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PREFACE

The objectives of the tomography research group are to develop a better theoretical understanding of tomography methods and to develop practical algorithms that can be usefully applied to crosswell, CDP and VSP data. This is the second year of our project and it has enjoyed the support of eight oil companies, ARCO, Amoco, Marathon Oil Co., Conoco, Phillips, SUN Oil Co., Standard Oil Co., and Texaco.

1988: Theoretical Accomplishments.

An analytical generalized inverse is derived for reflection+transmission traveltime tomography (paper 1). Analysis of the generalized inverse pinpoints the benefits and pitfalls of reconstructing 1-D velocity structure with transmission+reflection VSP data. The benefit over reconstructions using VSP transmission data is reduced model parameter variance; the pitfall is possible global coupling of model variances with one another. However, near zero-offset will eliminate global coupling of variances.

An analytic generalized inverse is derived for 2-D crosswell tomography (paper 2). Unfortunately, this inverse is restricted to a model gridded into a 4x4 grid of pixels. Results show that the best resolved pixel slownesses will be those nearest the source well and those adjacent to the surface. To some degree, these conclusions appear to be valid for larger models. This report will be distributed some time in 1989.

For forward modeling, an almost perfect absorbing boundary condition is developed for the acoustic and elastic wave equations (paper 3). The key idea is to use a retarded time wave equation at the boundary and match boundary solu-
tions with the interior solutions. Numerical tests indicate absorption of waves incident at all angles on the boundaries. However, no stability analysis has been carried out.


Paper (4) examines the effectiveness of a layer stripping algorithm for inverting 1-D CDP data. The layer stripping research complements Calnan's (1987 tomography annual report) 1987 research on sequential and simultaneous CDP inversion methods. Results show that the layer stripping method is about as effective as a simultaneous travelt ime inversion method, and appears superior to the sequential inversion. Simultaneous travelt ime inversion methods solve for reflector depths and velocities simultaneously at all depth levels, while the layer stripping approach simultaneously solves for velocity and depth layer by layer. Sequential inversion globally inverts for depths, then for velocities, and repeats this process.

Paper (5) develops an automatic travelt ime picking technique which has been successfully used to pick CDP and crosswell data. Over 10,000 crosswell travelt ime picks were automatically picked by this program for the crosswell inversion in paper (6).

Paper (6) theoretically examines the benefits of reflection-transmission crosswell tomography. Theory show that non-bending reflection raypaths will not decrease the null space size. In fact, there will be N-1 zero-eigenvalues associated with any crosswell experiment, where N is the number of pixels in the lateral direction. This assumes negligible raybending. Reflections from dipping layers, however, will decrease the null space size. The null-space can be completely eliminated if surface geophones are included. Some numerical tests show that
reflection+transmission data provide a better reconstruction, but only if the
reflection traveltime picks are of the same order accuracy as the transmission
data.

Preliminary inversion of real crosswell data (paper 6) demonstrates that the
model reconstruction for crosswell+surface data can be quite different compared
to that for the crosswell data. Reconstructed models did not match the sonic
logs very well, but further tests are in progress.

Paper (7) develops a new method for Q inversion in crosswell environ-
ments. Original tests using amplitudes failed due to the extreme sensitivity of the
inverted solution to the initial wavelet amplitude. Initial wavelet amplitudes are
not known a priori. Attempts were then initiated to reconstruct Q by using a
linear relation between Q and wavelet broadening. These tests also failed due to
the extreme sensitivity of the reconstructed Q to the initial wavelet width. How-
ever, it was found that the initial wavelet width could be eliminated by subtract-
ing the wavelet widths of adjacent traces. Using this method, crosswell Q recon-
structions using synthetic and real data show promise.

Paper (8) demonstrates the use of parallel computers in solving the 3-D
wave equation. A 20 node CSA parallel computer is used to solve 3-D elastic
and acoustic wave equations. A linear speed-up is achieved with an increase in
the number of nodes for a linear node topology. NSF is considering awarding
the University of Utah a 400 node Giga-byte computer, capable of performing at
1 Giga-flop. If received, we hope to test full wavefield inversion algorithms on
this computer.

Paper (9) develops a methodology for using Love and Rayleigh phase data
to invert for near-surface velocities (static corrections). Tests with synthetic data
show that the S-velocity is best resolved using simultaneous inversion of both
Rayleigh and Love waves. Phase data from the fundamental modes are sufficient for S-velocity reconstruction, while higher modes may help to constrain detailed information. Preliminary results show that P-velocities are more difficult to reconstruct than S-velocities.

Paper (10) is an initial attempt to examine a crosswell environment with both electromagnetic and seismic methods. Parameters best resolved by electromagnetic and seismic methods are delineated, and numerical forward and inverse modeling tests are initiated. A companion paper (11) performs elastic Biot modeling to delineate seismic properties of a reservoir. Results show that Biot's model may require updating to account for the actual attenuation observed in rocks.

Paper (12) describes a flexible raytracing technique which has proved to be very reliable. It proves to be as accurate and much faster than our standard ray tracer. It is applicable to surface, crosswell and VSP experiments and is user friendly in implementation.

Students

Students supported by the 1988 tomography development project include (expected graduation dates):

1. Chris Calnan (MS, 2/89)
2. Dan Trentman (MS, 89)
3. Mark Turner (MS, 10/89)
4. Yi Luo (PhD, '91)
5. Mary Murphy (MS, 3/89)
6. Kim Olsen (PhD, '91)

Former Students:

1. Ed Salo (MS, 12/86, Shell)
2) Dan Johnson (MS, 12/87, Chevron)
3) Zak Ahmed (MS, 87, U of U)
1989 Research

Tomography research in 1989 will extend some of the current projects as well as initiate new projects. Our research has been greatly aided by the recent acquisition of a Stellar GS1000 computer; the Stellar is completely devoted to our tomography research. In addition, we have high priority access to a 20 node parallel (30 Mflop) computer, which may soon be upgraded to the Gigaflap category. We find that powerful computers are mandatory for realistic simulations and inversion of large data sets.

Projects that we expect to continue or initiate include:

1). Q inversion.
2). Initiation of research on full-wavefield inversion.
3). Anisotropic parameter inversion.
4). Electromagnetic and seismic analysis of reservoirs.
5). Travelt ime inversion using the Eikonal equation.
6). Exact absorbing boundary conditions.
7). Surface wave inversion.

We are particularly concerned that large well offsets lead to ray traced traveltimes that are strongly non-linear with respect to model parameters. The ray trajectories can become very sensitive to slight changes in velocities so that a ray's arrival time may not represent the actual onset of energy. There may be more effective ways to compute "actual" initial traveltimes without having to resort to the full wave equation.
References

1988 Mid-Year and Final Reports

1. Analytic Generalized Inverses for Transmission+Reflection Tomography (Schuster).

2. Analytic Generalized Inverse for 2-D Crosswell Tomography (Schuster, in progress).


4. The Effectiveness of the Layer Stripping Method of Seismic Inversion on a 1-D Model (Murphy).

5. Automatic Picking and Mispick Detection of First Breaks in Reflection Seismic Records (Olsen).

6. Crosswell Tomography with Reflection+Transmission Data (Calnan).

7. Crosswell Attenuation Tomography (Olsen).

8. 3-D Modeling of Acoustic and Elastic Wave Propagation on Parallel Computers (Sikorski).


10. Prolegomeon for a Comparison between Seismic and Electric Reservoir Description and Monitoring (Tripp).

11. Poro-elastic Modeling in a Crosswell Environment (Luo)

Absorbing Boundary Condition Using A Retarded Time
Wave Equation

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Absorbing Boundary Condition Using A Retarded Time

Wave Equation

ABSTRACT

An absorbing boundary condition is derived by using a retarded time wave equation at the grid boundary. Theoretically, this boundary condition should perfectly absorb all waves incident at any angle. This prediction is verified by numerical simulations in elastic and acoustic media. Results show almost perfect absorption for all experiments. The advantages of this absorbing boundary condition are that it is only applied to the boundary nodes, it is simple to implement, and it absorbs almost all energy. The problems are that the absorbing boundary condition 1). may sometimes require a finer discretization of the mesh near the boundary, 2). has not been thoroughly tested and 3). has not been theoretically analyzed for stability.
INTRODUCTION

Solving the wave equation by finite differences requires implementation of boundary conditions which absorb incident energy at the sides and bottom of the gridded model. A popular absorbing boundary condition is based on approximations to the one-way wave equation (Clayton and Engquist, 1980; Keys, 1985). However, these absorbing boundary conditions are not perfectly absorbing for waves incident at any angle and require the extension of the grid boundaries to mitigate boundary reflections. It has been reported that instabilities arise in Clayton and Engquist's boundary condition when compressional to shear velocity ratios exceed 2.0 (Stacey, 1988). A more effective boundary condition is a combination of paraxial approximations at the boundary coupled with a boundary region of attenuation (Cerjan et al., 1985). While effective in damping boundary reflections, it requires an attenuative region 15 to 30 nodes wide around the boundaries. This padding is not very bothersome for 2-D models, but it can be fatal for large memory bound 3-D models. Hence, there is a need to develop an absorbing boundary condition which is localized at the boundary (no padding required), absorbs all waves at any incident angles, and is stable for a wide range of Poisson's ratios.

TIME RETARDED WAVE EQUATION

To overcome the problems with the previous absorbing boundary conditions, we propose an absorbing boundary condition based on the time retarded wave equation. A retarded time transformation is applied to the wave equation and this equation is used at the boundary. The boundary retarded time wave equation is then coupled to the interior wave equation by a simple interpolation scheme. Numerical results show almost perfect absorption at the boundaries.

The key idea is to use the wave equation at the boundary condition rather than some paraxial approximation. The problem, however, is that the central differencing formula for the Laplacian requires the field values one grid point outside the grid model. We thought this problem could be overcome by using, for the boundary at \( z_{\text{max}} \), a backward (rather than a central) differencing operator for the Laplacian's \( \frac{\partial^2}{\partial z^2} \) term. In this way, there is no requirement for exterior grid points and, theoretically, this scheme should perfectly absorb all waves. In actual practice, this scheme is unstable. Instability arises because the Laplacian differencing star for backward differencing is lopsided in the z-direction so that the CFL stability condition is violated.

To regain stability we need to devise a a somewhat centered differencing scheme which would not require exterior grid points at the boundary. Several clever schemes were devised, of which the retarded time wave equation scheme is presented.

Elastic Wave Equation Boundary Conditions. The 2-D elastic wave equation in homogeneous media is represented by
\[
\dot{\ddot{U}}_r = D_1 \ddot{U}_{xx} + H \dddot{U}_{xx} + D_2 \dddot{U}_{zz} \tag{1}
\]

where

\[
D_1 = \begin{bmatrix} \alpha^2 & 0 \\ 0 & \beta^2 \end{bmatrix} ; \quad D_2 = \begin{bmatrix} \beta^2 & 0 \\ 0 & \alpha^2 \end{bmatrix} ;
\]

\[
H = (\alpha^2 - \beta^2) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} ; \quad \dot{U} = \begin{bmatrix} u \\ w \end{bmatrix}
\]

and \((u, w)^T\) is the vector of horizontal and vertical particle displacements, and \(\alpha\) and \(\beta\) are the compressional and shear velocities, respectively.

The goal is to apply a transformation at the say, right-hand side \(x=x_{\text{max}}\) boundary, so that the wave equation does not demand field values outside the model. Such a transformation is given by

\[
\eta' = t - x/\alpha ; \quad t' = t ; \quad z' = z \tag{2a}
\]

applied to the first row equation in (1) and

\[
\eta'' = t - x/\beta ; \quad t'' = t ; \quad z'' = z \tag{2b}
\]

applied to the second row equation in (1).

Application of the transformations in (2a) to the derivatives in the first row equation of (1) yields

\[
u_{xx} = \frac{1}{\alpha^2} u_{\eta' \eta'} ; \quad u_{zz} = u_{z'z'} \tag{3}
\]

\[
u_{\eta' \eta'} = u_{\eta' \eta'} + 2u_{\eta' \eta'} + u_{\eta' \eta'} ; \quad w_{x'z'} = \frac{1}{\alpha} w_{\eta' \eta'}.
\]

Thus, the first row equation in (1) is transformed under (3) to
\[ u_{\eta'\eta'} = -2u_{\eta'\eta'} \frac{(\alpha^2 - \beta^2)}{\alpha} w_{\eta'\eta'} + \beta^2 u_{\eta'\eta'}. \] (4a)

Likewise the second row of equation (1) under transformation (2b) becomes

\[ w_{\eta'\eta''} = -2w_{\eta'\eta''} \frac{(\alpha^2 - \beta^2)}{\beta} u_{\eta''\eta'} + \alpha^2 w_{\eta'\eta''}. \] (4b)

Equations (4a) and (4b) are applied at the right-hand side boundary while equation (1) is used for the interior of the model. Appropriate transformations in \( z \), similar to those in equation (2), are applied at the \( z \) boundaries.

The benefit of applying equation (4) at the boundary is that the second-order derivative in (1) with respect to the coordinate direction perpendicular to the boundary plane is transformed to a first-order derivative in \( \eta' \). Under a suitable central differencing and interpolation scheme, this means that the discrete form of equation (4) does not demand extra field values outside the gridded model. In addition the geometry of the spatial differencing star in \( \frac{\partial^2}{\partial r^2} \) or \( \frac{\partial^2}{\partial z^2} \) is somewhat symmetrical. As demonstrated in the numerical results section, this pseudo-symmetry of the differencing star gives rise to an apparent stability. Note that no approximations have been made so that the boundary wave equation (4) should exactly satisfy solutions of the interior wave equation (1). Theoretically these boundary conditions should not give rise to artificial reflections at the grid boundaries.

**Coupling Interior Equation To Boundary Equation.** The interior wave equation (1) is valid for solutions in the cartesian coordinate system \((x,z,t)\), while the boundary wave equation (4) is valid for the coordinate system \((\eta', x', z')\). The transformation between these two sets of coordinates is given by equation (2).

Figure (1a) depicts the coordinate systems corresponding to \((x,z,t)\) and \((\eta', x', z')\). Note that \( \eta \) is constant if \( t \) changes as fast as \(-x\alpha\) changes; i.e., \( \eta \) is constant along the darkened coordinate axis in Figure 1. Note that the slant angle of the darkened axis increases as the velocity \( \alpha \) decreases.

To implement this method assume that the field values on the \( t-1 \) and \( t \) panels in Figure 1b are known and that the boundary field value (denoted by a darkened concentric circle) at \( t+1 \) is to be computed. This is calculated by discretizing equation (4) using a second-order central differencing stencil (denoted by dotted lines) centered at the star in the \( t \) panel. The grid points at time \( t \) corresponding to this stencil are the star and 4 filled circles. Field values at the filled circles at \( t \) and \( t-1 \) are found by interpolation of field values at the open circle grid points. The interpolation accuracy should be the same as the differencing accuracy; e.g., for second-order differencing of the wave equation, interpolation was achieved by quadratic Lagrange polynomials.

For corner points of the model, transforming \( z' = z \) to \( \eta'' = t - z / \beta \) in (2a), and transforming \( z'' = z \) to \( z'' = t - z / \alpha \) in (2b) will allow implementation of this absorbing boundary condition.
NUMERICAL TESTS.

Numerical solutions to the elastic and acoustic wave equations are computed for the time retarded absorbing boundary condition. Equations (4) are used at the boundary for the elastic tests, and a corresponding time retarded acoustic wave equation is used for the acoustic simulations. Models range from homogeneous to complexly layered media. All test results demonstrated the ability of the retarded time wave equation to almost perfectly absorb all waves at the boundaries.

Homogeneous Model. A homogeneous acoustic model is depicted in Figure 1c. The pressure field due to the line source in the Figure 1c model is computed by solving the acoustic wave equation with a second-order central differencing scheme (Kelly et al., 1976). Figure 2 depicts the seismograms interrogated at the top of the Figure 1c model where Dirichlet absorbing boundary conditions are imposed along the side boundaries. Resolving the wave equation with the retarded time wave equation at the boundaries yields the seismograms in Figure 3a. Figures 3b and 3c yield compare the seismograms at the tenth and twentieth traces, respectively, for Dirichlet and time-retarded boundary conditions. It is obvious that the time-retarded boundary condition almost perfectly absorbs incident energy.

Figure 4 depicts a thin rectangular model which is characterized by waves incident at very steep angles with respect to the side-boundaries. Figures 5 and 6 depict the seismograms interrogated at the top of the Figure 4 model with time-retarded and Dirichlet boundary conditions, respectively. Again, the absorbing boundary condition performs almost flawlessly. Figure 7 depicts SH waves in the Figure 1c model computed by solving two first-order equations with the staggered grid method of Virieux (1984) and imposing retarded time equations at the boundary. The tenth and twentieth traces are depicted in Figures 7b and 7c, respectively, and compare the retarded time results to the Dirichlet boundary condition results. Elastic wave results are computed for an elastic model corresponding to Figure 1c and using second-order differencing of equation 1. Figure 8 depicts the x and z component displacement seismograms; almost perfect absorption has occurred.

Heterogeneous Models. Figure 9 depicts a layered acoustic media, and Figure 10 depicts the seismograms computed by a second-order differencing scheme. Only reflections corresponding to expected layer reflections occur. It is interesting to note that the layer reflections recorded near the sides of the model were slightly different than analytic solutions. The explanation is that the energy reflecting near the truncated sides of the model are deprived of illuminating the full Fresnel zone on the interface.

An acoustic salt dome model is depicted in Figure 11, and the recorded seismograms are illustrated in Figure 12. Virieux’s (1984) staggered grid method is used and results show that the retarded time boundary conditions perform almost flawlessly.

A number of synthetic tests were conducted to bring this absorbing boundary condition to its knees. These tests included oscillating harmonic sources and time iterations greater than 3000 time steps. All tests failed to generate instabilities or side reflections.
DISCUSSION

A nearly perfect absorbing boundary condition is presented based on applying the retarded time wave equation at the boundary. The advantages of this boundary condition are that it is simple to implement, it is localized to a few nodes at the boundary, and it appears to absorb almost all seismic energy. It was observed that if the interpolation scheme was lower order than the differencing scheme of the wave equation, then reflection artifacts were induced at the side boundaries. A partial explanation for this might be in Oliger (1976). These artifacts, however, were eliminated when the interpolation and differencing accuracies were nearly the same. Some tests showed that a finer grid spacing (by almost half) was needed to eliminate boundary reflections. This might prove to be a severe handicap unless the finer discretization is restricted to the boundary.

Perhaps the most severe deficiency in our formulation is that a theoretical stability analysis has not been carried out. Time limitations prevented carrying out this needed step, but we hope to complete this step soon. If successful, then this retarded time equation may be very useful for 3-D forward modeling.

REFERENCES


Stacey, R., 1988, Improved transparent boundary formulations for the elastic wave equation: BSSA, 78, No. 6, 2089-2097.

Virieux, J., 1984, SH-Wave propagation in heterogeneous media; Velocity-stress
finite-difference method: Geophysics, 49, 1933-1957.
FIGURES

Figure 1. (a) grid points in cartesian coordinate system (open circles) and retarded time coordinate system (filled circles). Note that the t' axis is skewed with respect to the t axis. (b) dashed lines denote stencil, centered at the star, corresponding to a second-order differencing of equation 4. The open (closed) circles correspond to the grid points in the cartesian (retarded) coordinate system. (c) model A with the source centered in the middle of a model consisting of 31x31 grid points. The symbols '*' represent geophones.

Figure 2. The synthetic seismograms computed from Model A using Kelly's method (2nd order central differencing of the wave equation) without absorbing boundary conditions. In this case, as well as in the following examples, the #1 seismogram is the recording from the geophone at the left-side boundary and the last seismogram corresponds to the geophone recording at the right-side boundary.

Figure 3. The synthetic seismograms waves computed from Model A using Kelly's method with the new absorbing boundary conditions (figure 3a). In figure 3b and figure 3c, the solid lines are the , respectively, tenth and twentieth traces of figure 3a, the dotted lines are the tenth and twentieth traces of figure 2 (no absorbing boundary condition).

Figure 4. Model B, with a source centered in the middle of a 31x151 gridpoint model. From such a configuration, the incident wave angles are greater than 60 degrees. The symbols '*' represent geophones.

Figure 5. The synthetic seismograms computed from Model B using Kelly's method with the new absorbing boundary conditions. In this figure, it can be seen that even for large incidence angles, the new absorbing boundary conditions are still stable and effective. Stability was present for time steps greater than 2000 and a harmonically oscillating source.

Figure 6. The synthetic seismograms of SH waves computed from Model A using a second order staggered grid method (Virieux) without absorbing boundary conditions.

Figure 7. The synthetic seismograms of SH waves computed from Model A with the new absorbing boundary conditions (figure 7a). In figure 7b and 7c, the solid lines are , respectively, the tenth and twentieth traces in figure 7a, and the dotted lines are the , respectively, tenth and twentieth traces in figure 6 (no absorbing boundary conditions).

Figure 8. Elastic wave seismograms for model A using a second-order differencing scheme. The compressional velocity is 5.0 km/s and the shear velocity is 3.0 km/s. Figures 12a (vertical component) and 8b (horizontal component) show the synthetic seismograms using the new absorbing boundary conditions. Successful tests have also been conducted using a fourth-order Bayliss differencing scheme. In addition, limited numerical tests show that our new absorbing boundary condition is far superior to the classical second-order paraxial absorbing boundary conditions.
Figure 9. Model C where the top side is set to be the free surface. The Model consists of 31 columns of nodes and 51 rows of nodes. An interface is located at the thirty-fifth row.

