx2, nz2, dt, i)
c******

parameter(nx=700, nz=700, ntt=10000)
common/velo/sou(ntt), ca(nx,nz), cm(nx,nz), cl(nx,nz),
ldens(nx,nz)
common pxl(nx,nz), psl(nx,nz), xyl(nx,nz),
u1(nx,nz), v1(nx,nz)
c Add source into field
    if (lopian.eq.0) then
    endif
    if (lopsiq.eq.0) then
        ul(nx2,nz2) = ul(nx2,nz2) + sou(l)*dt/dens(nx2,nz2)
    else
        vl(nx2,nz2) = vl(nx2,nz2) + sou(l)*dt/dens(nx2,nz2)
    endif
else
endif

if (lopips.eq.0) then
    ul(nx2,nz2) = ul(nx2,nz2) + sou(l)*dt/dens(nx2,nz2)
else
    vl(nx2,nz2) = vl(nx2,nz2) + sou(l)*dt/dens(nx2,nz2)
endif

write(16) (nsnapa(j,kk),j=1,nx)
write(17) (nsnapa(j,kk),j=1,nx)

continue
write(15,*),'snappy printed at time step = ',i
endif
write(15,*) 'Time step computed = ',i
return
end

if (itt.eq.1 .and. i.gt.ntbeg) then
    nnsnapa(ii,kk) = 10000*ul(ii,kk)
    nnsnap (ii,kk) = 10000*vl(ii,kk)
endif

continue
write(15,*),itt,ii
write(15,*) 'snappy printed at time step = ',i
endif
write(15,*) 'Time step computed = ',i
return
end

 subroutine wretse(n3,i,nts,kp,nts,k,n,nts,mbeg,nx,nz)
 parameter(nx=700, nz=700, ntt=10000)
 common ps1(nx,nz),ps2(nx,nz),xyl(nx,nz),
 ul(nx,nz),vl(nx,nz)
 integer n2 nsnaps,nsnapa
 common nsnaps(nx,nz),nsnapa(nx,nz)

 c write seismogram information for receivers along depth level nz3
 c
    itt = ntskp*(i/nts,kp)
    if (itt.eq.1) then
        write(9) (ui(j,nz3),j=1,nx,ntskip)
        write(10) (vl(j,nz3),j=1,nx,ntskip)
    endif
 c
 c write snapshots information
 c
    itt = ntsnap*(i/ntssnap)
program formats are just like the example of the grid on the front page. it is saved in the
memory of the device. the program is executed during the execution of the program. it continues
to execute the instruction and returns to the program's place.

The grid model is used to represent the grid on the front page. it is saved in the
memory of the device. the program is executed during the execution of the program. it continues
to execute the instruction and returns to the program's place.

**TWO MODELS**

- The grid model will be used in the future. it is saved in the memory of the device. it is executed during the execution of the program. it continues to execute the instruction and returns to the program's place.

---

**FREE SURFACE**

- The grid model is used to represent the grid on the front page. it is saved in the
memory of the device. the program is executed during the execution of the program. it continues
to execute the instruction and returns to the program's place.

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**EXAMINE:** the model for a 2-layer fault model:

The grid model is used to represent the grid on the front page. it is saved in the
memory of the device. the program is executed during the execution of the program. it continues
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to execute the instruction and returns to the program's place.
```fortran
! Front.

c
real xmin, x0, nx, kmin
integer kal=1, it=0

! 1 n=ix0, nx-1
! 2 do 2 k=1, ka
! 3 if (f(n,k).eq.0) goto 1
! 4 if (t(n, f(n,k)).lt. tmin) then
! 5 tmin = t(n, f(n,k))
! 6 xmin = n
! 7 kal= k
! 8 kmin = f(n,k)
! 9 endif
! 10 continue
! 11 continue
! 12 continue
! 13 return
! 14 end

subroutine eband(xmin, x0, nx, kmin)
parameter(nx=801, np=801, ka=400)
common (f(nx,nap), s(nx,nap), t(nx,ka))
integer f

! This part expands the wavefront boundary from the minimum
! arriving time point.

k1 = xmin
if (t(nl+1).gt.0.0) then
  c*** find t(nl1, k1) & etc.
  c**** call trat4(nl1, knl1, h, nx, nz)
else
  c*** if t(nl, k1).gt.0.0 then
  c**** call trat1(nl1, knl1, h, nx, nz)
endif
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