Figure 10. Synthetic seismograms computed from Model C using Kelly's method with the new absorbing boundary condition at the vertical and bottom sides. In these seismograms, we can see three arrivals, labeled D, R and RR. D is the direct wave, R is the primary reflection, and RR is the multiple reflection between the free surface and interface.

Figure 11. Model D, a simplified salt dome model. It consists of 51 columns of nodes by 51 rows of nodes. The high frequency jitter at later times has been eliminated (not shown) by correcting an error in the Fortran code.

Figure 12. Synthetic seismograms of SH waves computed from Model D using a second order staggered grid (Virieux) method with the new absorbing boundary condition. Although the interpretation of the wave field is somewhat complex, it is clear that there is no reflection arising from the vertical or side boundaries.
Figure (1a).
Figure (1b).
Figure 11

SCURCE

\[ V_1 = 4 \text{ km/s} \]

\[ V_2 = 5 \text{ km/s} \]

MODEL D

GEOPHONES
Figure 12

Time (2 ms)
Analytic Generalized Inverses for Transmission+Reflection Tomography

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ABSTRACT

Analytic generalized inverses are derived for arbitrary combinations of transmission and reflection traveltime data. These inverses are valid for negligible ray bending and one-dimensional velocity structure. Results show that:

1). the slowness variance of a layer decreases with an increase in the number and length of rays which terminate in it.

2). the slowness variance associated with offset transmission VSP data will always be less than that associated with VSP reflection data.

3). The slowness covariance matrix for VSP transmission data is tridiagonal; i.e., the slowness errors in the $i^{th}$ layer only couple to those from the $i+1^{th}$ and $i-1^{th}$ layers. On the other hand, reflection VSP data will introduce a mutual coupling term between the slowness errors in the reflecting layer and the surface layer.

4). the slowness variance associated with transmission+reflection VSP data will generally be less than that from reflection or transmission VSP data. Moreover, the slowness covariance matrix for transmission+reflection data will be fully populated, meaning that there will be global coupling of slowness errors. However, if the reflecting segment lengths are somewhat similar then the covariance matrix will be essentially tridiagonal.

It is concluded from this analysis that the combination of reflection data with transmission data should most often improve the accuracy of slowness reconstruction. This is especially true if the reflection segment lengths are nearly the same and the traveltime picking errors for transmitted and reflected arrivals are similar.
INTRODUCTION

An analytic generalized inverse was recently derived (Schuster, 1988; Schuster et al. 1988) for VSP and CDP traveltime equations. This formula provides insights into tomographic resolution as a function of source-receiver geometry and model complexity. Careful examination showed that a layer's slowness variance is decreased with an increase in the number and length of rays terminating in that layer. Additionally, the averaging of random traveltime pick errors is enhanced if more rays intersect a layer.

A problem with this analytic inverse is that its validity is restricted to either one-dimensional transmission VSP data or one-dimensional reflection CDP data. It is not valid for arbitrary combinations of transmission and reflection data. If a more general analytic inverse were available, then the limitations and benefits of reflection+transmission tomography could be theoretically assessed; this would help to optimize the applications of VSP+CDP tomography. For example, Salo and Schuster (1988) empirically show that reflection+transmission VSP tomography provides a more accurate velocity reconstruction than transmission VSP tomography. However, their results do not clearly identify the underlying mechanism behind this improvement.

This paper derives an analytic generalized inverse for certain combinations of transmission and reflection traveltime equations associated with one-dimensional VSP and CDP data. The formula shows that a layer's slowness variance decreases with an increase in the number or length of reflected rays which terminate in that layer. However, this benefit may be offset by coupling between the slowness errors in the layer and the slowness errors of the layers where the reflected arrivals are received.
THEORY

The traveltime integral relates the slownesses, \( s(z) \), in a one-dimensional medium to the observed traveltimes, \( t(x,z) \), i.e.,

\[
t(x,z) = \int_{raypath} s(z') \, dl'.
\]

where \( x \) (or \( z \)) is the offset (depth) coordinate, and \( dl' \) is the differential raypath length. For an \( N \) layered medium with homogeneous velocities in each layer, the traveltime integral can be discretized into matrix vector notation

\[
t = \mathbf{L} \mathbf{s}, \tag{2a}
\]

where \( t \) is the \( M \times 1 \) vector of observed traveltimes, \( s \) is the \( N \times 1 \) vector of unknown layer slownesses, and \( \mathbf{L} \) is the \( M \times N \) matrix with elements \( [L]_{ij} = l_{ij} \) corresponding to segment lengths of the \( i^{th} \) ray in the \( j^{th} \) layer. Typically, there are more equations than unknowns (\( M \gg N \)) so that the least squares solution

\[
s = (\mathbf{L}^T \mathbf{L})^{-1} \mathbf{L}^T \mathbf{t}, \tag{2b}
\]

is used to iteratively reconstruct the slowness field.

For uncorrelated unit variance traveltime errors, \( (\mathbf{L}^T \mathbf{L})^{-1} \) corresponds to the slowness covariance matrix (Menke, 1984). The optimal covariance matrix is a diagonally dominant matrix with small values along the diagonal. A diagonal matrix means that the slowness errors in neighboring layers are (desirably) uncorrelated with one another; and smaller values of \( (\mathbf{L}^T \mathbf{L})^{-1}_{ii} \) allow us to more clearly peer through the data noise to see the uncorrupted model. Using the
covariance matrix, \((L^T L)^{-1}\), slowness resolution can be analyzed for different combinations of transmission and reflection data.

In the following sections, the covariance matrix (or \((L^T L)^{-1}\)) will be derived for each of the cases in Figures (1-3). The extension to more general inverse matrices is given in the appendix. In all of these cases, it is assumed that the rays suffer negligible raybending and that a geophone is located at each equally spaced interface. The extension to unequally thick layers and non-negligible ray bending is given in Schuster et al. (1988).

**Reflection+Transmission VSP Data.** Consider the VSP experiment in Figure (1a) where there is just one surface source over a five layer medium. Assume that the geophones at each interface only detect the transmitted arrivals as well as the reflected arrivals emanating from the last interface. The traveltime \(L\) matrix associated with these transmitted+reflection data is given in Figure (1b), where \(l_i (L_i)\) is the segment length of the reflected (transmitted) ray terminating at the \(i^{th}\) interface. The transmission traveltime equations give rise to the lower triangular matrix in Figure (2b), and the reflection traveltime equations give rise to the fully populated matrix in Figure (3b).

It is desirable to derive the covariance matrix \((L^T L)^{-1}\) associated with Figure (1b). This is facilitated by recognizing that the normal form (\(L^T L\)) of the rectangular matrix \(L\) in Figure (1b) can be expressed as

\[
(L^T L) = L^T, L_r + L^T, L_t
\]  

where \(L_t\) is the 5x5 transmission data matrix shown in Figure (2b) and \(L_r\) is the 5x5 reflection data matrix shown in Figure (3b).

Equation (A-1) in the appendix shows that the transmission \(L_t = S^{(S)} D^{(S)} Q\)
matrix can be decomposed into a product of a 5x5 integration matrix \( Q \), a 5x5 diagonal matrix \( D^{(5)} \), and a 5x5 shift matrix \( S^{(5)} \). It can also be shown that the normal form of the reflection matrix, \( L^T L_r \), can be decomposed into the following form

\[
L_r^T L_r = Q^T \left[ \begin{array}{cccc}
S^{(0)T} D^{(0)} & S^{(1)T} D^{(1)} & S^{(2)T} D^{(2)} & S^{(3)T} D^{(3)} & S^{(4)T} D^{(4)} \\
D^{(0)} S^{(0)} & D^{(1)} S^{(1)} & D^{(2)} S^{(2)} & D^{(3)} S^{(3)} & D^{(4)} S^{(4)}
\end{array} \right] Q
\]

\[
= Q^T \left( \sum_{s=0}^{4} [S^{(s)T}]^2 [D^{(s)}] \right) Q,
\]

where each of the shift \( S^{(s)} \) and diagonal \( D^{(s)} \), matrices is of order 5x5, and the \( Q \) matrix is given in equation (A-1). All of the element values in \( D^{(s)} \) are zero except for \( [D^{(s)}]_{55} = I_s \), and

\[
S^{(0)} = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix};
S^{(1)} = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix};
S^{(2)} = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix}
\]

\[
S^{(3)} = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix};
S^{(4)} = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix}.
\]

For example, the elements in the last row of Figure (3b) may be expressed as

\[
[D^{(4)} S^{(4)} Q]_{5i} = (1 + \delta_{5i}) l_4 
\]

for \( i = 1, 2, \ldots, 5 \); likewise the elements in the first row in equation (3b) may be expressed as

\[
[D^{(0)} S^{(0)} Q]_{5i} = 2l_0 
\]

for \( i = 2, 3, 4, 5 \), and so on.
Inserting equations (4), (5) and (A-1) into equation (3) yields the following equation

\[
[L^T L] = Q^T \left( \sum_{s=0}^{5} [S(s)^T] [D(s)]^2 [S(s)] \right) Q =
\]

\[
= Q^T \begin{bmatrix}
 l_1^2 + L_1^2 & 0 & 0 & 0 & -2l_1^2 \\
 0 & l_2^2 + L_2^2 & 0 & 0 & -2l_2^2 \\
 0 & 0 & l_3^2 + L_3^2 & 0 & -2l_3^2 \\
 0 & 0 & 0 & l_4^2 + L_4^2 & -2l_4^2 \\
 -2l_1^2 & -2l_2^2 & -2l_3^2 & -2l_4^2 & 4 \sum_{i=0}^{4} l_i^2 + L_i^2
\end{bmatrix} Q
\]

(6)

By Cholesky factorization the expression above can be rewritten as

\[
[L^T L] =
\]

\[
Q^T \begin{bmatrix}
 1 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 \\
 -2\alpha_1 & -2\alpha_2 & -2\alpha_3 & -2\alpha_4 & 1
\end{bmatrix} \begin{bmatrix}
 l_1^2 + L_1^2 & 0 & 0 & 0 & 0 \\
 0 & l_2^2 + L_2^2 & 0 & 0 & 0 \\
 0 & 0 & l_3^2 + L_3^2 & 0 & 0 \\
 0 & 0 & 0 & l_4^2 + L_4^2 & 0 \\
 0 & 0 & 0 & 0 & \beta
\end{bmatrix} \begin{bmatrix}
 1 & 0 & 0 & 0 & -2\alpha_1 \\
 0 & 1 & 0 & 0 & -2\alpha_2 \\
 0 & 0 & 1 & 0 & -2\alpha_3 \\
 0 & 0 & 0 & 1 & -2\alpha_4 \\
 0 & 0 & 0 & 0 & 1
\end{bmatrix} Q
\]

(7a)

where \( R^T, D, \) and \( R \) represent, respectively, the lower triangular, diagonal, and upper triangular matrices explicitly shown in equation (7). In addition

\[
\alpha_i = \frac{l_i^2}{l_i^2 + L_i^2}, \quad \text{and} \quad \beta = L_1^2 + 4l_0^2 + 4 \sum_{i=1}^{4} \frac{l_i^2 L_i^2}{l_i^2 + L_i^2}.
\]

(7b)

The inverse to \( R^T DR \) is \( \frac{[R]}{-1}[D]^{-1}[R^T]^{-1} \) where
\[
[R^T]^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
2\alpha_1 & 2\alpha_2 & 2\alpha_3 & 2\alpha_4 & 1
\end{bmatrix}.
\] (8)

From equation (7) the inverse to \([L^T L]^{-1}\) is \([Q]^{-1}[R]^{-1}[D]^{-1}[R^T]^{-1}[Q^T]^{-1}\), which can be rewritten explicitly in terms of equations (8) and (A-2) as

\[
[L^T L]^{-1} = [L_1]^{-1} + [L_2]^{-1},
\] (9a)

where \([L_1]^{-1}\) is the tridiagonal matrix

\[
[L_1]^{-1} = \begin{bmatrix}
\frac{1}{l_1^2 + l_1^2} & \frac{1}{l_1^2 + l_1^2} & 0 & 0 & 0 \\
\frac{1}{l_1^2 + l_1^2} & \frac{1}{l_2^2 + l_2^2 + l_3^2 + l_3^2} & \frac{1}{l_2^2 + l_2^2} & 0 & 0 \\
0 & \frac{1}{l_2^2 + l_2^2} & \frac{1}{l_3^2 + l_3^2} & \frac{1}{l_3^2 + l_3^2} & 0 \\
0 & 0 & \frac{1}{l_3^2 + l_3^2} & \frac{1}{l_4^2 + l_4^2} & \frac{1}{l_4^2 + l_4^2} \\
0 & 0 & 0 & \frac{1}{l_4^2 + l_4^2}
\end{bmatrix},
\] (9b)

and \([L_2]^{-1}\) is a fully populated coupling matrix

\[
[L_2]^{-1} = \frac{1}{\beta} \begin{bmatrix}
\Delta_{10} & \Delta_{21}\Delta_{10} & \Delta_{32}\Delta_{10} & \Delta_{43}\Delta_{10} & \Delta_{54}\Delta_{10} \\
\Delta_{10}\Delta_{21} & \Delta_{21}^2 & \Delta_{32}\Delta_{21} & \Delta_{43}\Delta_{21} & \Delta_{54}\Delta_{21} \\
\Delta_{10}\Delta_{32} & \Delta_{21}\Delta_{32} & \Delta_{32}^2 & \Delta_{43}\Delta_{32} & \Delta_{54}\Delta_{32} \\
\Delta_{10}\Delta_{43} & \Delta_{21}\Delta_{43} & \Delta_{32}\Delta_{43} & \Delta_{43}^2 & \Delta_{54}\Delta_{43} \\
\Delta_{10}\Delta_{54} & \Delta_{21}\Delta_{54} & \Delta_{32}\Delta_{54} & \Delta_{43}\Delta_{54} & \Delta_{54}^2
\end{bmatrix},
\] (9c)

where

\[
\Delta_{i,i-1} = 2\left(\frac{l_i^2}{l_i^2 + l_i^2} - \frac{l_{i-1}^2}{l_{i-1}^2 + l_{i-1}^2}\right) \quad \text{for } i = 2, 3, 4
\]
\[ \Delta_{10} = \frac{2l_1^2}{l_1^2 + L_1^2}; \quad \Delta_{54} = 1 - \frac{2l_4^2}{l_4^2 + L_4^2}, \]

and \( \beta \) in equation (7b) is at least twice greater than the sum of the squared layer thicknesses.

Equation (9) reveals that the fully populated nature of \( [L]_2^{-1} \) introduces a widespread coupling of slowness errors among the layers. This means that a large traveltime pick error in one layer can adversely affect the slowness estimate in all layers. However, the magnitude of the elements in tridiagonal \( [L]_1^{-1} \) usually dominates those in \( [L]_2^{-1} \) because of the small values of \( \frac{1}{\beta} \) and \( \Delta_{i-1} \).

The values of \( \Delta_{i-1} \) for \( i=2,3,4,5 \) are almost zero when the segment lengths have similar length for rays which terminate at the \( i^{th} \) and \( i-1^{th} \) interfaces. For example, a zero-offset experiment over an evenly layered media will insure decoupling of slowness covariances (i.e., \( \Delta_{i-1} = 0 \), for \( i=2,3,4,5 \)) between the remote layers. Also, \( \frac{1}{\beta} \) becomes smaller with an increase in layer thickness, source offset and depth to the deepest interface.

Therefore, \( [L^T L]^{-1} = [L]_1^{-1} \) is effectively a tridiagonal matrix when the ray-paths have similar segment lengths and the layers are reasonably thick and deep. Slowness resolution will tend to improve when the source offset increases, ray segments have similar lengths, and the model is parameterized into thicker and deeper layers.

**Reflection VSP Data.** The \( [L^T L]^{-1} \) matrix in equation (9) reduces to that for VSP reflection data if there are no transmission data. That is, when \( L_i = 0 \) in equation (9) then \( [L^T L]^{-1} \) reduces to
\[
\begin{bmatrix}
\frac{1}{l_1^2} & \frac{1}{l_0^2} & \frac{1}{l_1^2} & 0 & 0 & \frac{-5}{l_0^2} \\
\frac{1}{l_2^2} & \frac{1}{l_1^2} & \frac{1}{l_3^2} & \frac{1}{l_2^2} & 0 & 0 \\
0 & \frac{1}{l_2^2} & \frac{1}{l_3^2} & \frac{1}{l_4^2} & \frac{1}{l_3^2} & 0 \\
0 & 0 & \frac{1}{l_3^2} & \frac{1}{l_4^2} & \frac{1}{l_3^2} & \frac{1}{l_4^2} \\
\frac{-5}{l_0^2} & 0 & 0 & \frac{1}{l_4^2} & \frac{1}{l_5^2} & \frac{25}{l_0^2}
\end{bmatrix}
\]

Comparing equation (10) to (9), we conclude that the slowness variance for reflection VSP data is at least twice as great as for transmission-reflection data. This disparity can become even more pronounced as the source offset increases (i.e., as \( L_i \) and \( l_i \) increase). On the other hand, reflection VSP data (equation 10) compared to reflection-transmission data shows much less coupling of slowness errors between remote layers. The \([L^T L]^{-1}\) matrix for reflection VSP data is mainly tridiagonal, with coupling terms only at the \([L^T L]^{-1}\) elements. In general, this means that slowness errors in the reflecting layer are mutually coupled to those from the shallowest layer; otherwise, the slowness errors in the \( i^{th} \) layer are only coupled to those in the \( i-1^{th} \) and \( i+1^{th} \) layers.

It is appropriate to ask why the slowness errors in the second, third, and fourth layers are coupled only to their immediate neighbors, while the errors in the shallowest (first) layer are coupled to the remote non-neighboring deepest (fifth) layer. The explanation arises from the fact that reflected rays in Figure (3a) terminate in the first, second, third, and fourth layers, while no ray terminates in the fifth layer. If rays terminate just above and below a layer then the slowness estimate, e.g. \( s_3 \), can be given by

\[
\frac{t_2}{l_2} - \frac{t_3}{l_3} =
\]
\[
\frac{l_2(s_1 + s_2 + 2s_3 + 2s_4 + 2s_5)}{l_2} - \frac{l_3(s_1 + s_2 + s_3 + 2s_4 + 2s_5)}{l_3} = s_3
\]

where \(t_i\) is the traveltime recorded for the ray terminating at the \(i^{th}\) geophone, and \(l_i\) is that ray's segment length.

The above equation can be interpreted as estimating a layer's slowness by subtracting the weighted average slownesses (total traveltime divided by segment length of a ray in a layer) of the two rays which terminate adjacent to each other. The difference should be equal to the average slowness of the layer between the adjacent geophones, which is exactly equal to that layer's slowness. This information, namely traveltimes terminating at adjacent geophones, is available for the first through fourth layers and explains the local coupling of slowness errors.

On the other hand, the fifth layer is not bracketed by terminating rays. In this case, the estimate for \(s_5\) can be determined by subtracting the average slowness for the rays terminating at the \(0^{th}\) and \(4^{th}\) geophones, i.e.,

\[
\frac{l_4}{l_4} - \frac{l_0}{2l_0} =
\]

\[
\frac{l_4(s_1 + s_2 + s_3 + s_4 + 2s_5)}{l_4} - \frac{l_0(2s_1 + 2s_2 + 2s_3 + 2s_4 + 2s_5)}{2l_0} = s_5
\]

The \(1/2\) factor reduces the average slowness of the shallowest terminating ray to
that of a ray which terminates at point A in Figure 3a. Therefore, rays are now available which terminate just above and below the fifth layer. This explains the remote coupling in slowness variances between the fifth and first layers in equation (10). The previous two equations are consistent with the terms in the corresponding \( \mathbf{L}^{-1} \) matrix, and the associated \( [\mathbf{L}^T \mathbf{L}]^{-1} \) matrix is exactly that given in equation (10).

**Transmission VSP Data.** The \( [\mathbf{L}^T \mathbf{L}]^{-1} \) matrix in equation (9) reduces to that for VSP transmission data when there are no reflections. That is, if \( l_i = 0 \) in equation (9) then \( [\mathbf{L}^T \mathbf{L}]^{-1} \) reduces to

\[
[\mathbf{L}^T \mathbf{L}]^{-1} = \begin{bmatrix}
\frac{1}{L_1^2} & -\frac{1}{L_1^2} & 0 & 0 & 0 \\
-\frac{1}{L_1^2} & \frac{1}{L_1^2} + \frac{1}{L_2^2} & -\frac{1}{L_2^2} & 0 & 0 \\
0 & -\frac{1}{L_2^2} & \frac{1}{L_2^2} + \frac{1}{L_3^2} & -\frac{1}{L_3^2} & 0 \\
0 & 0 & -\frac{1}{L_3^2} & \frac{1}{L_3^2} + \frac{1}{L_4^2} & -\frac{1}{L_4^2} \\
0 & 0 & 0 & -\frac{1}{L_4^2} & \frac{1}{L_4^2} + \frac{1}{L_5^2}
\end{bmatrix}
\]  

(11)

In comparison to equation (10) for offset VSP reflection data, equation (11) for transmission VSP data shows less variance. This is because transmitted segment lengths, \( L_i \), are longer than reflected segment lengths, \( l_i \), so that \( \frac{1}{L_i^2} < \frac{1}{l_i^2} \).

In addition, the \( [\mathbf{L}^T \mathbf{L}]^{-1} \) matrix in equation (11) is sparser than equation (10) so there is less coupling among slowness errors for transmitted data. Hence, transmission VSP data is preferred over reflection VSP data.

On the other hand, transmission-reflection VSP data (equation 9) shows less
slowness variance than does transmission VSP data (equation 11). For a zero-offset source, the slowness variance for transmission data is at least twice as great as that for transmission+reflection data. Although the $[L^T L]^{-1}$ for transmission VSP data is strictly tridiagonal, the $[L^T L]^{-1}$ for transmission+reflection data is often dominantly tridiagonal. Hence, it is suggested that transmission+reflection VSP data might be preferred over transmission VSP data or reflection VSP data. This is especially true if the traveltime picking errors for both reflected and transmitted data are comparable. It is also suggested that VSP transmission data is to be preferred over VSP reflection data.

**Extension To N-Layers And Multi-Source Experiments.** By an inductive process it can be shown that the matrix structures of $[L^T L]^{-1}$ in equations (9), (10) and (11) remain the same for an increase in the number of layers.

By property 2 in the appendix, the $[L^T L]^{-1}$ matrices can be derived for multi-source VSP experiments. For example, if the VSP experiment in Figure (1a) was repeated $S$ times at different source locations, then the corresponding $L$ matrix would be of order $S10x5$. The corresponding $[L^T L]^{-1}$ matrix is in the same form as equation (9) except that $l_i^2$ is replaced by $\sum_{s=0}^{S} l_i^{(s)2}$ and $L_i^2$ is replaced by $\sum_{s=0}^{S} L_i^{(s)2}$. This follows from property 2. Note that $l_i^{(s)}$ (or $L_i^{(s)}$) is defined to be the segment length of the reflected (transmitted) ray emanating from the $s^{th}$ source and terminating at the $i^{th}$ interface.

**Rank Deficient Matrices.** If a geophone is removed from an interface in Figures (1-4), then the resulting $[L^T L]$ matrix will be rank deficient. This means that there will be zeroes in the spectrum of $[L^T L]$, or that element values in $[L^T L]^{-1}$ will become infinite. It can be shown that geophone removal at the, say
i=2, interface in Figure (1a) is equivalent to setting $l_2=L_2=0$ in equation (9); thus the layer slownesses are indeterminate. If only transmission data were used then $L_i=0$ in equation (11) implies indeterminate slownesses for the second and third layers and well resolved slownesses for the first, fourth and fifth layers. Indeterminacy is still present no matter how many sources are used.

Using a similar analysis for CDP data, this means that layer slownesses are not resolvable unless there is at least one reflecting horizon for each slowness layer. The slowness in a non-reflecting layer can be resolved if VSP data is used (with a geophone at that interface) in conjunction with CDP data. These last statements assume negligible ray bending.

To recover slowness determinacy, raybending must be taken into account. It can be shown by Calnan (1989) that raybending in a non-reflecting layer will allow for its unique reconstruction. However, the non-reflecting layer (or layer without a geophone) may be associated with an eigenvalue very near zero so that any noise may seriously degrade the accuracy of slowness reconstruction.

**SUMMARY**

A method is described which allows for the derivation of analytic generalized inverses associated with arbitrary combinations of 1-D reflection and transmission traveltime data. Arbitrary combinations include data with transmitted arrivals, primary reflections, multiples, peg-leg multiples etc. These generalized inverses can be used to analyze the resolution properties associated with different data sets, including VSP data, CDP data, and VSP+CDP data. This analysis can also be extended to earthquake data where both direct arrivals and multiply reflected arrivals are used in 1-D tomographic reconstruction of velocities.
Some conclusions drawn from analyzing generalized inverses for transmission+reflection data include:

a). slowness variance in a layer is decreased by an increase in the number of and length of ray segments terminating in that layer. Terminating rays are transmitted or reflected rays that terminate (or are recorded) in a layer.

b). transmission VSP data provides the narrowest effective bandwidth of \((L^T L)^{-1}\), i.e., the least degree of slowness error correlation among neighboring layers.

c). reflection VSP data widens the bandwidth of \((L^T L)^{-1}\). The slowness errors in the reflecting (deepest) layer are correlated with the slowness errors of the shallowest layer in which reflections are recorded. Slowness errors in the other layers only correlate with the slowness errors of their neighbors.

d). reflection+transmission VSP data gives rise to a \((L^T L)^{-1}\) matrix with the widest bandwidth; i.e., \((L^T L)^{-1}\) is a composite of a fully populated coupling matrix and a tridiagonal matrix. The elements in the coupling matrix are mostly negligible if the segment lengths of the ray paths are similar, or if the ray path lengths are very long. In this case, the covariance matrix is dominantly tridiagonal and the resulting slowness variances should be less than that for VSP transmission or reflection data.
REFERENCES


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APPENDIX

A general procedure is described to derive $[L^T L]^{-1}$ for some arbitrary combinations of transmission and reflection data.

The key to deriving $[L^T L]^{-1}$ is recognizing that the square $L$ matrices in Figures (1b), (2b), (3b) and (4b) can be decomposed into a concatenation of a diagonal matrix, $D$, a lower triangular shift matrix, $S$ (to be discussed later), and a lower triangular integration matrix, $Q$, i.e., $L = DSQ$. As an example, the $L$ matrix in Figure (2b) can be decomposed into

$$L = DSQ = \begin{bmatrix} L_1 & 0 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 & 0 \\ 0 & 0 & L_3 & 0 & 0 \\ 0 & 0 & 0 & L_4 & 0 \\ 0 & 0 & 0 & 0 & L_5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}.$$  (A-1)

where the $(D)_{ii}$ element is the segment length of the transmitted ray which terminates in the $i$th layer, $S$ reduces to an identity matrix, and $Q$ corresponds to a discrete integration operator (similar to a convolution with a heavyside function). This is to be expected since $L$ is the discrete approximation to the traveltime integral.

The inverse matrix to equation (A-1) is $([Q]^{-1} [D]^{-1})_{ij} = \frac{(\delta_{ij} - \delta_{ij+1})}{L_i}$, where $[Q]^{-1}$ is a first-order backward difference operator

$$[Q]^{-1}_{ij} = \delta_{ij} - \delta_{ij+1},$$  (A-2)

which is not too surprising since $Q$ is an integration matrix. Therefore, the
generalized inverse associated with equation (A-1) is given by
\[
(L^T \mathbf{L})^{-1} L^T = [Q]^{-1} [D]^{-2} [Q^T]^{-1} L^T.
\]

Shift Matrices. A shift matrix will be defined as a square lower triangular matrix where the element values along the diagonal are equal to one another. In addition, only one of the sub-diagonals can have non-zero elements such that the non-zero elements are equal to one another. For the example in Figure (2b), the shift matrix is an identity matrix. This will not be the case when VSP reflection data is used as in Figure (3).

As an example, consider the VSP experiment in Figure (4) where the data is restricted to reflection traveltimes recorded by the second geophone above a reflecting interface. It is assumed that the rays suffer negligible raybending, that a geophone is located at each equally spaced interface, and that each interface gives rise to a reflection. Since raybending is negligible then a ray's segment length is the same in each layer and \( l_i \) is defined as the segment length of the ray which reflects from the \( i^{th} \) interface. Thus the \( L \) matrix for this 5 layer example becomes

\[
\begin{bmatrix}
l_1 & 0 & 0 & 0 & 0 \\
l_2 & 2l_2 & 0 & 0 & 0 \\
l_3 & 2l_3 & 2l_3 & 0 & 0 \\
l_4 & l_4 & 2l_4 & 2l_4 & 0 \\
l_5 & l_5 & l_5 & 2l_5 & 2l_5 \\
\end{bmatrix}
\]
\[
\begin{pmatrix}
  l_1 & 0 & 0 & 0 & 0 \\
  0 & l_2 & 0 & 0 & 0 \\
  0 & 0 & l_3 & 0 & 0 \\
  0 & 0 & 0 & l_4 & 0 \\
  0 & 0 & 0 & 0 & l_5
\end{pmatrix}
\begin{pmatrix}
  2 & 0 & 0 & 0 & 0 \\
  0 & 2 & 0 & 0 & 0 \\
 -1 & 0 & 2 & 0 & 0 \\
  0 & -1 & 0 & 2 & 0 \\
  0 & 0 & -1 & 0 & 2
\end{pmatrix}
\begin{pmatrix}
  1 & 0 & 0 & 0 & 0 \\
  1 & 1 & 0 & 0 & 0 \\
  1 & 1 & 1 & 0 & 0 \\
  1 & 1 & 1 & 1 & 0 \\
  1 & 1 & 1 & 1 & 1
\end{pmatrix}
= D^{(1)} S^{(1)} Q,
\]

where \( l_i \) is the segment length of the \( i^{th} \) ray which terminates at the \( i^{th} \) interface, \( S^{(1)} \) is the scaling-shift matrix which transforms the diagonal and subdiagonals of \( Q \) into either 1's or 2's, \( D^{(1)} \) is a diagonal matrix which scales the \( i^{th} \) row by \( l_i \), and \( Q \) is a lower triangular integration matrix with lower triangular element values equal to 1. Note that if only the third geophone above a reflector recorded reflected arrivals then the sub-diagonal of -1's in \( S^{(1)} \) would be displaced to the next left sub-diagonal. Conversely, if only the first geophone above a reflecting interface recorded a reflected arrival then the sub-diagonal of -1's in \( S^{(1)} \) would be just to the left of the diagonal. And if only transmitted arrivals were recorded then \( S^{(1)} \) would reduce to the identity matrix. The \( n^{th} \) sub-diagonal of -1's in \( S^{(1)} \) will "shift" the lower triangular elements of \( Q \) \( n \) rows lower.

The inverse to this shift matrix \( S^{(1)} \) is given by

\[
[S^{(1)}]^{-1} = \begin{pmatrix}
  \frac{1}{2} & 0 & 0 & 0 & 0 \\
  0 & \frac{1}{2} & 0 & 0 & 0 \\
  \frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 \\
  0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 \\
  \frac{1}{8} & 0 & \frac{1}{4} & 0 & \frac{1}{2}
\end{pmatrix},
\]
and the inverse for any shift matrix of arbitrary order can easily be derived.

**Rectangular Matrices.** If there are more equations than unknowns then \( \mathbf{L} \) becomes rectangular (e.g. Figure 1b). This situation can arise when there are a multitude of offset sources or by combining reflection data with transmission data. In these cases we rely upon the following two properties to derive \( (\mathbf{L}^T \mathbf{L})^{-1} \).

**Property 1.** If \( \mathbf{L} = [\mathbf{L}^{(1)}, \mathbf{L}^{(2)}, \ldots, \mathbf{L}^{(S)}]^T \), is a rectangular matrix where each \( N \times N \) \( \mathbf{L}^{(s)} \) matrix can be decomposed into the product of three \( N \times N \) matrices \( \mathbf{D}^{(s)} \mathbf{S}^{(s)} \mathbf{Q} \), then

\[
[\mathbf{L}^T \mathbf{L}] = \mathbf{Q}^T \left( \sum_{s=1}^{S} [\mathbf{S}^{(s)}]^T [\mathbf{D}^{(s)}]^2 [\mathbf{S}^{(s)}] \right) \mathbf{Q}.
\]

(A-3)

**Property 2.** From equation (A-3), the inverse to \( [\mathbf{L}^T \mathbf{L}] \) is

\[
[\mathbf{L}^T \mathbf{L}]^{-1} = [\mathbf{Q}]^{-1} \left( \sum_{s=1}^{S} [\mathbf{S}^{(s)}]^T [\mathbf{D}^{(s)}]^2 [\mathbf{S}^{(s)}] \right)^{-1} [\mathbf{Q}^T]^{-1},
\]

(A-4a)

where \( [\mathbf{Q}]^{-1} \) is given by equation (A-2). If the shift matrices are identical to each other, say \( \mathbf{S}^{(s)} = \mathbf{S} \), then

\[
[\mathbf{L}^T \mathbf{L}]^{-1} = [\mathbf{Q}]^{-1} [\mathbf{S}]^{-1} \left( \sum_{s=1}^{S} \frac{\delta_{ij}}{[\mathbf{D}^{(s)}]_{ii}^2} \right) [\mathbf{S}^T]^{-1} [\mathbf{Q}^T]^{-1}.
\]

(A-4b)

Shift matrices remain the same when the same experiment is repeated at a
different source location; in this case, only the $D^{(s)}$ are distinct from one another. An example is given in Schuster (1988) where the data consists of traveltimes from a multi-source VSP experiment and the shift matrices reduce to identity matrices. Another example is the combination of CDP reflection data and VSP transmission data where the shift matrices reduce to identity matrices.
Fig. (1a).
Fig. (1b). \[ L = \begin{bmatrix} 2l_0 & 2l_0 & 2l_0 & 2l_0 & 2l_0 \\ l_1 & 2l_1 & 2l_1 & 2l_1 \\ l_2 & l_2 & 2l_2 & 2l_2 \\ l_3 & l_3 & l_3 & 2l_3 \\ l_4 & l_4 & l_4 & l_4 & 2l_4 \\ L_1 & 0 & 0 & 0 & 0 \\ L_2 & L_2 & 0 & 0 & 0 \\ L_3 & L_3 & L_3 & 0 & 0 \\ L_4 & L_4 & L_4 & L_4 & 0 \\ L_5 & L_5 & L_5 & L_5 & L_5 \end{bmatrix} \]
VSP TRANSMISSION DATA

Fig. (2a).
Fig. (2b). \[ L = \begin{bmatrix} L_1 & 0 & 0 & 0 & 0 \\ L_2 & L_2 & 0 & 0 & 0 \\ L_3 & L_3 & L_3 & 0 & 0 \\ L_4 & L_4 & L_4 & L_4 & 0 \\ L_5 & L_5 & L_5 & L_5 & L_5 \end{bmatrix} \]
Fig. (3a).
Fig. (3b). \[ L = \begin{bmatrix}
2l_0 & 2l_0 & 2l_0 & 2l_0 & 2l_0 \\
l_1 & 2l_1 & 2l_1 & 2l_1 & 2l_1 \\
l_2 & l_2 & 2l_2 & 2l_2 & 2l_2 \\
l_3 & l_3 & l_3 & 2l_3 & 2l_3 \\
l_4 & l_4 & l_4 & l_4 & 2l_4 
\end{bmatrix} \]
THE EFFECTIVENESS OF THE LAYER-STRIPPING
METHOD OF SEISMIC INVERSION
ON A ONE-DIMENSIONAL MODEL

by
Mary Murphy
ABSTRACT

The efficiency of the layer-stripping method is measured by looking at the effects of noisy data and initial guess on a simple, one-dimensional (1-D) layered model. The Marquardt method of non-linear inversion is used to find each layer's velocities and depths, given arrival times and receiver offsets for each layer.

Overall, a closer initial guess results in more accurate parameters than a more distant guess, although an individual layer's parameters may be worse. Inverting noisy data with 2 millisecond (ms) standard deviation results in slightly less accurate parameter estimates, while using data with 5 ms standard deviation results in much poorer parameter estimates. The combination of noisy data and distant initial guess leads to great error in the parameter estimates.
INTRODUCTION

The layer-stripping method of seismic inversion assumes the earth consists of homogeneous discrete layers. The method entails solving for the layer thickness and velocity of the top layer; fixing these values, and then solving for the next lower layer's thickness and velocity. These values are then fixed, and the inversion continues until all the layers' thicknesses and velocities are solved. The advantage of this method is that only two parameters, velocity and thickness, are solved in each iteration, so the inversion matrices are small. The disadvantage is that the number of iterations to solve a multi-layer problem can become large, depending on the stopping criteria.

The data for this study consists of traveltime arrivals from synthetic 48-fold common depth point (CDP) seismic reflection data generated from a 9 layer over a half-space, 'one-dimensional (1-D) model (Figure 1). To simulate 48-fold data, a 1-D raytracing program (Johnson, 1987) is used to trace rays from a surface source to 97 receivers, which are evenly spaced from 0.0 to 1.98 km. The 1-D true model is estimated by picking reflector depths and velocities from a velocity log (Figure 2).

A simple 1-D, homogeneous, discrete layer case is used so that the effects of noisy data and of the initial guess can be more easily separated and identified. With synthetic data the arrival times are known, unlike with CDP reflection data which has complications like noise and S-wave, multiple, and converted phase arrivals. The simplicity of the model allows the effectiveness of the method to be examined without the complicating factors found in real-life CDP data.
THEORY

The forward problem for seismic reflection data is

\[
t = \sum_{j=1}^{layer} \left[ \frac{(x_j^2 + 4h_j^2)^{1/2}}{v_j} \right]
\]

where:

\(t\) = arrival time

\(x\) = horizontal distance that wave travels in layer \(j\)

\(layer\) = number of layers

\(h_j\) = thickness of layer \(j\)

\(v_j\) = velocity of layer \(j\)

The inverse problem of solving for velocity and thickness, given \(t\) and \(x\), is non-linear, so the Marquardt-Levenberg method of inversion is used for this problem. The Marquardt method, also known as ridge regression, is an iterative damped least-squares approach which combines both Gaussian-Newton least-squares and the gradient, or steepest descent, method.

The Gaussian-Newton least-squares method is given by

\[
\tilde{m}_{k+1} = \tilde{m}_k + (G'G)^{-1}G'\Delta d
\]

where:

\(\tilde{m}\) = the parameter vector \(\tilde{m} = [v \ h]\)

\(k\) = iteration number

\(G = G(\tilde{m}_k) = \text{sensitivity or Jacobian matrix calculated at } \tilde{m}_k\)

\[
G = \begin{bmatrix}
\frac{\partial t_1}{\partial v} & \frac{\partial t_1}{\partial h} & \cdots & \frac{\partial t_n}{\partial v} & \frac{\partial t_n}{\partial h}
\end{bmatrix}
\]
$\Delta d = \text{true traveltime - predicted traveltime from model}$

The difficulty with the Gaussian-Newton method, however, is that $G^tG$ is nearly singular and the solution diverges if the initial guess is far away from the true value. The advantage is that if the guess is close to the true value, the solution converges very quickly.

The gradient method is given by

$$\hat{m}_k^{\Delta} = \hat{m}_k^{\Delta} + G^t\Delta d$$

The parameter vector is in the direction of greatest change of the error contours. The advantage of this method is that the solution always converges, although the solution may converge slowly.

The parameter vectors $\hat{m}^\Delta$ and $\hat{m}^\imath$ are often perpendicular to one another. The Marquardt method combines both Gaussian-Newton and gradient methods by calculating a parameter vector in between $\hat{m}^\imath$ and $\hat{m}^\Delta$. The Marquardt method is given by

$$\hat{m}_{k+1}^{\imath} = \hat{m}_k^{\imath} + (G^tG + \mu_kI)^{-1}G^t\Delta d$$

where:

$I = \text{identity matrix}$

$\mu_k = \text{damping factor}$

If $\mu_k = 0$, then $\hat{m}_{k+1}^{\imath} = \hat{m}_{k+1}^{\Delta}$, so the new Marquardt parameter vector equals the new parameter vector of the least-squares method. As $\mu_k \rightarrow \infty$, the new Marquardt parameter vector $\hat{m}_{k+1}^{\imath}$ approaches the direction of the gradient parameter vector $\hat{m}_{k+1}^{\Delta}$. The algorithm starts out with $\mu_k = 0.001$. If an iteration succeeds, then the damping factor $\mu_k$ is divided by 10. If an iteration fails, then $\mu_k$ is multiplied by 10.

The success or failure of an iteration is estimated by $\chi^2$. $\chi^2$ is a merit function given by

$$\chi^2(\hat{m}_{k+1}) = \sum_{i=1}^{n} \left[ \frac{(t_{i \text{true}} - t_{i \text{pred}}(\hat{m}_{k+1}))}{\sigma_i} \right]^2$$
where:

\[ n = \text{number of data points} \]

\[ t_{i}^{\text{true}} = \text{true arrival times} \]

\[ t_{i}^{\text{pred}} = \text{arrival times calculated using } m_{k+1} \text{ parameters} \]

\[ \sigma_{i} = \text{standard deviation of } t_{i}^{\text{true}} \]

The program iteratively minimizes \( \chi^{2} \) by comparing \( \chi^{2}(m_{k+1}) \) with \( \chi^{2}(m_{k}) \). If \( \chi^{2}(m_{k+1}) < \chi^{2}(m_{k}) \), then \( m_{k+1} \) is closer to the true value than \( m_{k} \), so \( \mu_{k} \) is decreased by a factor of 10. This reduction in \( \mu_{k} \) weights the parameter vector toward the least-squares method vector. If \( \chi^{2}(m_{k+1}) > \chi^{2}(m_{k}) \), then \( m_{k} \) is a better guess than \( m_{k+1} \), and \( \mu_{k} \) is increased by a factor of 10. As a result, the new parameter vector is weighted toward the gradient method vector. The iterations continue until they reach the specified maximum number or \( \chi^{2}(m_{k+1}) - \chi^{2}(m_{k}) \) is less than a specified value.

To get around the difficulty of calculating confidence intervals for a non-linear problem, the problem is assumed to be linear close to the solution. 90% confidence intervals were calculated for all inverted models, and are given by

\[ \text{confidence interval} = m_{i} \pm z\sigma_{m_{i}} \]

where:

\[ m_{i} = \text{estimated velocity or layer thickness parameter} \]

\[ z = \text{constant} \]

\[ \sigma_{m_{i}} = \text{parameter variance} \]

The constant \( z \) is picked from the student t-distribution chart and is dependent on both the percent confidence interval desired and the degrees of freedom (\( n-m \)) of the problem. For 90% confidence intervals with 95 degrees of freedom, \( z \) is interpolated to be 1.664. \( \sigma_{m_{i}} \) is the param-
eter variance, which is the square root of the $i^{th}$ diagonal element of the covariance matrix.

A 90% confidence interval means the probability is 90% that the true parameter falls within a specified distance around the estimated parameter. Since only two parameters are solved for at a time in the layer-stripping method, the confidence intervals of layer thickness and velocity were plotted against each other, resulting in a rectangle for each layer. The true value will fall within a rectangle only if it is within the confidence intervals of both estimated parameters. The confidence interval is strictly a 1-D measurement; plotting two confidence intervals against each other is merely a convenience.
DISCUSSION

The effectiveness of the layer-stripping method is measured by examining the effects of the initial guess and of noisy data on the quality of the inverted model. Three different initial guesses are used in combination with data of three varying degrees of noise. The three initial guesses are labeled a close guess, a coarse guess, and a poor guess, while the data contains 0.5, 2, and 5 millisecond (ms) of noise.

The close guess (Figure 5) varies not more than 0.1 km from true layer thickness, and not more than 0.5 km/s from true velocity. The coarse guess (Figure 8) appears to be a layer over a half-space model; however, layer thicknesses are not more than 0.1 km from the true values, and the velocities vary no more than 1.3 km/s from the true values. The poor guess (Figure 11) thicknesses vary as much as 0.39 km from the true values, while the velocities vary as much as 5.3 km/s from the true values.

The data for the inversions consists of synthetic arrival times generated from the 1-D ray-tracing program. No noise was added to these traveltimes. The measurement uncertainty in the true arrival times is the standard deviation. The standard deviation of the clean data is estimated to be 0.5 ms.

As expected, the inversion of clean data for the close initial guess works very well (Figure 6). The calculated thickness are often exact, with the greatest difference being 0.015 km for layers 4 and 5. Layer 4 is underestimated by 0.015 km, which is 4% of the true value, while layer 5 is overestimated by the same distance, which is 11% of the true value. The velocities are also estimated very accurately with the greatest error being 0.07 km/s (2% relative error).

The 90% confidence intervals for the close initial guess inverted model (Figure 7) illustrate the excellent results. The confidence intervals are very small. Although the axes have different units, the longer confidence intervals along the velocity axis show the greater uncertainty of the velocity estimates, and illustrate the accuracy of the thickness estimates. Figure 7 also shows
that the true parameter need not fall into the interval, as indicated in layers 4 and 5. Both true velocity values fall within the velocity confidence intervals, but the true thickness values do not fall within the thickness intervals. This result is expected, since the thicknesses are incorrectly estimated in layers 4 and 5. In general, the confidence intervals increase with depth, because the parameter variances increase with depth.

The second inversion of noise-free data uses the coarse initial guess. The result of the inversion (Figure 9) is still rather good, but not as accurate as the inverted model using the close initial guess. The estimated thicknesses are still more accurate than the estimated velocities. The thickness estimates are all correct except in layers 4 and 5 where they are incorrect by 0.013 km (4% relative error) and 0.008 km (6% relative error), respectively. The velocities are well estimated, except in layers 4 and 5, where layer 4 is overestimated by 0.22 km/s (4% relative error) and layer 5 is underestimated by 0.48 km/s (11% relative error).

The confidence intervals (Figure 10) also reflect the inaccuracy of the layer 4 and 5 estimations. The confidence intervals are very small, and the true thickness values of layers 4 and 5 lie just outside the intervals, while the true velocity values lie a good deal outside the intervals.

This pattern of overestimating and underestimating, or vice versa, of consecutive layers occurs often in the layer-stripping method. If a layer is assigned an estimated velocity greater than the true velocity, the calculated travel times through this layer will be less than the true travel times. As a result, the calculated travel times for the next lower layer will be larger, so the velocity guess will be lower than the true velocity. The location of incorrectly estimated layer parameters appears to be random.

The poor guess inverted model (Figure 12) is excellent, and more accurate than either the close or coarse guess inverted models, up until layer 7. The errors for layers 7 and 8 are greater than either the close or coarse guess inverted model errors. The thickness error of layer 7 is overestimated by 0.024 km (17% relative error), and layer 8 is underestimated by 0.039 km (30% relative error). The velocity error of layer 7 is overestimated by 0.67 km/s (17% relative
error), while the error of layer 8 is underestimated by 0.99 km/s (30% relative error).

The confidence intervals for this model (Figure 13) reflect the excellent accuracy of the inversion up until layer 7, and also reflect the poor inversion of layers 7 and 8. The confidence intervals of layers 1 through 6 are extremely small, and the true values of all parameters fall within these intervals. The large errors of layers 7 and 8 are reflected in the much larger confidence intervals, and note the true parameter values still fall outside the confidence intervals.

The inversion of the poor model is correct until layer 7, while the close and coarse guess inversions deviate at layer 4. The traveltime curves (Figure 3) show that the hyperbolic moveout in traveltimes of layers 1, 2, and 3 is good, while the curves of layers 4 through 9 are very flat. This means that the raypaths in each of the lower layers don’t vary much from 0.0 to 1.92 km offset. In addition, the lower layer thicknesses are relatively thin so changes in velocity won’t alter the arrival times by much. Layers 1, 2, and 3 have good moveout, and their parameters are accurately estimated for all initial guesses and data quality.

In the inversion of noise-free data for all three initial guesses, the thicknesses are estimated much more accurately than the velocities, and the magnitude of velocity error may be a function of both initial guess and raypath coverage.

Next, the effect of using noisy data is examined. The same three initial guesses are inverted with two types of noisy data. A random number generator was used to add 2 ms uncorrelated, random noise (Figure 3), and 5 ms uncorrelated, random noise (Figure 4) to the clean data.

The 2 ms noisy data was used with the close initial guess to invert for velocity and thickness. The resultant inverted model (Figure 14) is good, but obviously not as accurate as the clean data inverted model for velocity, especially of the lower thin layers. The thicknesses are still more accurately modeled than velocities, but errors for both parameters with 2 ms noisy data are greater than with clean data. The thickness is underestimated by 0.014 km (4% relative error) for layer 4, and overestimated by 0.013 km (9% relative error) for layer 5, although the
velocity estimations are good for those two layers.

Adding noise to the data results in a larger error in the inversion for velocity in the lower layers. The velocity error in layers 7, 8, and 9 is over 0.10 (3% relative error), over 0.13 (5% relative error), and under 0.29 km/s (7% relative error), respectively. The added noise prevents a quick correction with an overestimation followed by an underestimation of the same amount, as in the clean data case. Also, the noise adds to velocity miscalculations in the lower thin layers, where greater changes in velocity don’t affect traveltimes significantly. The error in thicknesses of layers 4 and 5 is just about the same as the error of the clean data case. The most significant difference in comparing noisy and clean data was in the lowest three layers.

The noise substantially enlarges the confidence intervals (Figure 15). Due to the error in thickness of layer 5, the true values of layer 5 lie just outside the confidence intervals. The greater errors in velocity result in greatly increased velocity confidence intervals, so that even though substantial errors in velocity exist, all true velocity values lie within their confidence intervals. The addition of noise to the data greatly increases the velocity parameter variance.

This error in lower layers due to noise is also exhibited in the coarse guess inversion (Figure 16). Again, the thickness estimates are much more accurate than the velocity estimates. The greatest absolute thickness error is 0.012 km (3% relative error) in layer 4. The velocity inversion of layers 4 and 5 are overestimated by 0.23 km/s (5% relative error) and underestimated by 0.52 km/s (12% relative error), respectively. Because these differences are almost exactly the same for the inversion with clean data, they must be due to the effect of initial model, not the effect of 2 ms noisy data.

The effect of noisy data can be seen in layers 7, 8, and 9 where the velocity is over by 0.09 km/s (2% relative error), over by 0.18 km/s (5% relative error), and under by 0.28 km/s (7% relative error), respectively. The magnitudes of these errors due to noise are not as great as the errors in layers 4 and 5, due to the coarse initial guess. These lower layer errors, as in the close guess case, are due to noise because velocity changes in the lower thin layers don’t affect
traveltimes as much because of the noisy data.

The confidence intervals using 2 ms noisy data (Figure 17) are not much larger for the coarse guess in comparison with the close guess. The velocity confidence intervals are quite long, indicating large parameter variances, and increase with depth. The thickness confidence intervals increase rapidly from layers 3 to 4, but tend to remain constant length throughout the rest of the model, which indicates that the thickness parameter variances do not change significantly for layers 4 through 9.

2 ms noise has little effect on the inverted model parameters of the poor initial guess (Figure 18) until below layer 6. For layers 1 through 6, the greatest error is 0.03 km/s (0.7% relative error) for velocity, and 0.002 km (0.7% relative error) for thickness. These results are better than for the coarse initial guess, which illustrate that a closer initial guess does not necessarily result in a more accurate inversion for each individual layer.

The error of layers 7, 8, and 9 of the poor initial guess is substantial. Layer 7 is overestimated by 0.96 km/s (24% relative error) for velocity and 0.034 km (24% relative error) for thickness. Layer 8 velocity and thickness errors are underestimated by 1.15 km/s (35% relative error) and 0.045 km (35% relative error), while those of layer 9 are underestimated by 0.28 km/s (7% relative error) and 0.01 km (7% relative error). These errors, however, are just a little worse than for the inversion of clean data, which again indicates that noisy data may not be as important as the initial guess.

Although the inverted model with 2 ms noise is just a little worse than with clean data, the confidence intervals are substantially worse (Figure 19 - note increase in axes length). The velocity confidence intervals of layers 7 and 8 are very large. The layer 9 confidence intervals are substantially smaller than those of layers 7 and 8, perhaps indicating that the underestimation of layer 8 had reversed a good deal of uncertainty resulting from the overestimation of layer 7.

Due to the good results of the inversion of data with 2 ms added noise, data with 5 ms added noise was inverted. As expected, the inversion results are poorer than for 2 ms noisy data,
although the thickness estimates are still very good.

For the close guess (Figure 20), the errors in thickness usually increase with increasing depth, with the largest error being 0.034 km (23% relative error) for layer 9. As seen before, the velocity error is dominant. Velocity errors begin in layer 4, and continue down to the lowest layer. This result differs from the very accurate results of the clean data inversion and small errors of the 2 ms noisy data inversion. The velocity errors of layers 4 through 9 are 0.06 (1%), 0.14 (3%), 0.33 (7%), 0.36 (9%), 0.16 (5%), and 0.92 km/s (22% relative error), respectively. Apparently, adding substantial noise greatly affects the velocity inversion of layers having flatter traveltime curves.

The confidence intervals (Figure 21) dramatically illustrate the effect of adding noise. The intervals plotted for layers 1, 2, and 3 are still quite small, however, the rest of the intervals are very large. Even though the thickness estimates are very close to the true values, the large confidence intervals show how strongly the noise affects the parameter variance.

The inversion of the coarse guess (Figure 22) also shows accurate thickness estimates, with errors not more than 0.028 km (19% relative error). Similar to the close guess model, the velocities are off an average of 0.33 km/s for layers 4 through 9. The velocity error for layers 4 and 5 are about the same as for the coarse model inversion with clean and 2 ms noisy data, so that the effect of 5 ms noisy data is not apparent until below layer 5. As with the close guess with 5 ms noise case (Figure 21), the coarse guess confidence intervals (Figure 23) are narrow until layer 4, where they greatly increase.

The results of the poor initial guess (Figure 24) are good for layers 1 through 4, but they deteriorate for the lower layers. The velocity errors for layers 5 through 9 are 0.13 (3%), 0.32 (7%), 1.58 (40%), 2.13 (65%), and 1.24 km/s (30% relative error), respectively. The thickness estimates are more accurate, with the greatest absolute error being 0.083 km (64% relative error) for layer 8.
The errors of the inverted poor guess with 5 ms noise manifest themselves in the confidence intervals (Figure 25). As with the confidence intervals for the poor guess with 2 ms noise (Figure 19), the axes have been changed to accommodate the large confidence intervals. The parameters are not constrained in any way, so the confidence intervals reach into physically impossible negative thicknesses and velocities.

Figures 26 - 32 show the relative error of each of the layers’ parameters due to the different initial guesses and noisy data. The parameter estimation accuracy using the clean data (Figures 26, 27) is good for both the close and coarse guesses, with little difference between the two. The poor guess shows greater relative error in the lower layers than the better guesses.

The parameter estimation accuracy using the 2 ms noisy data (Figures 28, 29) shows little change from that of the clean data. The relative errors are just a little worse, which illustrates that most of this error is still due to initial guess and not the addition of 2 ms noise to the data.

The relative errors of the parameter estimates using 5 ms noisy data (Figures 30, 31) are substantially higher than for the clean and 2 ms noisy data. The estimates for both the close and coarse model are still about the same, and are both still fairly good. The estimates for the poor initial guess are good until layers 7 through 9, where the estimates are much worse.
CONCLUSION

The layer-stripping method using the Marquardt method to invert synthetic reflection data for 1-D layer velocities and thicknesses works very well. One advantage in inverting seismic data is that velocities and, to a lesser extent, thicknesses have physical limits. Velocities have upper and lower bounds, and thicknesses can never be negative. Because the Marquardt method finds a local minimum, these intuitive constraints mean that even a bad initial guess may be close enough to the real solution to converge to it. The non-linear problem examined in this report is overdetermined with 95 degrees of freedom which allows better accuracy of parameter estimations, even at depths greater than the offset.

The use of clean data results in excellent inverted thicknesses and velocities. The estimates of thicknesses are much more accurate than the velocities. With clean data the initial guess dominates the accuracy of the inversion. In general, the closer the guess to the true value the better the estimate. The clean data allows for the quick correction of any miscalculated parameters by immediately overestimating or underestimating the true value so that the error is not propagated through subsequent layers.

The effect of noise is to reduce the ability of the parameters to quickly adjust themselves, so that the error is not completely corrected in the next layer, and may take another layer to adjust itself. This is especially true in the lower layers, where small changes in velocity have little effect on the traveltimes. Even using noisy data, the thickness parameters are still more accurate than the velocity. 2 ms noise does not greatly affect the accuracy of the inversion; it does however greatly increase the parameter variance, and therefore increases the confidence intervals. 2 ms noise results in slightly less accurate models, and much worse confidence intervals. The effect of 5 ms noise is dramatic. The velocity errors begin in higher layers and are larger. The thickness estimations are still very good, but not as accurate as with cleaner data. The confidence intervals are sometimes so large they extend into physically impossible values.
Layers 1, 2, and 3 are well estimated for all initial guesses and all noise levels because of their excellent hyperbolic moveout. Layers 4 through 9, with their flatter travelt ime curves, have parameter estimation accuracies dependent on initial model and on data noise level; the initial model affects the accuracy of all these layers, while noise has a greater effect on the lower layers.
REFERENCES


Figure 1

Figure 2

Reflecting interface locations =

Depth (meters)
True Model vs Close Initial Guess

Figure 5
True Model vs Coarse Initial Guess

Figure 8
Figure 9

90% Confidence Intervals (Coarse Guess, 0.5ms std)

Figure 10

True Model vs Inverted Model (Coarse Guess)
True Model vs Poor Initial Guess

velocity (km/s)

True Model  /  depth (km)  /  Initial Guess

Figure 11
Velocity Estimation Accuracy Using 0.5 ms Noisy Data

Figure 26

Thickness Estimation Accuracy Using 0.5 ms Noisy Data

Figure 27
Velocity Estimation Accuracy Using 5 ms Noisy Data

Layer
Figure 30

Close Initial Guess
Coarse Initial Guess
Poor Initial Guess

Thickness Estimation Accuracy Using 5 ms Noisy Data

Layer
Figure 31
### Percent Relative Error for Velocity Parameter, 0.5 ms Noise

<table>
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<th>Layer</th>
<th>Close Guess</th>
<th>Coarse Guess</th>
<th>Poor Guess</th>
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<tbody>
<tr>
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<td>0%</td>
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<td>0%</td>
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</tr>
<tr>
<td>9</td>
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### Percent Relative Error for Thickness Parameter, 0.5 ms Noise

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<thead>
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<th>Close Guess</th>
<th>Coarse Guess</th>
<th>Poor Guess</th>
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### Percent Relative Error for Velocity Parameter, 2 ms Noise

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### Percent Relative Error for Thickness Parameter, 2 ms Noise

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### Percent Relative Error for Thickness Parameter, 5 ms Noise

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Figure 32
Automatic picking and mispick detection of first breaks in reflection seismic records

Kim Bak Olsen
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Institute of Geology
DK-8000 Aarhus C.
Abstract

A computerized method for the picking of first arrivals from the records of multiple coverage reflection seismic surveys is presented. The picking is performed as a single-trace operation, evaluating the energy in a small running window. The sensitivity of the algorithm to the choice of the length of the picking window is estimated. A test on the candidates of arrivals is defined from the subsequent seismic energy. The threshold value for the picking procedure as well as the test routine is determined by the noise level preceding the selected picking window.

A mispick detection technique for the output of the picking algorithm is established by means of a running median filter. The selection of the length of the filter and the threshold value for the rejection of a sample in the travel time sequence are discussed.

Finally, the picking method and the mispick detection filter are applied to the field records of a land and a marine reflection seismic survey in Denmark and a crosswell experiment in Texas, USA.
Introduction

The rapidly varying conditions in the near-surface zone are, with the available advanced line and recording instrumentation, one of the most important sources of uncertainty in the final interpretation of seismic reflection records. Small variations in thickness of a shallow low velocity layer (LVL) are able to blur much deeper structures.

One way to gain information of the near-surface conditions is to involve the first breaks from seismic reflection or refraction records in a tomography-type modelling experiment (e.g., Amorim 1987, Olsen 1988). An obvious advantage of these methods is the ability to evaluate several observations at each station so that the effect of random noise is minimized. However, in order to perform a routinely extraction of a large amount of first breaks from the seismic records, a computerized method is required.

Several procedures have been suggested for an automatic picking of first arrivals. Coppens (1985) and Peraldi & Clement (1972) apply correlation techniques for the picking procedure. However, such routines are expected to be unstable when the signature of the first arrival is changing abruptly between two adjacent traces.

Hatherly (1982) evaluates the seismic amplitude, normalized with the standard deviation of the previous samples. The method is accompanied by a number of tests applied to the candidates of arrivals, detected by the picking routine. The sensitivity to noise seems to decrease when the average of several amplitudes is included in the test (e.g., Gelius et al. 1984, Ramanantoandro 1987). Moreover, the evaluation of the squared amplitude (energy) is expected to enhance the signal to noise ratio at the first arrival, and Coppens (1985) suggests a convolution of the trace with the energy in a small window normalized with the previous squared amplitudes. However, picking is not possible until the normalizing energy is significantly larger than the value in the numerator of the convolution operator.

This paper suggests a method for the automatic picking of first arrivals by the energy in a small running window without any normalization. A test of the candidates similar to the approach of Hatherly (1982) is incorporated in the procedure.

Picking algorithm

The picking algorithm analyzed evaluates the energy in a running window of $m$ samples, $E(n,n+m)$

$$E(n,n+m) = \sum_{l=n}^{n+m} A(l) \cdot A(l)$$  \hspace{1cm} \text{(1)}$$

where $A(l)$ is the amplitude at the $l$th position in the trace.

The criterion for the picking of a sample is defined from a threshold value of (1). In order to improve the stability of the algorithm when the noise level is expected to vary spatially, the threshold value for the picking of $i$th trace ($T_p^i$) is selected proportional to the standard deviation of the samples preceding the window expected to contain the first arrival

$$T_p^i = C_p \cdot \sigma_i [1 \cdot p_i]$$  \hspace{1cm} \text{(2)}$$

where $\sigma_i [1 \cdot p_i]$ is the standard deviation of the samples preceding sample no. $p_i$ and $C_p$ is a constant. The value of $C_p$ is conveniently selected from a few trial-and-error pickings.
of a limited number of shots in the survey.

The length of the picking operator (\( m \)) is essential for the stability of the algorithm. Fig. 1 shows the response of a specific trace for evaluation of three different values of \( m \) (1/10 \( \cdot \lambda \), 1/2 \( \cdot \lambda \), 1.5 \( \cdot \lambda \)). The small value of \( m \) defines a steep and oscillating response at the arrival compared to a slower raise of energy and smooth response for the large value. Obviously, there is a trade-off between the accuracy and stability in the choice of operator length. However, a value of \( m \) as one half of the wavelength is found to satisfy the demands of precision and stability in most cases.

When high-frequency noise is present previous to the picking window, the threshold value from (2) is not a stable criterion for the selection of first arrivals. In order to reject the erroneous candidates of arrivals, a test on the energy of the subsequent \( nt \) samples after the candidate of arrival at position \( nc \), \( ET(nc, nc+nt-1) \)

\[
ET(nc, nc+nt-1) = \sum_{i=nc}^{nc+nt-1} A(l) A(l)
\]

is performed. The criterion for passing the test is defined similarly to the approach of the picking algorithm. A threshold value for the \( i \)th trace (\( T_i \)) is selected by

\[
T_i = C_i \cdot \sigma_i [1, p_i]
\]

An optimal value of \( nt \) involved in the test is selected by considering the frequency of the noise and the expected minimum distance between adjacent arrivals in each trace. Generally, a sensible choice of \( nt \) seems to be a few wavelengths of the first arrival.

**Mispick detection**

The picking algorithm described is developed assuming random noise and a S/N ratio greater than one at the first arrival. When these assumptions are violated, however, a number of mispicks will be delivered by the procedure. To the extreme, the main part of the picked values will be in error, and no information is extracted from the seismograms. Generally, a limited number of erroneously picked travel times are obtained due to correlated noise in a number of traces.

In order to reduce the number of mispicks, delivered by the picking routine, the outliers are assumed to be limited in amount (< 20%) and located randomly in the travel time sequence. A running median filter is selected for the operator of mispick detection. This filter is closely related to the \( L_1 \)-norm, that is known to be fairly insensitive to outliers (Menke 1984). An odd number of samples (NF) is chosen to be considered by the filter, and a threshold value \( \Delta t \) of the difference between the predicted and picked arrival at position (NF+1)/2 is estimated.

A limited amount of randomly distributed mispicks in error of several times the shifts between the arrivals between two adjacent traces are expected to be detected by the median filter. However, these assumptions are often not obeyed, and the number of retained mispicks and rejected true arrivals is determined by the selection of NF and \( \Delta t \).

Sensible values of NF and \( \Delta t \) are estimated from the amount and quality of data and the characteristics of the desired method of interpretation. A sequence containing a small percentage of travel times in error is expected to be insensitive to the choice of the filter length. In order to retain stability, however, a small value of NF should be selected for a time sequence containing a large amount of mispicks. A \( L_1 \)-norm inversion or a manual interpretation of the data would be fairly robust to a limited number of outliers (evaluated
with a large value of $\Delta t$ in the data compared to a least-squares estimate. A sensible choice of $\Delta t$ seems to be at the order of the estimated average time shift of the arrival time between two adjacent traces.

Picking of field data

The picking procedure has been applied to field data from a land and marine reflection seismic survey. Fig. 2 shows the picked arrivals from a reflection record (fig. 3), gathered in northern Jutland, Denmark, 1984. A 96 channel asymmetric split spread was used in the survey. The values of the parameters involved in the picking were $m = 1/2 \cdot \lambda$, $nt = 3 \cdot \lambda$, $C_p = 3.0$ and $C_r = 10.0$.

Fig 4 shows the result of applying a median mispick detection filter to the travel times shown in fig. 2. A filter length of seven samples and a threshold value of 1.0 times the average time shift between the expected first arrivals of two adjacent traces were used. 15 values, mainly in the vicinity of arrival no. 50, no. 90 and to some extent between no. 5 - 25 are rejected.

Fig. 4 shows the effect of decreasing the threshold value to 75 % of the average time shift. 26 values from the picking result were rejected. Obviously, almost all the mispicks in the first part of the sequence are removed to the expense of the rejection of an increased number of apparent true arrivals.

A total amount of 13248 channels from the survey were picked, applying the parameters listed above. An average of 4.0 % of mispicks were detected by a seven sample median filter and a threshold value of one average time shift. However, the mispick detection failed for the 24 channels of largest distance to the shots due to a concentration of cycle skippings, and the values had to be rejected manually.

Fig. 6 and 7 show a 48 channel marine seismic record from Kattegat, Denmark, 1981 and the associated travel times obtained from the picking algorithm, respectively. The parameters applied are equivalent to the values from the previous experiment, and no mispicks were detected for threshold values above one average time time shift.

Fig. 8 and 9 represent another common shot gather from the Kattegat Survey and the picked arrivals from the record. Due to a gradually decreasing amplitude of the first arrival for increasing distance from the shot, the picked times of the traces 27 - 48 represent secondary arrivals (multiples). Due to the distribution of the mispicks the median filter is unable to detect the values in error.

With the application of the parameters involved for the land survey, an average of 1.1 % of mispicks were detected from the picking of 3936 traces of the Kattegat Survey. However, visual control of the travel times was required to detect the errors due to the vanishing amplitude of the arrivals.

Finally, fig. 10 shows a common shot point gather from a crosswell experiment in Texas, USA. Fig. 11 represent a comparison between the travel times, manually ($\circ$ symbol) and automatically (+ symbol) picked from the record shown in fig. 10. Apparently, the automatical procedure is able to extract a higher number and, especially in the near-surface (LVL) zone, more consistent travel times than obtained from the manual picking. This is basically due to the higher dynamic range of representation of the samples in the computer compared to the monitor plot.

98 shots of 96 traces each from the survey were picked by the technique described, involving parameters similar to the surface experiments. A relative large amount of mispicks (10.0) was obtained due to the effect of attenuating zones in the target area.
Conclusion

A computerized routine for the picking of first arrivals from the seismic energy is shown to deliver an acceptable low amount of travel times in error for the application to seismic field records. The definition of the threshold value for both the picking algorithm and the test routine as proportional to the standard deviation prior to the first arrival decreases the sensitivity of the procedure to varying noise level of the data. The application of a small running window of half the estimated wavelength of the first arrival seems to be an optimal choice for the picking operator. Moreover, a test on the seismic energy a few wavelengths adjacent to the candidate of arrivals is found to improve the stability of the picking technique.

A median filter involving an odd number of samples seem to be able to decrease a random distribution of mispicks in a travel time sequence. A filter length of seven samples and a threshold value on the order of the average time shift between the arrivals of two adjacent traces is shown to be a sensible choice of parameters for the application to reflection seismic records. However, phenomena such as cycle skipping and vanishing amplitude of the arriving wavelet are not considered by the automatic procedure, and a visual control of the travel time sequence obtained is essential for the further data processing.

Acknowledgements

The author would like to thank the Britoil group on license 9/84 (comprising Britoil, Amerada Hess, DLG, Danoil and Dansk Olie- og Gasproduktion A/S) for permission to evaluate and publish the field data shown.
References

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Fig. 1 The response of varying length (m) of the picking operator for a field seismogram.

Fig. 2 Common shot point gather from the Saebby reflection seismic Survey, northern Jutland, Denmark (1984).

Fig. 3 Travel times picked from the common shot point gather shown in fig. 1.

Fig. 4 The result of the application of a seven sample median rejection filter to the travel times shown in fig. 3. The threshold value is one average time shift of the arrivals between two adjacent traces.

Fig. 5 The result of the application of a seven sample median rejection filter to the travel times shown in fig. 3. The threshold value is 0.75 % of the average time shift of the arrivals between two adjacent traces.

Fig. 6 A common shot point gather from the Kattegat marine seismic Survey, Denmark (1981).

Fig. 7 Travel times picked from the common shot point gather shown in fig. 6.

Fig. 8 A common shot point gather from the Kattegat marine seismic Survey, Denmark (1981).

Fig. 9 Travel times picked from the common shot point gather shown in fig. 8.

Fig. 10 A common shot point gather from a crosswell experiment in Texas, USA.

Fig. 11 Comparison of manually (° symbol) and automatically (+ symbol) picked travel times from the common shot point gather shown in fig. 10.
Fig. 1

- $m = \frac{1}{10} \cdot \lambda$
- $m = \frac{1}{2} \cdot \lambda$
- $m = 1.5 \cdot \lambda$
Crosswell Tomography with
Reflection + Transmission Data

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Salt Lake City, Utah 84112
Crosswell Tomography with Reflection + Transmission Data

ABSTRACT

The value of reflection data in crosswell tomography is analyzed. Reflector positions are assumed known. Analysis with straight rays suggests that, with a coarse parameterization, reflections from horizontal interfaces do not decrease the dimension of the null space and thus can not be expected to improve reconstructions beyond those possible with transmission data only. With finer parameterizations some improvement can be expected, however. Reflections from dipping interfaces can eliminate the null space altogether, theoretically allowing a perfect reconstruction. Synthetic tests with bending rays show that bends alleviate problems with the null space at the expense of introducing nonlinearity problems. Reflections in this case, with their higher amount of noise and longer, less certain ray paths, do not significantly improve reconstructions. More valuable is vertical seismic profile data which, when included with the crosswell data, permits detection of lateral changes in velocity. Preliminary inversion of a real data set exhibits a poor correlation with a corresponding sonic log that may be indicative of a borehole damaged by the explosives set off in the experiment. Several apparent artifacts are also present very near one borehole and may be the result of poorly-located receivers.
1. INTRODUCTION

A tomography problem exists when there is a line integral relationship between some observed data and a physical property of some medium. For example, in medical applications measurements of x-ray amplitude are related, through a line integral, to x-ray opacity. Similarly, in geophysical exploration the relationship exists between seismic traveltimes and the seismic velocity of a medium or between the amplitude of seismic energy and the seismic absorption characteristics of a medium. Geophysical tomography is also possible with electromagnetic energy.

Seismic methods have proven to be the most effective geophysical methods for oil exploration. Rock properties that can be determined with seismic data include seismic velocity, seismic absorption, and anisotropy. The value of seismic data in determining these properties depends largely on two factors: the frequency content of the data and its signal-to-noise ratio. High frequency content is necessary for resolution of small features and high signal-to-noise ratio enables interpretations to be made more accurately. As both frequency content and signal-to-noise ratio of seismic energy fall off rapidly with distance (Telford et al., 1976), shot-receiver spacing is an important consideration when a seismic experiment is being designed. Also to be considered, when shots and receivers are being positioned, is the fact that near-surface material is typically very attenuative to high frequencies.

Surface seismic data (recorded with shots and receivers at or near the ground surface) suffers from both of the above problems: reflected energy recorded with this deployment propagates large distances and usually passes through the attenuative near-surface layer twice. The vertical seismic profile (VSP) deployment is an improvement over the surface deployment because receivers are located at depth within boreholes; thus, the energy travels shorter distances and passes through the near-surface material only once. Furthermore, a useful direct wave with high signal-to-noise ratio penetrates rocks at depths of interest and is recorded with the VSP receivers.

In the crosswell deployment, shots and receivers are all positioned within boreholes at the depths of interest (Figure 1). Distances traveled are, therefore, typically less than those in both the VSP and surface deployments, and transmission through the near-surface can be avoided entirely. As with the VSP geometry, a useful direct wave (or "transmitted arrival") is recorded. For these reasons, the frequency content and signal-to-noise ratio of crosswell data are usually very high, allowing accurate resolution of features less than a meter in extent.

In addition to the transmitted arrival, reflections can arise from the free surface between boreholes or from velocity interfaces at depth and can often be seen appearing after the transmitted arrival in a crosswell data record. The goal of this paper is to determine the benefits, if any, of including these reflection data with the transmitted arrivals in crosswell tomography. It is assumed that interface
locations are known between the two boreholes; this is often a good assumption when well logs are available. The paper is organized into three chapters: the first discusses previous work and some of the theory of crosswell tomography; the second contains the results of some simple straight-ray modeling; and the third contains the results of modeling with refracted rays and a test on real data.

**PREVIOUS WORK**

Bois et al. (1972) performed some of the earliest research with crosswell data. By using primacord charges to generate seismic energy in one borehole and timing first arrivals in boreholes 500m to 700m distant, they were able to detect what they believed to be a structural event. In 1979, Dines and Lytle described a straight-ray reconstruction algorithm appropriate for use with electromagnetic data and seismic data in weakly refractive media. They later extended the method to situations in which rays are strongly refracted (Lytle and Dines, 1980). Butler and Curro elaborated on some principles of crosswell experiment design in 1981.

Researchers at Los Alamos National Laboratory used transmitted crosswell arrivals to monitor changes in elastic wave velocities and amplitudes during operation of a hot dry rock geothermal system (Pearson et al., 1983; Fehler and Pearson, 1984). They discovered a general decrease in velocities and amplitudes that they attributed to the forming and subsequent fluid filling of fractures in the granitic rock. Wong et al. (1983 and 1984) also conducted a crosswell survey in crystalline rock. Using amplitudes, transmitted traveltimes, and a straight-ray assumption, they succeeded in locating fracture zones of low velocity and low transparency. Bregman (1986) used the same data sets in a study of bent-ray tomography with the same two-dimensional parameterization that is used in this paper. Paulsson et al. (1985), as part of a nuclear waste research program, used crosswell measurements of P-wave traveltimes and amplitudes and S-wave traveltimes in a small-scale experiment to successfully monitor elastic property changes during heating of a granitic rock mass.

Devaney (1984) proposed a new method, diffraction tomography, which uses an entire scattered wavefield rather than just traveltimes. Due to approximations involved, the method works best for reconstructions of weakly inhomogeneous media. In a series of ultrasonic laboratory tests, Lo et al. (1988) compared results with ray (traveltime) tomography to results with diffraction tomography. For the crosswell geometry they concluded that ray tomography performs as well as diffraction tomography when bodies of interest are larger than one wavelength. Diffraction tomography was found to be superior when the bodies of interest are smaller.

Tomography with reflected arrivals in surface seismic data was attempted by Bishop et al. (1985) as they inverted for both velocity and reflector location in laterally varying media. They successfully used their method to compute a model which satisfied seismic data recorded over a laterally-varying surface layer of permafrost. Bording et al. (1987), also working with surface seismic data, proposed "iterative tomographic migration" as a scheme in which migration for reflector location follows tomographic inversion for medium velocity. This method, when tried on synthetic data, performed better than conventional common depth point
processing methods.

Ivansson (1985) performed several synthetic tests of crosswell tomography in the presence of slow velocity zones. He found reconstructions with many parameters and sufficient damping of solution length to be superior to reconstructions with fewer parameters and little damping. McMechan et al. (1987) tried crosswell tomography with reflected arrivals, assuming known reflector locations and allowing for ray bending. They found reflections to be helpful in improving both synthetic and scale model reconstructions. Most recently, migration of reflections in crosswell data has been addressed in the literature (for example, Zhu and McMechan, 1988; Hu et al., 1988); however, a study of the value of reflected arrivals when reflector locations are known has not yet been done.

**CROSSWELL GEOMETRY AND THEORY**

The crosswell geometry (Figure 1) in its simplest form consists of sources and receivers in two vertical boreholes separated by the region of interest. This simple geometry is assumed for this section, but the implications of some variations of this geometry (e.g., deviated boreholes, sources or receivers on the ground surface between the two boreholes) are discussed in other chapters.

The seismic source used for a shot in crosswell tomography is usually a small explosive charge or a downhole mechanical impulse device. Transmitted seismic energy propagates along the ray paths from the shot in borehole A to the receivers in borehole B (Figure 1); the recorded data for one shot comprise one "scan". The source is then moved to a new position in its borehole and a new scan recorded. This continues until scans corresponding to several source positions have been recorded.

Physical properties are most commonly represented, or parameterized, in tomography by a two-dimensional (2-D) grid of small cells ("pixels"). Figure 1 illustrates such a parameterization for the area between two boreholes. Within each pixel, physical properties are assumed to be constant. For amplitude tomography, the property represents the absorption characteristics of the rock, and for traveltime tomography it is seismic velocity (or its inverse, "slowness").

Regardless of the parameterization, tomography requires a functional relationship between the physical parameters and the data. In traveltime tomography, as with most geophysical inversion, the relationship is a nonlinear one:

\[ t = \int_L s(x,z) \, dl, \]  

(1)

where \( t \) is the traveltime of a single ray through the medium and \( s \) is the position-dependent seismic slowness of the medium. The integral is over the raypath \( L \), and the relationship is nonlinear because \( L \) is a function of \( s \). \( s \) is the parameter which we are seeking to determine, given the data \( t \). For a pixel parameterization, the traveltime equation above can be written in matrix form as

\[ t = Ls, \]  

(2)
where \( \mathbf{t} \) is a vector of data (traveltimes) for several rays, \( \mathbf{s} \) is a vector of slownesses of individual pixels, and \( \mathbf{L} \) is a matrix of ray segment lengths within individual pixels.

In tomography, the nonlinear inversion for the slowness model \( \mathbf{s} \) is carried out by expanding Equation (2) in a Taylor's series about some estimate of the slownesses and truncating the series to first order in slowness perturbation. This linearizes the problem within an iteration by neglecting the change in ray path that occurs with a perturbation in slowness. The result is

\[
\Delta \mathbf{t} = \mathbf{L} \Delta \mathbf{s},
\]

(3)

where \( \Delta \mathbf{t} \) is a vector of predicted data subtracted from observed data (traveltime residuals) and \( \Delta \mathbf{s} \) is a vector of slowness corrections for individual pixels. For the pixel parameterization discussed here, \( \mathbf{L} \) consists of the same ray segment lengths used in Equation (2).

The slownesses are solved for in an iterative manner, with each iteration consisting of a forward step (represented by Equation (2)) followed by an inverse step (represented by Equation (3)). In the forward step, predicted data are generated with the most recent estimate of slownesses. The inverse step involves solving for \( \Delta \mathbf{s} \), which allows a new slowness estimate to be made. The least squares solution of Equation (3) is usually sought:

\[
\Delta \mathbf{s} = \left( \mathbf{L}^T \mathbf{L} + \epsilon^2 \mathbf{I} \right)^{-1} \mathbf{L}^T \Delta \mathbf{t}.
\]

\( \epsilon \) is a damping parameter that is necessary because \( \mathbf{L} \) is usually ill-conditioned, a fact that will be discussed in some detail in the next chapter. Iterations continue through several updates of \( \mathbf{s} \) until some stopping criterion has been met: either the slowness estimate sufficiently satisfies the observed data or the slowness estimate is no longer changing significantly with successive iterations.
2. BASIC PRINCIPLES

This chapter presents some tomographic reconstructions of synthetic crosswell data in which it is assumed, in all but two sections, that all rays are straight, that the data are noise-free, and that reflectors are perfectly horizontal. These assumptions simplify the inversion problem drastically, and the goal of this work was to take advantage of this simplicity and draw some conclusions concerning the strengths and limitations of the crosswell geometry. The 1-D tomography problem is addressed first, and this is followed by an analysis of the 2-D problem. The following conclusions, subject to the above assumptions, are discussed:

(1) For a fine parameterization, inclusion of reflection data may decrease the dimension of the null space of the problem, allowing a more accurate reconstruction;

(2) for a crosswell parameterization of $M \times N$ pixels (as in Figure 1), the inversion problem has at most $MN - (N - 1)$ nonzero eigenvalues; and

(3) for either the transmission or the transmission+reflection problem, it is possible to devise an experimental geometry and/or 2-D parameterization for which the inversion problem will have no null space.

Additionally, two conclusions are discussed in which one of the above assumptions is not necessary:

(1) Ray bending due to Snell's law decreases the dimension of the problem's null space, thus providing another reason for including highly-refracted reflections in the inversion; and

(2) inclusion of reflections from dipping interfaces can eliminate the null space altogether, theoretically allowing perfect reconstruction of the slowness structure.

ONE-DIMENSIONAL RESULTS

Figure 2 illustrates three straight rays from a hypothetical crosswell experiment. The ground surface gives rise to two reflected rays. As shown in the figure, the reflected rays can actually be thought of as transmissions through a mirror image of the slowness structure, with the reflector being the reflection axis. Thus, when we make the above assumptions (straight rays, noise-free data, and horizontal reflectors), reflection data in the experiment are equivalent to transmission data that were recorded with a more extensive array of sources and receivers. Furthermore, the length of both reflected and transmitted rays is dependent only upon shooting angle, or the angle the ray makes with the horizontal at the shot point. Depth of shot does not influence ray length. These ideas will be important in the following discussions.

A one-dimensionally (1-D) varying geology is probably an unrealistic simplification in any real-data crosswell tomography problem; however, it is a
useful starting point for synthetic tests. Most of the insights drawn from the 1-D case are applicable to the more complicated two-dimensional problem.

**Horizontal layers**

In Appendix A, the slice projection theorem and its application to crosswell tomography are presented. An important result is that slowness variations in a direction perpendicular to rays are resolved best, while slowness variations in a direction parallel to rays are not resolvable (Fawcett and Clayton, 1984). In Figure 2, geology varies only in the vertical direction. This results in slowness variations most favorable for crosswell transmission tomography, since a majority of the transmitted rays are parallel or roughly parallel to slowness interfaces. In fact, a single horizontal ray (for example, ray 1 in Figure 2) in a layer allows that layer's velocity to be exactly calculated; it is, simply, the ray's length divided by its traveltime.

Reflected rays, on the other hand, are equivalent to transmitted rays oriented at high angles to horizontal slowness interfaces (rays 2 and 3 in Figure 2) and are, therefore, more nearly parallel to the direction of slowness variation. By the slice projection theorem, these rays are less favorable for calculation of these slownesses.

The simple argument above indicates that, for the case of straight rays and noise-free data, reflections do not allow a more accurate determination of the velocities of horizontal layers. This conclusion holds for the 2-D case, also, since 2-D parameterization can be viewed as reducing a 1-D layer into a composite of 2-D pixel components.

**Vertical layers**

**Straight rays.** Figure 3 depicts a 1-D layered model in which the slowness variation is in the horizontal, rather than vertical, direction. The following simple argument reveals that direct traveltimes from a crosswell deployment of sources and receivers are insufficient for resolving this type of slowness variation. Equation (2) written out explicitly for rays 1 and 2 of the figure gives the traveltimes for each:

\[ t_1 = l_1 s_1 + l_1 s_2 + l_1 s_3 + l_1 s_4 \]

and

\[ t_2 = l_2 s_1 + l_2 s_2 + l_2 s_3 + l_2 s_4, \]

where \( l_i \) is the length of the \( i \)th ray in a layer and \( s_i \) is the slowness of the \( i \)th layer. Since the rays do not bend, each ray has an equal segment length in each layer. Since \( l_2 \) differs from \( l_1 = l_2 \cos \theta \) by a multiplicative constant, the two equations are linearly dependent:
\[ t_1 = (l_2s_1 + l_2s_2 + l_2s_3 + l_2s_4) \cos \theta = t_2 \cos \theta. \]

All other possible direct-ray traveltime equations are also linearly dependent by the same argument. Unique determination of the four velocities requires a rank 4 system of equations, and such a system of equations cannot be constructed with straight rays. In fact, the system of equations will have rank equal to 1 regardless of the number of rays or the number of layers. Equivalently, if \( N \) equals the number of unknown slownesses, then \( \mathbf{L} \) of Equation (2) has \( N-1 = 4-1 = 3 \) eigenvalues equal to 0. This means that we can only hope to calculate an average slowness for the medium as a whole between the two boreholes; the four individual velocities cannot be correctly determined.

Including reflection data from horizontal interfaces does not improve this situation because, as shown earlier, reflection data in this case are equivalent to additional transmission data. Traveltime equations for rays 3 and 4 (Figure 3) can be expressed as a constant multiplied by any of the other traveltime equations.

This 1-D result can be extended to the case of a 2-D pixel parameterization with the following argument. Elements of the partial derivatives matrix for the 2-D case, \( \mathbf{L}_{2D} \), are ray path segment lengths within pixels. The \( i \)th column consists of segment lengths rays which intersect the \( i \)th pixel. If an \( M \times N \) grid of pixels (as in Figure 1) is re-parameterized as a \( 1 \times N \) system of \( N \) vertical layers, rows of the new partial derivatives matrix (\( \mathbf{L}_{1D} \)) are calculated simply by summing appropriate rows of \( \mathbf{L}_{2D} \). For example, the column of \( \mathbf{L}_{1D} \) corresponding to the \( i \)th vertical layer can be constructed by summing columns of \( \mathbf{L}_{2D} \) associated with pixels in the \( i \)th vertical layer. \( \mathbf{L}_{1D} \) has \( N \) columns, and we saw that it has at least \( N-1 \) eigenvalues equal to 0. It follows, therefore, that \( \mathbf{L}_{2D} \) (which has \( M \times N \) columns) also has at least \( N-1 \) eigenvalues equal to 0. This result is not surprising: resolution of vertically-oriented features is at least as bad for 2-D tomography as for 1-D tomography. This is consistent with the prediction from the slice projection theorem (Appendix A).

**Bent rays.** For this and the following sections, singular value decomposition (SVD) is used to analyze certain properties of \( \mathbf{L} \) in Equation (2). A review of SVD is presented in Appendix B. Briefly, it allows one to inspect the singular values and related singular vectors of a matrix. The singular values (closely related to eigenvalues) give a sense of the degree of linear independence of a system of equations; e.g., if some change in the matrix \( \mathbf{L} \) causes its smallest singular values to increase in magnitude while the large singular values are relatively unchanged, then \( \mathbf{L} \) has become, for purposes here, more linearly independent.

When Snell's law is honored at interfaces, rays bend and the linear dependence of the system of traveltime equations (Equation (2)) is reduced because the equations no longer differ by a single multiplicative constant. The greater the ray bending, the greater the linear independence. Snell's law is obeyed in the tracing of the rays in Figure 4, which is otherwise similar to Figure 3. Since bending is greatest at high angles of incidence and reflected rays have the highest angles of incidence to the slowness interfaces, addition of reflected energy might greatly reduce the linear dependence of the traveltime equations. Plotted in Figure 5 are
the singular values for the traveltime equations associated with rays of Figure 4. All four singular values are nonzero, unlike the straight-ray case with its single nonzero singular value. The bent-ray traveltime equations comprise a full rank system in contrast to the partial rank system for the straight rays; in fact, the four nonzero singular values theoretically allow a unique slowness function to be calculated for the four vertical layers.

**Bent rays and nonlinearity.** Inversion for the unique slowness function of the previous section requires at least one assumption: that the ray paths are known. The assumed paths could, in fact, be incorrect. For instance, layers 1 and 3 in Figure 4 could be exchanged with one another, resulting in no change in travel times but a small change in ray paths (shown in Figure 6). Therefore, the existence of a unique solution does not ensure that the solution is the correct one. As discussed in the introduction, when ray bending is taken into account the inversion problem is a nonlinear one that must be solved iteratively. Any one iteration may entail inverting a system with a unique inverse (due to ray bending), but the accuracy of the final reconstruction depends upon whether or not the assumed ray paths were accurate throughout the iterations. This in turn depends largely on the accuracy of the starting model's velocities. Therefore, it seems that unless the starting model is similar to the true model, ray bending in itself will not permit any improvement in the accuracy of slowness reconstructions.

In summary, the analysis with 1-D geologies allows several conclusions to be made. First, results with the slice projection theorem indicate that reflections may not contribute to a more accurate determination of velocities of horizontal layers which are, in principle, perfectly determined with transmission data alone. Second, velocities of vertically-oriented features cannot be determined when straight rays are assumed for transmissions and reflections from horizontal interfaces. Third, ray bending due to Snell's law may allow unique determination of velocities, but may also require an accurate initial guess at the correct slowness structure. Accurately-known reflected ray paths, with their high angles of incidence to vertical features, might be especially valuable in determining slowness variations in the horizontal direction.

**TWO-DIMENSIONAL RESULTS**

One of the main limitations of the crosswell geometry is its inability to yield a unique velocity reconstruction. This is unlike the vertical seismic profile (VSP) experiment, which has been shown by others (e.g. Ivansson, 1987) to theoretically allow for a unique reconstruction. Medical applications of tomography also suffer no nonuniqueness problems. The nonuniqueness of the crosswell inversion problem is a result of that method's incomplete source-receiver coverage (Appendix A), and causes the partial derivatives matrix \( \mathbf{L} \) to contain a null space associated with unresolvable features of the model. This is equivalent to saying that \( \mathbf{L} \) is ill-conditioned, is rank-deficient, or has singular values equal, or nearly equal, to 0. This problem, and the value of reflections in alleviating this problem, are the subjects of this section. Also examined are possibilities for eliminating the null space. The synthetic data used here are generated with straight rays traced through 2-D
grids of pixels. SVD is then performed on the resulting $L$, allowing features of the null space to be examined. SVD also yields tomographic reconstructions through application of the natural generalized inverse (Menke, 1984). The straight rays make the inversion problem a linear one: Equation (2) is solved for $s$ in a single iteration.

**Null space of the 2-D problem**

As discussed in Appendix B, a model vector $s$ (consisting of slownesses of individual pixels) can be written as a sum of two component vectors:

$$s = s_p + s_0,$$

where $s_p$ is the component of the model that is resolvable, or "illuminated", with a given experimental geometry and $s_0$ is the component that is not illuminated. "Experimental geometry" here refers to ray coverage as well as source-receiver deployment. $s_0$ itself is composed of linear combinations of eigenvectors within the null space of $L$, while $s_p$ is composed of linear combinations of eigenvectors in the non-null space. It is hoped, of course, that the component $s_0$ constitutes only a small portion of $s$; when this is the case, a tomographic reconstruction is likely to bear a strong resemblance to the true model. Some generalizations about the null space and $s_0$ will now be made.

The dimension of a null space is equal to the number of eigenvectors comprising it. With the assumptions made so far in this chapter and an additional one assuming continuous source-receiver coverage in the two boreholes, it is possible to derive an expression for the dimension of the null space of the crosswell problem. This is done in Appendix C for the case of a parameterization of $M \times N$ pixels ($N$ pixels separating the boreholes). The result is that the dimension of the null space is $N-1$ or, equivalently, the rank of $L$ is $MN-(N-1)$. It will be shown that the null space eigenvectors are associated with the horizontal variations (vertical layers) of the model. This knowledge is potentially useful when designing a parameterization for an experiment, and it will also prove useful when the value of reflections is analyzed in the next section.

The requirement in Appendix C of continuous source-receiver coverage is not realistic—in practice, shots and receivers have a finite spacing in the two boreholes. For this reason, the actual dimension of the parameter null space for the case of straight rays may be larger than $N-1$. It will depend on the parameterization, or the number of pixels in the horizontal direction and the number of shot-receiver positions per pixel side in the vertical direction. It is desirable in any inversion procedure, of course, to keep the dimension of the null space to a minimum. It will be shown here that the minimum-dimension null space of $N-1$ can be achieved for a finite shot-receiver spacing if the parameterization is sufficiently coarse (or, equivalently, if the ray coverage is sufficiently dense).

Figure 7 shows a hypothetical crosswell geometry in which two boreholes, 600 feet deep and 600 feet apart, each have 12 uniformly spaced shots or receivers. The parameterization is such that there are 36 pixels of unknown slowness. 144 transmitted rays were generated to construct the partial derivatives matrix $L$. SVD
was then performed on $\mathbf{L}$, and the resulting singular values are plotted in Figure 8a. According to the rule given above, sufficient ray coverage should result in a null space of dimension 5 for this problem. The last 5 singular values in Figure 8a are (to the precision of the machine) equal to 0. Of course, a finer parameterization (or sparser ray coverage) might result in a null space dimension greater than 5, but as long as there are 6 pixels in every row and no new data are added to the problem the null space can not have a dimension less than 5.

Analysis with 1-D parameterizations revealed that vertically-oriented slowness features are the ones most likely to be found within the null space of $\mathbf{L}$. The 5 null space singular vectors for this problem are contoured in Figure 8b; not surprisingly, all 5 represent purely vertical features. In the rest of this thesis this condition of a minimum-dimension null space comprised of eigenvectors representing purely horizontal variations will be called the "minimum null space".

As discussed in Appendix B, SVD allows natural generalized inversion of traveltime data and results in the one least-squares solution that has no component in the parameter null space. For this problem synthetic straight-ray data can be generated by assuming some "true model" for the parameterization of Figure 7. If the data are noise-free, then operating on them with the natural generalized inverse yields

$$s_T = s - s_0 = s_p,$$

where $s_T$ is the tomographic reconstruction of the model. Figures 9a and 9c give two possible models for the parameterization discussed here; both have a single-pixel low velocity zone. Figures 9b and 9d are $s_p$ for each model, the SVD reconstructions. Both reconstructions are of similar quality. This indicates that anomalies are equally-well reconstructed, regardless of their locations in the model, when the condition of minimum possible null space is met. Note also that the error in the reconstructions is evident as vertical layers, which are precisely features of the null space depicted in Figure 8b.

It was shown in the section on 1-D modeling that straight-ray reflection traveltimes do not resolve slowness variations such as those represented by the eigenvectors of Figure 8b. Therefore, the addition here of reflections from horizontal interfaces cannot be expected to reduce the dimension or change the nature of the null space when the condition of minimum null space is met. They will not improve upon the reconstructions in Figures 9b and 9d. To confirm this, reflected straight rays from the top and bottom boundaries of the model in Figure 7 were added to the direct rays, giving a total of 432 rays. The results of performing SVD on the enlarged $\mathbf{L}$ matrix are given in Figure 10. Again, 5 singular values equal to 0 are present (Figure 10a), and the 5 corresponding null eigenvectors represent purely horizontal variations in slowness (Figure 10b). SVD reconstructions of the same two true models are given in Figure 11; as predicted, the addition of straight ray reflections does not improve the accuracy of the reconstructions.
The value of reflections

If either $M$ (number of pixels in the vertical direction), $N$ (number of pixels in the horizontal direction), or both are increased with no corresponding increase in rays, then the parameterization becomes finer and the number of rays penetrating any one pixel decreases. The dimension of the null space may increase, and often it will expand to include variations in other than the horizontal direction. An example is given in Figure 12, where the parameterization of Figure 7 (6 x 6 pixels) has been further subdivided into 12 x 8 pixels. This increases the number of unknowns from 36 to 96. With a finer parameterization resolution of smaller details may be possible, especially if the null space has not expanded to include non-horizontal slowness variations. However, Figure 13a shows that the null space has a dimension of 14, rather than the minimum possible dimension of 7. Some of the eigenvectors within this null space are plotted in Figure 13b, and it is apparent that the null space now includes slowness variations that are not purely horizontal. If no data in addition to the direct ray data were available, it might be wise to return to a coarser parameterization.

However, when we include reflection data from both the top and the bottom boundaries of the model, SVD results in the singular values given in Figure 14a. Only 7 are equal to 0, and the null space eigenvector plots in Figure 14b confirm that, even with this finer parameterization, the reflections allow the minimum null space to be achieved. The reflected rays contribute an increased "angular variety" to the inversion, meaning that a greater number of slopes is present in the ray coverage.

The above result can largely be explained with a "pixel stripping" argument similar to the one used in Appendix C. The argument is made in analogy to the layer stripping method of determining 1-D velocity structure with surface seismic data. Figure 15a shows the top two rows of a parameterization in which $N=4$. If we assume that three of the top four slownesses are known, the fourth can be determined by measuring the traveltime of the top ray in the figure. The top row of pixels is then effectively "stripped" from the model, and the slowness of a pixel ($s_{2,4}$) in the second row can be determined, with the traveltime of a single ray segment, as shown in Figure 15b. This pixel is then stripped, and the slowness of the next pixel to the left ($s_{2,3}$) determined as in Figure 15c. The remaining slownesses in the row are determined in the same manner (Figures 15d and 15e). This can be carried out all the way down the model until all slownesses have been determined. The assumption concerning the three known slownesses of the top row corresponds to the null space dimension (which equals 3 in this case).

If we now consider a more finely parameterized model, the value of reflections becomes apparent. Figure 16a shows a parameterization in which source-receiver coverage is unchanged but the number of pixels in the horizontal direction has been doubled. Two reflected rays have also been plotted with the five transmitted rays. If we again assume $N-1=7$ known slownesses for the top row, the top row can be stripped away with the traveltime of the horizontal ray. To begin the stripping of the second row, however, requires more rays than just the transmitted ones; one of the reflected rays is needed for calculation of $s_{2,8}$ (Figure 16b). A transmitted ray then gives $s_{2,7}$ (Figure 16b). Moving to the left, the next two
slowness ($s^{2.6}$ and $s^{2.5}$) can then be calculated with a reflected and transmitted ray (Figure 16c). A symmetric set of rays, shot from the other side of the model, can then be used to determine the four remaining slownesses of the second row. In this way all slownesses of the model can be solved for. The increased angular variety in the ray set given by the reflected rays allows slowness reconstruction for a finer parameterization.

In summary, for the case of straight rays, horizontal reflectors, and noise-free data, addition of reflections serves to decrease the dimension of the null space when the condition of minimum null space is not met with transmission data alone. However, the null space eigenvectors representing pure horizontal variations will still be present; reflections, in this case, do not permit resolution of these features.

Elimination of the null space

The minimum null space for the crosswell geometry and 2-D parameterizations as discussed thus far will always have nonzero dimension if rays are straight and reflectors are horizontal; however, by manipulating the experimental geometry, the 2-D parameterization, or both, the null space can be eliminated, theoretically allowing a unique reconstruction. That is the topic of this section.

As discussed above, SVD gives a reconstruction that is equal to the true model minus its null space components ($s_T = s_p$). A certain linear combination of the null eigenvectors (namely, $s_0$) can be added to $s_T$ to return to the true model. It was also seen that the eigenvectors of the minimum possible null space all correspond to purely horizontal slowness variations. For these two reasons it is necessary that each column of pixels of the SVD reconstruction have associated with it a single constant that, when added to slownesses of that column, results in the slownesses of the true model for that column. For example, in Figure 9b an SVD reconstruction is presented for a model containing a low velocity pixel in the fourth column of pixels. The reconstructed slowness for that pixel differs from its true slowness by $7.67 \times 10^{-3}$ ms/ft. All the other pixels in this column also differ from their true values by this same amount. The same is true of the other columns, although a different constant applies. Therefore, within any column, relative slownesses are accurately known, but absolute slownesses are uncertain. Knowledge of the absolute slowness of just one pixel per column would enable us to determine all absolute slownesses, allowing a unique reconstruction and eliminating the null space.

One way in which this can be accomplished is by using some sort of a priori information to justify a modification of the parameterization. Based on well logs or knowledge of the local geology, we may be able to assume that the pixels in one or more rows of the model are of a constant slowness. If this assumption is valid then each of these rows can be reparameterized as a single long pixel extending from one borehole to the other. Each long pixel's slowness can then be assumed known (as from well log information) or solved for as part of the inversion. Either way, the ambiguity in absolute slowness is alleviated for all columns and the null space should be eliminated. As a test, the 12 by 8 pixel parameterization of Figure 12 was modified such that the eleventh row from the top comprises a single pixel of uniform slowness (Figure 17). A full set of direct and reflected ray
traveltimes was generated for the model of Figure 18a and SVD was performed, resulting in the reconstruction of Figure 18b. The true model and the reconstruction are indistinguishable. Figure 18d, a plot of the singular values for the new parameterization, confirms that the null space has been eliminated. Figure 18c is the reconstruction for the conventional parameterization of Figure 12; it is obviously not as accurate as that for the new parameterization.

Of course, the assumption that a layer’s velocity is constant is not always a good one. An example is given in Figure 19a, where two pixels in the constant velocity row of the last example now have velocities that differ from the rest of that row by positive and negative 8 percent. Travel times were generated for this model, and inversion was carried out again with that layer parameterized as a single wide pixel. The result is given in Figure 19b. The reconstruction with the conventional parameterization is shown in Figure 19c. The reconstructions are of similar accuracy; the bad assumption concerning the layer’s velocity still permits a somewhat accurate reconstruction.

Another way to eliminate the null space is through a modification of the geometry of the experiment. For example, placing sources (or receivers) on the ground surface between the two boreholes, as well as within both boreholes, results in a geometry equivalent to a vertical seismic profile (VSP) experiment. As shown by others, including Ivansson (1987), this geometry results in a partial derivatives matrix ( $L$ ) that has no null space. A perfect reconstruction would theoretically be possible.

**Dipping reflectors and deviated boreholes**

The dimension of the null space is also reduced if one or more reflectors are dipping, rather than horizontal. In this situation, reflected ray path length is a function of source and receiver depth as well as shooting angle. As a result, the linear dependence of the system of equations is reduced. When boreholes are not perfectly parallel, transmitted ray path length and reflected ray path length both become functions of depth and shooting angle. Therefore, this perturbation in geometry should also reduce linear dependence.

An example of the effect of a dipping reflector is given in Figures 20 and 21. Figure 20a shows a crosswell geometry of $12 \times 12$ pixels with 12 shots or receivers in each borehole. 432 straight transmitted and reflected rays were generated, with reflecting interfaces being the top and bottom of the model. As shown in Figure 20b, this geometry results in 11 eigenvalues close to or equal to 0. The bottom reflector was then given a small amount of dip, as shown in Figure 21a. Again, 432 rays were generated. The resulting singular values are plotted in Figure 21b. The small amount of dip significantly reduces, or even eliminates, the null space.
3. SYNTHETIC TESTS AND REAL DATA

This chapter presents some synthetic tests with Snell (bent) rays and the latest results of an ongoing attempt to invert real data. Since the ray paths are velocity-dependent, the inversions are now nonlinear and solved iteratively. Many results of this chapter are, consequently, quite possibly model-dependent, but an attempt is made to draw some conclusions that will hold in general. Conclusions with the synthetic tests include

(1) the largest improvement in the best estimate of the model parameters is typically made in the first iteration, with little or no improvement made in following iterations.

(2) presence of a low velocity zone (LVZ) causes ray coverage to be uneven, making this type of velocity anomaly more difficult to detect and delineate than a high velocity zone (HVZ);

(3) with "clean" (low noise) data, reflections from horizontal interfaces enhance the ability of crosswell tomography to detect horizontal variations in velocity and the presence of low and high velocity zones;

(4) if reflections are assumed to be twice as noisy as transmissions, including them in the inversion appears to only slightly improve the reconstruction of a model with a low or high velocity zone embedded in a strong velocity gradient;

(5) the addition of surface-to-well (VSP) data can greatly improve reconstructions, especially horizontally-varying features; and

(6) regardless of the particular data sets included, noise in the data usually prevents the best reconstruction from having the minimum traveltime residual; i.e., the last reconstruction in a sequence of iterations is often not as accurate a representation of the true model as some previous reconstruction in the same sequence of iterations.

ITERATIVE RAY TRACER

The Fortran ray tracing program used in this chapter is a modification of one written by Weber (1988). In particular, it was modified to attempt to trace rays to specific receiver locations and to calculate partial derivatives of traveltimes for those rays with respect to velocity. Bregman (1976) derives most of the equations used here for traveltimes and partial derivatives. An outline of these derivations is given in Appendix D.

The ray tracer uses a parameterization in which models are constructed as combinations of triangles, with velocities specified at triangle corners ("nodes"). As discussed in Appendix D, the velocity within any one triangle varies as a uniform linear gradient, causing a ray path segment within one triangle to be either a circular arc or a straight line. Best results were observed when ray tracing was carried
out in double precision.

**LEAST-SQUARES INVERSION**

Velocities were used as the inversion parameters (rather than slownesses) because they vary in a straightforward fashion within triangles. As discussed in the introduction, the least-squares solution to the traveltime equations is sought:

\[
\Delta v = \left[ L^T L + \varepsilon^2 I \right]^{-1} L^T \Delta t.
\]  \hspace{1cm} (4)

However, when inversion of real data is attempted later, \(L\) becomes very large (about 12,000 \(\times\) 1500 elements) and calculation of \(L^T L\), followed by solution of Equation (4), is not feasible. An iterative least-squares solver, LSQR (Paige and Saunders, 1982), is therefore used in an attempt to alleviate the problems associated with Equation (4). Synthetic tests in this chapter and real data inversion both were done with LSQR.

LSQR, called as a Fortran subroutine, is able to operate directly on the matrix of partial derivatives \(L\). The choice was made, however, to first scale \(L\) with a diagonal scaling matrix \(W\). Therefore, LSQR solves

\[
W \Delta t = W L \Delta v.
\]

The effect of \(W\) is to scale \(L\) such that all its rows have a norm equal to 1. This increases the influence of shorter ray paths, which are presumably more accurately known than longer ray paths. Additionally, in all iterations it was necessary to use a damping parameter with the same effect as the diagonal damping (\(\varepsilon\)) in Equation 4. effect of ray bending on inversion with crosswell reflection data.

**SYNTHETIC TESTS**

Synthetic tests were run on a few selected models to determine the effect of ray bending on inversion with crosswell reflection data. Synthetic data were generated with an assumed "true" model \((v_{\text{true}})\), and the nonlinear inverse problem was solved iteratively beginning with an initial "guess" \((v_0)\). At each iteration, knowledge of the true model, compared with the result of the current iteration, was used to guide action in the subsequent iteration. Of course, with real data the true model is not known; however, working with this "best case" situation permits the definition of some limitations of crosswell tomography with and without reflections or VSP data.

**Definitions**

Some more terms need to be defined before the procedure can be presented in detail. \(\delta\) will be used to represent a lower limit on \(|\Delta v_i|\), the
magnitude of the solution vector. If $|\Delta v_i|$ is found to be smaller than $\delta$, then $\Delta v_i$ is judged to give no significant change in the current best estimate of parameters, $v_{i-1}$.

Two measures of error will be referred to. The first one, $E_{m_i}$, is a measure of RMS model error for an iteration:

$$E_{m_i} = \frac{|v_{true} - v_i|}{M^{1/2}},$$

where $M$ is the number of nodes. The other measure of error, $E_{t_i}$, is an RMS measure of the residual traveltime error, calculated by comparing the synthetic or real data with predicted data given by rays traced through the velocity reconstruction of the $i$th iteration:

$$E_{t_i} = \frac{|W_i \Delta t_i|}{N^{1/2}}.$$

Here $W_i$ is the same diagonal scaling matrix described in the last section and $N$ is the number of data values. $E_{t_i}$ is the value used in judging the success or failure of an iteration: if $E_{t_i} \geq E_{t_{i-1}}$, then this attempt at the $i$th iteration has failed and a different reconstruction must be tried. The size of $E_{t_i}$ is also used as a stopping criterion when inversions with real data are attempted later.

**Procedure**

Figure 22 is a flowchart illustrating the procedure used for the synthetic tests. Iterations begin with an initial guess and end when no further reduction in $E_{t_i}$ appears possible with a $\Delta v_i$ of reasonable size; i.e., when a reduction of $E_{t_i}$ requires that $|\Delta v_i| < \delta$. The $i$th iteration is judged to be successful, therefore, when both

$$E_{t_i} < E_{t_{i-1}}$$

and

$$|\Delta v_i| \geq \delta.$$  

With these two constraints, the iterations proceed through two distinct phases. In the first (depicted in Figure 22a), $E_{m_i}$ is required to decrease with each iteration. Each successive reconstruction is a more correct, in an RMS sense, estimate of the true model. When $E_{m_i}$ can no longer be reduced, then the second phase of iterations begins (Figure 22b). $E_{m_i}$ is now allowed to increase with increasing iterations while $E_{t_i}$ is still forced to decrease. Presumably, at this stage reduction of $E_{t_i}$ requires a solution with a significant contribution from eigenvectors composed largely of poorly-illuminated parameters. Inversion in the second phase can,
therefore, be thought of as allowing the model estimate to expand into the null space of the inversion problem. Requiring $E_i$ to decrease with each iteration keeps the synthetic tests similar to inversion with real data, where the traveltime residual and visual inspection are usually the only criteria for judging the result of an iteration. Synthetic tests carried out in this fashion should give some indication of how crosswell reflections, when included with transmission data and/or VSP data a) affect the null space and thus the accuracy of the final reconstruction, and b) influence the number of iterations required to reach the final reconstruction.

**HVZs versus LVZs**

Many of the following synthetic tests suggest that models with a low velocity zone can be less accurately reconstructed than models with a high velocity zone. This can be explained by examining "hit count", or ray density, plots generated by tracing rays through the two different types of models. Figure 23 depicts a crosswell parameterization of $13 \times 13$ nodes which was used in a test of ray densities for two different models. 12 sources and 12 receivers were equally spaced in the two boreholes, and four complete sets of rays were traced: 144 transmitted rays through a HVZ, 288 reflected rays through a HVZ, 144 transmitted rays through a LVZ, and 288 reflected rays through a LVZ. Reflectors in both cases were assumed to be the top and bottom of the model, and both the LVZ and HVZ contrasted with the background velocity by 25 percent and were located in the center of the model. Ray segment lengths within individual triangles were summed and each triangle assigned a shade of grey depending upon the magnitude of its summed ray segment lengths: light colors represent high ray density, while dark colors represent low ray density. Results are shown in Figure 24 (transmitted rays only) and Figure 25 (transmitted plus reflected rays). These two figures clearly show that the LVZ repels rays and the HVZ attracts them. Also, the HVZ ray density plots suggest that ray coverage is much more uniform in the presence of a HVZ than in the presence of a LVZ for both of the two different sizes of data sets. Ray coverage is most uniform for the set of 432 rays traced through the HVZ (Figure 25a). Both LVZ plots (Figures 24b and 25b) display conspicuous "holes" in the area of the LVZ, and this lack of ray coverage, as will be seen in following sections, degrades tomographic reconstructions of such models. For these reasons, the following synthetic tests treat LVZs and HVZs as separate cases.

**Parameterization and true models**

The majority of synthetic tests were run with a parameterization that, hopefully, approximates the condition of minimum null space (Chapter 2) when reflections are included, but has a null space of greater dimension when reflections are excluded. In inversions with such a parameterization, addition of reflections should prove most beneficial. This might give some idea of the maximum improvement we can expect when including reflection data with transmission data in crosswell tomography.

The parameterization tested and chosen for the synthetic tests (Figure 26) is one of $13 \times 10$ nodes and measures 600 ft in both depth and width. This
approximates the geometry of the real data experiment discussed later. 12 source-
receiver positions were spaced at 50 ft intervals starting at a depth of 25 ft in both
boreholes. Reflectors were again taken to be the top and bottom horizontal bound-
daries of the model. This fictitious geometry yields synthetic data sets, therefore, of
144 transmitted traveltimes or 144 transmitted traveltimes plus 288 reflected travel-
times.

As a test of this parameterization and for comparison against the results of
Chapter 2, two reconstructions were attempted with a model (Figure 27a) that
causes several, but not all, rays to bend. The model consists of a HVZ with max-
imum velocity of 7500 ft/s within a uniform background medium of 6000 ft/s.
Both iterative inversions began with an initial "guess" of a uniform background
medium with correct velocity. The first inversion used only transmitted arrivals and
continued for 17 iterations (Figure 27b), and the second included reflections in addi-
tion to the transmissions and was carried out for 7 iterations (Figure 27c). The
reflections are clearly beneficial in this case; furthermore, the reconstruction with
reflections (Figure 27c) exhibits vertical features similar to those seen in Chapter 2
for reconstructions under the condition of minimum null space. This parameteriza-
tion was, therefore, judged to be one in which reflections might produce maximum
benefit, and the following synthetic tests are parameterized in the same way.

The two assumed "true" models for the following synthetic tests are given
in Figures 28a and 29a. Both are composed of an anomalously fast (Figure 28a) or
slow (Figure 29a) body with 13 percent velocity contrast embedded in a velocity
gradient with both horizontal and vertical components. The gradient ensures that
nearly all rays bend due to Snell's law. Velocity in the upper right corner is 5950
ft/s, while velocity in the lower left corner is 7000 ft/s. Synthetic data, generated
by tracing rays through these two models, is used as input to the synthetic test flow
of Figure 22.

Noise-free tests

Noise-free data generated with the HVZ model of Figure 28a were
inverted with an initial "guess" of a 6500 ft/s uniform velocity model. The final
result after 7 iterations for transmissions-only data is given in Figure 28b, and the
final result after 11 iterations for transmissions plus reflections is given in Figure
28c. The larger data set gives a better reconstruction, as seen in a plot of $E_m$
(RMS model error) versus iteration number (Figure 29a). Figure 29b is a plot of
$E_t$ (scaled RMS traveltime error) versus iteration number; for both data sets, the
largest drop in $E_t$ comes in the first iteration. It will be seen that this is a charac-
teristic of all the synthetic tests.

Final reconstructions for two corresponding LVZ tests are given in Figure
30. The same starting model was used. The transmissions plus reflections result
(Figure 30c) once again is better than the transmissions-only result (Figure 30b), but
this is readily apparent only on the corresponding graph of $E_m$ (Figure 31a). Both
final reconstructions are worse than their HVZ counterparts. Reductions in $E_t$ (Fig-
ure 31b) are similar in both cases.
The reconstructions in Figures 28 and 30 suggest that the inversions in all four cases succeeded somewhat in detecting the horizontal component of the velocity gradient. This contradicts one of the conclusions of Chapter 2: even with a poor starting model, ray bending apparently permits the detection of some horizontal velocity variations.

Noise tests

In order to test the effect of noise on crosswell reconstructions, zero-mean random noise with a normal distribution and a standard deviation of .5 ms was added to the synthetic transmission data, and noise with a standard deviation of 1 ms was added to the synthetic reflection data. Additionally, some tests in this section include a set of 216 surface-to-hole (double VSP) traveltimes that were generated with 9 evenly-spaced source-receiver positions on the top horizontal surface of the model (Figure 26). These data will be referred to as "surface data", and the noise added to them has the same standard deviation, mean, and distribution as that added to the crosswell transmission data. Noise levels used were estimated by inspecting the real data set.

Six data sets are considered for the noisy synthetic tests: two sets of transmissions (well-to-well), two sets of transmissions plus reflections (reflections emanating, as before, from the top and bottom horizontal boundaries of the model), and two sets of transmissions plus reflections plus surface data. Given in Figure 32 are final results of the inversions with noisy HVZ data sets. Inversion with the largest data set (transmissions plus reflections plus surface) continued for 7 iterations (Figure 32b), inversion with the transmissions only continued for 8 iterations (Figure 32c), and inversion with the transmissions plus reflections continued for 7 iterations (Figure 32d). Figure 33a shows the corresponding plots of $E_{mv}$ and reveals that the data sets which include reflections and reflections plus surface data yield final reconstructions which are more accurate than the final reconstruction with transmissions-only data. Also, Figure 32b suggests that the addition of the surface data permits the detection of the horizontal component of the velocity gradient, a result consistent with Chapter 2. Only the vertical component is apparent in Figures 32c and 32d. The plot of $E_{lv}$ (Figure 33b) reveals that traveltine error is again greatly reduced on the first iteration with much smaller reductions on successive iterations. As one might expect, the residuals are larger for the noisy data than for the noise-free data (Figure 29a). Also, residuals are significantly less for the transmissions-only data because there are less data values, and therefore less traveltime equations, constraining the solution (Salo and Schuster, 1989).

Figure 33b shows that for all 3 tests the most accurate model is not the result of the last iteration, even though this is the reconstruction which yields the minimum traveltine residual. This is especially apparent for the largest data set. Noise in the data possibly causes convergence to a local, rather than global, minimum. The best reconstructions, as given by the lowest points of the curves in Figure 33b, are shown in Figure 34. The anomalous body is not as well-defined in these reconstructions as in the final ones of Figure 32, but the background medium appears smoother and more accurate.
Final reconstructions for corresponding LVZ tests are given in Figure 35 and the appropriate plots of $E_m$ and $E_t$ are in Figure 36. The curves are very similar to those of the HVZ tests (Figure 33). The transmissions-only reconstruction appears to be slightly worse than the reconstruction with reflections added (Figure 35d), but Figure 36a reveals that, in terms of $E_m$, the opposite is true. Apparently, addition of the very noisy reflection data in this case worsened, rather than improved, the end result of the iterations. Once again the large data set including the surface data yields the best final reconstruction and succeeds in detecting the horizontal component of the velocity gradient.

Figure 37 is a plot of the best reconstructions in terms of $E_m$. As with the corresponding HVZ results, the anomalous body is poorly defined in all three cases but the background medium is smoother and more accurate.

In summary, comparison of the LVZ results with the HVZ results suggests that, in the presence of noise, LVZs are no less resolvable than HVZs. With noise-free data the LVZ model reconstructions were worse. Inversions of the noisy data all required a similar number of iterations; addition of distinctly different data sets to the process did not significantly increase or decrease this number. The largest decrease in $E_m$ and $E_t$ was always in the first iteration, and $E_m$ often increased in later iterations (especially when noisy surface data were included). Reconstructions with minimum $E_t$, therefore, do not necessarily correspond to reconstructions which best fit the model. Iterations with real data might best be terminated when some sort of a priori information (such as well logs) no longer supports the inversion results.

**REAL DATA RESULTS**

A high-quality crosswell data set was obtained from an anonymous oil company and some preliminary attempts at inverting it are presented here. The survey geometry is characterized by two vertical, parallel boreholes which are about 1000 ft deep and separated by 600 ft. Energy from 98 shots in one borehole was recorded by 96 receivers in the other borehole. Both shot and receiver spacing were 10 ft. The shots were small explosive charges, and the receivers used were hydrophones. Additionally, 23 shots and 24 geophones were positioned on the ground surface between the two boreholes, yielding another set of data equivalent to a double VSP experiment. This second data set will be referred to as the "surface data". Both sets of data were used in the inversion result presented here.
Parameterization

Figure 38 depicts the parameterization used for the inversion. There are 49 nodes in the vertical direction and 31 nodes in the horizontal direction, giving 1519 total unknowns. There are approximately 2 source/receiver positions per node in the vertical direction and about .75 source/receiver positions per node in the horizontal direction.

Preliminary inversion results

An automatic first arrival picking program (Olsen, 1989) was used to pick the first breaks in most shot gathers of the crosswell and surface data sets. Some manual picks of the surface data were also made, and the total number of data values inverted is 12,141 (8104 crosswell data values and 4037 surface data values). The same ray tracer used in the synthetic tests of the previous section was used here, and the iterative inversion was carried out in a similar fashion: again, LSQR was used with several different damping parameters. The actual solution kept as the reconstruction for a given iteration was chosen by visual inspection. Rays were then traced through this latest reconstruction and traveltimes compared with the real data. Iterations were halted when it was found that any change in parameters, other than an insignificant one, caused an increase in the scaled traveltime residual.

The starting model used for the inversion is shown in Figure 39a. It is a weak vertical velocity gradient, chosen by examining the sonic log (Figure 39b). Only 3 successful iterations were carried out, and the result of the last one is given in Figure 39c. The sonic log was obtained in the borehole on the left side of the reconstruction; the sonic log and reconstruction obviously do not agree. However, this borehole is the one in which the shots were set off, and the sonic log was obtained before this shooting. The supplier of the data believes the shooting may have somewhat damaged the borehole, possibly explaining the discrepancy. Another problem is apparent in the other borehole (the receiver borehole): several small, localized pockets of low velocity material appear between depths of about 600 ft to 1000 ft. These may be due inaccurately-located receivers (Bregman, 1986).

The most conspicuous feature of the reconstruction in Figure 39c is a surface LVZ. Figure 40 shows how this feature, and others, changed with each iteration. As with the synthetic tests, the largest change came in the first iteration; most features present in the final reconstruction are present in this first one.

Future work

Figure 41 is a representative unprocessed shot gather of the crosswell data. In addition to the direct arrival, there is marked in the figure a strong reflection emanating from the free surface. Traveltimes for this arrival have already been picked manually and future work will concentrate on investigating the effect of including these traveltimes in the inversion. Also, the effect of starting model on final reconstructions will be studied.
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APPENDIX A

THE SLICE PROJECTION THEOREM
As mentioned in the introduction, the traveltime for a single ray is a line integral of the slowness function over a ray path:

\[ t = \int_{L} s(x, z) \, dl. \quad (i5) \]

For straight ray paths and the crosswell geometry of Figure A-1a

\[ dl = \frac{dx}{\cos \theta} \]

and

\[ z = z_0 + px, \]

where \( p \) is the slope of a ray in the figure's coordinate system and \( z_0 \) is the depth at which that ray intersects the left borehole. Now Equation (1) can be recast as

\[ t(z_0, p) = \hat{s}(z_0, p) = \int_{-\infty}^{\infty} \frac{s(x, z_0 + px)}{\cos \theta} \, dx. \quad (A-1) \]

The measurements of \( t \) for all values of \( z_0 \) and a constant value of \( p \) comprise a single "projection" of the slowness field. The parallel ray paths shown in Figure A-1a are an example of a subset of one such projection. Data for all projections, taken together, form a transformation of the slowness field, \( \hat{s}(z_0, p) \), called the Radon transform (Deans, 1983).

It will now be shown that the slice projection theorem provides a simple relationship between the Radon transform of the slowness field and its Fourier spectrum. A one-dimensional spatial Fourier transform and its inverse will be defined as

\[ F_x \left\{ f(x) \right\} = \tilde{f}(k_x) = \int_{-\infty}^{\infty} f(x) \, e^{-i k_x x} \, dx \]

and

\[ F_x^{-1} \left\{ \tilde{f}(k_x) \right\} = f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(k_x) \, e^{i k_x x} \, dk_x, \]

where \( x \) is a spatial variable and \( k_x \) is wavenumber in the direction of \( x \). The inverse two-dimensional Fourier transform is thus

\[ F_{k_x, k_z}^{-1} \left\{ \hat{f}(k_x, k_z) \right\} = f(x, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(k_x, k_z) \, e^{i(k_x x + k_z z)} \, dk_x \, dk_z. \]
This is used to rewrite $s(x,z_0+px)$ in Equation (A-1):

$$\hat{s}(z_0,p) = \frac{1}{\cos \theta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{S}(k_x,k_z) e^{ik_x z_0} e^{ik_z (z_0 + px)} \, dk_x \, dk_z \, dx$$

$$= \frac{1}{(2\pi)^2 \cos \theta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{S}(k_x,k_z) e^{ik_x z_0} \left[ \int_{-\infty}^{\infty} e^{ix(k_x + k_z p)} \, dx \right] \, dk_x \, dk_z.$$

Carrying out the integration with respect to $x$ and then with respect to $k_z$:

$$\hat{s}(z_0,p) = \frac{1}{2\pi \cos \theta} \int_{-\infty}^{\infty} \hat{S}(k_x,k_z) e^{ik_x z_0} \delta(k_x + k_z p) \, dk_x \, dk_z$$

$$= \frac{1}{2\pi \cos \theta} \int_{-\infty}^{\infty} \hat{S}(-k_z p, k_z) e^{ik_z z_0} \, dk_z.$$

This simplified expression is just the inverse Fourier transform of $\hat{s}(-k_z p,k_z)$ divided by the cosine; therefore

$$\hat{s}(z_0,p) = \frac{1}{\cos \theta} F_{k_z}^{-1} \left\{ \hat{s}(-k_z p,k_z) \right\}. \quad (A-2)$$

According to Equation (A-2), a one-dimensional Fourier transform of the slowness function yields the Fourier spectrum of the slowness function. In particular, a single projection ($p=$constant) maps into a single straight line, or "slice", that goes through the origin in Fourier space. Shown in Figure A-1b is the slice of the spectrum corresponding to the projection in Figure A-1a. If the parallel rays of a projection make an angle $\theta$ with the horizontal, the corresponding slice makes the same angle $\theta$ with the $k_z$ axis in Fourier space. An immediate consequence of this fact is that horizontal projections yield the spectrum of vertical variations in the slowness function and vice versa; in general, a ray's strongest contribution is to that portion of the spectrum representing slowness variations perpendicular to that ray. A straight ray contains no information about slowness variations parallel to it. For instance, a suite of horizontal rays between the two boreholes of Figure A-1a would yield, via the slice projection theorem, data along the vertical line that is the $k_z$ axis of Figure A-1b. These horizontal rays resolve the model's vertical slowness variations, but contain no information about horizontal slowness variations.

If a dense array of sources and receivers is situated such that the medium is completely surrounded (as in medical applications of tomography), data for a very large variety of projections can be recorded and a near-complete Fourier spectrum of slowness can be generated. A two-dimensional inverse Fourier transform of the spectrum will, in this case, yield a very accurate tomographic reconstruction of the slowness function. In crosswell tomography, however, the experimental geometry is very limited, resulting in fewer projections and a correspondingly
poorer representation in Fourier space. An inverse Fourier transform of this incomplete spectrum gives a reconstruction that is a smeared version of the true model. The biggest problem is with horizontal slowness variations; purely horizontal variations are completely unresolvable, due to a complete lack of vertical rays.
APPENDIX B

SINGULAR VALUE DECOMPOSITION
The $m \times n$ partial derivatives matrix $L$ of Equation (2) can be factored into a product of three matrices:

$$L = UV^T,$$

where the columns of $U$ are the $m$ eigenvectors of $LL^T$, the columns of $V$ are the $n$ eigenvectors of $L^TL$, and $\Sigma$ is a diagonal matrix of dimension $m \times n$ (van der Sluis and van der Vorst, 1987). This factorization is called the singular value decomposition (SVD) of $L$. Its value is that it yields orthonormal bases in data space ($U$) and parameter space ($V$), and the mapping from one to another is achieved with the diagonal matrix $\Sigma$.

The diagonal elements of $\Sigma$ ($\sigma_j$, $j=1,\ldots, \min(m,n)$) are arranged in order of decreasing magnitude. These elements, called "singular values", are the positive square roots of the eigenvalues of $L^TL$ and $LL^T$. In the problems dealt with in the second chapter, there will be at least as many data values as there are parameters; therefore, the maximum number of nonzero singular values is $n$. This permits Equation (B-1) to be rewritten as $L = U_p\Sigma_p V_p^T$, where $p$ refers to the number of nonzero eigenvalues. $V_p$ and $U_p$ consist of the first $p$ columns of $V$ and $U$, respectively, and $\Sigma_p$ is a square diagonal matrix with the nonzero singular values on its diagonal.

If the number of nonzero singular values ($p$) is less than $n$, there exists within the parameter space a subspace (a "null space") spanned by eigenvectors representative of features not resolvable with the given experimental geometry and ray coverage. Each singular value equal to 0 corresponds to one eigenvector in the null space. For instance if $\sigma_j=0$, the eigenvector corresponding to the $j$th column of $V$ is in the null space. Any model vector $s$ can be expressed as a sum of the eigenvectors in $V$. Therefore, $s$ can be expressed as

$$s = s_p + s_0,$$

where $s_0$ is a linear combination of null space eigenvectors and $s_p$ is a linear combination of non-null space (resolvable) eigenvectors.

If $s_0$ is nonzero, we cannot hope to perfectly reconstruct $s$ (without the addition of some a priori constraints). Furthermore, an infinite number of least-squares solutions to the inverse problem exists. The best choice, then, is to take the one least-squares solution of minimum length—the solution with no components in the null space. This solution is obtained by operating on the data vector with the natural generalized inverse operator (Hatton, et al., 1986) $V_p^{-1} \Sigma_p^{-1} U_p^T$. 
APPENDIX C

PROOF OF MINIMUM DIMENSION OF NULL SPACE
Theorem: Assume a crosswell geometry of two vertical boreholes and that the region between the two boreholes is parameterized into $M$ rows of pixels by $N$ columns of pixels. For continuous source-receiver coverage with straight rays, the model null space has a dimension of $N-1$.

Proof: The proof will be made in two parts, with a construction argument similar to that used by Bishop et al. (1985) being used for the first part.

Figure C-1a shows the area between two boreholes as being divided into six rows of four pixels each. Also in the figure are four rays originating in the top row at the left borehole and terminating in the second row at the right borehole. The rays were chosen such that within every column of pixels there is one ray which intersects the boundary between the top and second rows. For this reason, this group of rays is linearly independent: no ray's traveltine equation can be expressed as a linear combination of the other three. An attempt to form such a combination for any one ray would have to include traveltimes for all four rays, including the one which is the only one to penetrate pixel 5.

A similar group of four rays can be traced one row lower, giving rays that originate in row 2 and terminate in row 3. These four are also independent of one another. Furthermore, this group of rays is independent of the first group because no rays in the first group penetrate the pixels of the third row. This process can be carried out all the way down the stack of pixels. With $M$ rows and $N$ columns, $(M-1)N$ independent equations can be generated. A last ray, traced straight through the top row, gives one more independent equation (this is independent because this is the only ray to penetrate the pixel in the top right corner). Therefore, there are at least $(M-1)N+1 = MN-(N-1)$ independent equations involving the $MN$ unknowns, and

$$\text{rank}(L) \geq MN-(N-1). \quad (C-1a)$$

A second argument can be used to show that $\text{rank}(L) \leq MN-(N-1)$. If we require that each column of pixels have a constant slowness, then we are essentially reparameterizing the problem as in Figure C-1b. A new partial derivatives matrix for the reparameterized problem, $\tilde{L}'$, can be constructed from $L$. Each column of $\tilde{L}'$ consists of ray segment lengths in a column of the new model, so each column of $\tilde{L}'$ is just a sum of appropriate columns in $L$. The results of the section on 1-D models indicate that $\tilde{L}'$ has a rank equal to 1. The $N$ columns of $\tilde{L}'$ are, therefore, linearly dependent. Since each column of $\tilde{L}'$ is a sum of $M$ columns of $\tilde{L}$, $\tilde{L}$ has at least $N-1$ linearly dependent columns. Therefore,

$$\text{rank}(L) \leq MN-(N-1). \quad (C-1b)$$

Equations (C-1a) and (C-1b) imply that $\text{rank}(L) = MN-(N-1)$ and the number of eigenvectors in the null space is $N-1$. 
APPENDIX D

RAY TRACING AND PARTIAL DERIVATIVES CALCULATIONS
Most of the ray tracing and partial derivatives theory used in Chapters 3 and 4 of this thesis is due to Bregman (1986). The following is an outline of her development.

CURVED RAYS

As discussed in the text, models are constructed of triangles with the velocity function within any one triangle defined by its three corners, or knot points. Unless all three knot points are of equal velocity, velocity varies as a linear gradient ($\nabla v$) throughout that triangle. For this reason, any ray's path within that triangle is an arc of a circle. The geometry of this problem makes it convenient to carry out some calculations in a local coordinate system, illustrated in Figure D-1, that is oriented such that the local vertical axis extends opposite in direction to $\nabla v$. Thus, the coordinates $(x, z)$ are transformed into the coordinates $(\eta, \zeta)$. The results of calculations made in this local coordinate system are easily transformed back into the global coordinate system.

A ray's horizontal component of slowness in the local coordinate system, $p$, is constant along its path. The local vertical component of slowness, $q$, varies such that the following equality is true at all points along the path:

\[
\frac{1}{v^2} = \frac{1}{p^2} + \frac{1}{q^2}.
\]

$q$ is equal to 0 at the turning point (t.p.). If * is used to denote the point at which the ray enters the triangle and + denotes the point at which it exits, the traveltime through the triangle is given by

\[
T = \int_{\text{t.p.}}^{\text{t.p.} +} \frac{dl}{v} \tag{D-1}
\]

where the two terms are added if the ray turns and subtracted if it does not turn. Bregman shows that the incremental length of ray path, $dl$, can be expressed as

\[
dl = \frac{|\Delta v_j|}{|q| B v}
\]

where $B = |\nabla v|$. Equation (D-1) then becomes

\[
T = \int_{\text{t.p.}}^{\text{t.p.} +} \frac{|dv|}{B v^2 q
\]

\[
= \frac{1}{B} \ln \left[ \frac{\frac{1}{p} \sqrt{\frac{1}{p^2} - \nu^+ \nu^+}}{\nu^+} \right] \pm \ln \left[ \frac{\frac{1}{p} \sqrt{\frac{1}{p^2} - \nu^* \nu^*}}{\nu^*} \right].
\]
In the global coordinate system, velocity at any point within a triangle is given by
\[ v = v_o + v_x(x-x_0) + v_z(z-z_0), \quad (D-2) \]
where \(v_o\) is the velocity at some point \((x_0,z_0)\) within the triangle and \(v_x\) and \(v_z\) are the \(x\)- and \(z\)-components of the velocity gradient. Tomographic inversion requires that partial derivatives of travel times with respect to the parameters (velocities) be calculated. This amounts to differentiating Equation (D-1) with respect to velocities at each of three knot points for every triangle penetrated by a ray. This can be done analytically with Equation (D-2) and the following argument.

Since the problem is nonlinear, a perturbation in velocity gives a perturbation in ray path; however, according to Fermat's principle this travel time perturbation due to ray path perturbation is zero to first order, allowing the derivatives to be calculated as
\[
\frac{\partial T}{\partial v_i} = \frac{1}{L} \frac{\partial (1/v)}{\partial v_i} \, dl
\]
\[
= \frac{-1}{L} \frac{\partial v}{v^2} \frac{\partial v}{\partial v_i} \, dl
\]
for \(i=1,2,3\). \(L\) is the ray path within a triangle. Using Equation (D-2), the above equation can be expanded to
\[
\frac{\partial T}{\partial v_i} = \frac{\partial v_0}{\partial v_i} \frac{dl}{L \, v^2} - \frac{\partial v_x}{\partial v_i} \frac{(x-x_0)}{v^2} \, dl - \frac{\partial v_z}{\partial v_i} \frac{(z-z_0)}{v^2} \, dl. \quad (D-3)
\]

\[
\frac{\partial v_0}{\partial v_i}, \frac{\partial v_x}{\partial v_i}, \quad \text{and} \quad \frac{\partial v_z}{\partial v_i}
\]
are constants within any one triangle that depend only upon the positioning of the knot points.

Bregman defines the three integrals of Equation (D-3) as
\[
I_0 = \int \frac{dl}{L \, v^2}, \quad I_x = \int \frac{(x-x_0)}{L \, v^2} \, dl, \quad \text{and} \quad I_z = \int \frac{(z-z_0)}{L \, v^2} \, dl.
\]
The analytic solution of \(I_0\), which is independent of coordinate system, is
\[
I_0 = \frac{1}{B} \left| q^+ \right| \left| \pm q^* \right| \left| \right|
\]
\(I_x\) and \(I_z\) are easier solved in the local coordinate system as \(I_\eta\) and \(I_\xi\), respectively:
\[
I_\eta = \int \frac{(\eta-\eta_0)}{L \, v^2} \, dl
\]
\[
\frac{1}{B^2 \rho} \left[ \frac{1}{v^+} \frac{1}{v^*} \right] - \frac{q^* v^*}{B \rho} I_0 + (\eta^* - \eta_0) I_0
\]

and
\[
I_\zeta = \int_{L}^{(\xi - \xi_0)} \frac{dl}{v^2} = \frac{v_0}{B} I_0 - \frac{1}{B} T.
\]

\(I_\eta\) and \(I_\zeta\) are then transformed back into the global coordinate system where they can be used, along with \(I_0\), to solve Equation (D-3).

**STRAIGHT RAYS**

There are two cases in which the ray segment within a triangle will be a straight line rather than a circular arc. The first is when a ray enters the triangle parallel to the velocity gradient. In this case, \(dl = |dv| / B\) and
\[
T = \int_{L}^{(\xi)} \frac{dl}{v} = \frac{1}{B} \int \frac{|dv|}{v} = \frac{1}{B} \left[ \ln \left( \frac{v^+}{v^*} \right) \right].
\]

\(I_0\) is again independent of coordinate system:
\[
I_0 = \int_{L}^{(\xi)} \frac{dl}{v^2} = \frac{1}{B} \int \frac{|dv|}{v^2} = \frac{1}{B} \left[ \frac{1}{v^+} - \frac{1}{v^*} \right].
\]

The two other integrals are
\[
I_\eta = \int_{L}^{(\eta - \eta_0)} \frac{dl}{v^2} = (\eta^* - \eta_0) I_0
\]

and
\[
I_\zeta = \int_{L}^{(\xi - \xi_0)} \frac{dl}{v^2} = \frac{v_0}{B} I_0 - \frac{1}{B} T.
\]

After the appropriate coordinate transformation for \(I_\eta\) and \(I_\zeta\), Equation (D-3) can be solved.

The other straight ray case arises when all three knot points have the same velocity, resulting in a constant velocity throughout the triangle. The travel-time \(T\) is then trivial and
\[ I_0 = \int_{L} \frac{dl}{v^2} = \frac{1}{v} T. \]

The transformed coordinate system is not needed for calculation of \( I_x \) and \( I_z \):

\[ I_x = \int_{L} \frac{(x-x_0)}{v^2} \, dl = I_0 \left[ \frac{x^*+x^+}{2} - x_0 \right] \]

\[ I_z = \int_{L} \frac{(z-z_0)}{v^2} \, dl = I_0 \left[ \frac{z^*+z^+}{2} - z_0 \right] \]

Equation (D-3) can then be solved for the partial derivatives.
FIG. 1. Crosswell geometry and straight ray paths for one scan. Small boxes in Borehole B mark receiver positions.
FIG. 2. One-dimensional, horizontally-layered crosswell model and its mirror image. Rays reflecting from the ground surface have traveltimes equal to those for transmitted rays passing through the mirror image.
FIG. 3. One-dimensional, vertically-layered crosswell model and its mirror image. Snell’s law is neglected at all interfaces except the ground surface, causing rays to be straight.
FIG. 4. Bending rays traced through the model of Figure 3.
FIG. 5. Singular values for system of equations given by rays of Figure 4.
FIG. 6. Bending rays traced through modified model of Figure 3.
FIG. 7. Crosswell parameterization of $5 \times 6$ pixels. Small boxes mark source/receiver locations.
FIG. 8. (a) Singular value decomposition of 144 traveltime equations for straight transmitted rays traced through the parameterization of Figure 7. Singular values, (b) singular vectors corresponding to five smallest singular values.
(b)

FIG. 8. (continued)
FIG. 10. Singular value decomposition of 432 traveltime equations for straight transmitted and reflected rays traced through the parameterization of Figure 7. (a) Singular values, (b) Singular vectors corresponding to five smallest singular values.
FIG. 10. (continued)
FIG. 12. Crosswell parameterization of $12 \times 8$ pixels.
FIG. 13. Singular value decomposition of 144 traveltime equations for straight transmitted rays traced through the parameterization of Figure 12. (a) Singular values, (b) singular vectors corresponding to six small singular values.
FIG. 13. (continued)
FIG. 14. Singular value decomposition of 432 traveltime equations for straight transmitted and reflected rays traced through the parameterization of Figure 12. (a) Singular values, (b) singular vectors corresponding to seven smallest singular values.
FIG. 14. (continued)
FIG. 14. (continued)
FIG. 15. Pixel stripping with transmitted rays only. (a) Parameterization and rays. (b) Determination of slowness $s_{2,4}$. (c) $s_{2,3}$, (d) $s_{2,2}$, and (e) $s_{2,1}$. 
FIG. 15. (continued)
FIG. 16. Pixel stripping with transmitted and reflected rays. (a) Parameterization and rays. (b) Determination of slowness $s_{2,8}$ and $s_{2,7}$. (c) $s_{2,5}$ and $s_{2,6}$. 
FIG. 17. Crosswell parameterization with one row parameterized as a single pixel.
1.16
FIG. 18. (continued)
FIG. 20. (a) Crosswell parameterization of $12 \times 12$ pixels. (b) Singular values for 432 transmitted and reflected rays.
FIG. 21. (a) Crosswell parameterization of $12 \times 12$ pixels and a dipping reflector. (b) Singular values for 432 transmitted and reflected rays.
\[ i = \text{iteration number} \]
\[ v_i = \text{velocity result of } i \text{th iteration} \]
\[ \Delta v_i = \text{velocity update for } i \text{th iteration} \]
\[ L_i = i \text{th partial derivatives matrix} \]
\[ E_{w_i} = \text{RMS travelt ime residual} \]
\[ \epsilon = \text{damping parameter} \]
\[ \delta = \text{lower-limit threshold for } |\Delta v_i| \]

**Flowchart of first phase of synthetic test procedure**

1. **\( i = 0 \)**: Trace rays through \( v_0 \) to generate \( L_i \) and \( E_{w_i} \).

2. **\( i = 1 \)**: Compute \( \Delta v_1 \) with several different values of \( \epsilon \); choose \( \Delta v_1 \) for which
   \[ v_1 = v_0 + \Delta v_1 \]
   is closest to \( v_{\text{true}} \).

3. **\( i = 2 \)**: Trace rays through \( v_1 \) to generate \( L_2 \) and \( E_{w_1} \).
   - **yes**
     - Compute \( \Delta v_i \) with several different values of \( \epsilon \); choose \( \Delta v_i \) for which
       \[ v_i = v_{i-1} + \Delta v_i \]
       is closest to \( v_{\text{true}} \).
     - Is \( |\Delta v_i| \geq \delta \)?
       - **yes**
         - Recompute \( \Delta v_i \) with larger value of \( \epsilon \) to give new \( v_i \).
       - **no**
         - Trace rays through \( v_i \) to generate \( L_{i+1} \) and \( E_{w_{i+1}} \).
           - **yes**
             - **(next page)**
           - **no**

**FIG. 22.** (a) Flowchart of first phase of synthetic test procedure. (b) Flowchart of second phase of synthetic test procedure.
Recompute $\Delta v_i$ with smaller value of $\epsilon$ to give new $v_i$

Trace rays through $v_i$ to generate $L_{i+1}$ and $E_{i+1}$.

Is $E_{i+1} < E_{i+1-1}$?

$\nu_i = \nu_{i-1} + \Delta \nu_i$

Compute $\Delta \nu_i$ with several different values of $\epsilon$; choose $\Delta \nu_i$ such that $|\Delta \nu_i| \geq |\Delta \nu_{i-1}|$

$i = i + 1$

Is $|\Delta \nu_i| \geq \delta$?

$\nu_{\text{final}} = \nu_i$

(b) Recompute $\Delta \nu_i$ with larger value of $\epsilon$ to give new $v_i$

FIG. 22. (continued)
FIG. 23. Parameterization for ray density tests
FIG. 24. Ray density diagrams for 144 transmitted rays through (a) a HVZ and (b) a LVZ.
FIG. 25. Ray density diagrams for 432 transmitted and reflected rays through (a) a HVZ and (b) a LVZ.
FIG. 29. Error plots for inversion with noise-free HVZ data set. (a) $E_{m_i}$ versus iteration number. (b) $E_{\lambda_i}$ versus iteration number.
FIG. 31. Error plots for inversion with noise-free LVZ data set. (a) $E_m$ versus iteration number. (b) $E_t$ versus iteration number.
FIG. 33. Error plots for inversion with noisy H/V data set. (a) $E_m$ versus iteration number. (b) $E_v$ versus iteration number.
FIG. 36. Error plots for inversion with noisy LVZ data set. (a) $E_m$ versus iteration number. (b) $E_t$ versus iteration number.
FIG. 38. Parameterization for inversion of real crosswell plus surface data.
FIG. 41. Real data shot gather.
FIG. A-1. Parallel ray paths comprising a subset of one projection in a crosswell experiment (a) and its slice in Fourier space (b).
FIG. A-1. (continued)
FIG. C-1. Crosswell parameterization and one set of linearly independent rays (a). Reparameterization of columns into single pixels (b).
FIG. D-1. Ray tracing within a triangle of constant velocity gradient.
Crosswell Attenuation Tomography

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Crosswell Attenuation Tomography

ABSTRACT

This paper analyzes the possibilities of estimating attenuation from seismic travel time, amplitude and wavelet broadening information. The forward problem is solved by a two-point ray tracing algorithm using bending rays in a tomography approach. The inverse problem is formulated in terms of the inverse of the discretized quality factor \( Q^{-1} \) and is carried out as a linear estimate by damped least squares technique. The velocity distribution is considered known from tomography travel time inversion.

Modelling of synthetic data shows that the quality factor is extremely sensitive to the noise level of the initial amplitude and wavelet width at the source location. In order to eliminate the initial parameters an inversion technique using differences between adjacent data values is presented. Synthetic tests suggest that reliable inversion results are attainable with realistic noise levels in the data.

Finally, the methods discussed are applied to field data from a crosswell experiment in Texas, USA. Amplitudes and wavelet widths were picked from the seismic records and inverted for a distribution of \( Q_r \). A tomogram of the quality factor from the target area was extracted by the wavelet width inversion. Comparison with the associated velocity distribution indicate that wavelet broadening tomography is a possible source of information about attenuation. It was not possible to obtain useful results from the inversion of amplitudes. The main problems with the amplitudes are expected to be a crucial dependency on the receiver response.
INTRODUCTION

It has been shown that the presence of volatiles in rocks have substantial influence on the velocity ($V_p$) of elastic P-waves (Mavko & Nur, 1978). Velocity data are ambiguous, however, when lithological variations such as lenses of sandy material in shaley sequences are introduced.

It is a fact that the dissipation (quality) factor ($Q$) of a rock is decreased, when volatiles are introduced (Clark & Tittmann, 1980; Mochizuki, 1982; Tittmann et al., 1980). Expressions for the quality factor in terms of the amplitude, width of the wavelet and the rise time have been established (Telford et al., 1976; Kjartansson, 1979).

Previous attenuation modelling experiments have been performed by determining a mean value of $Q$ ($\bar{Q}$) from estimates of the slope and intercept of the linearized expression for the amplitudes (Albright et al., 1988). Ho-lin et al. (1988) formulated the problem using amplitudes as a tomography approach that has proved to be a valuable technique for estimating sub-surface parameter distributions. The parameter vector was discretized into a model composed of rectangular cells, each of a constant value of $Q_i$. Using the ratio of the observed amplitudes for the P- and S-waves the unknown values of the initial amplitudes, radiation pattern and receiver response were eliminated from the equations. In dealing with partially saturated rocks, however, the attenuation coefficients of the P- and S-waves are not expected to be related in a straight-forward manner.

The objective of this paper is to develop a reliable method for inverting $Q$ from travel times, amplitudes and wavelet widths. Linear expressions are established for both amplitude and wavelet broadening data. The sensitivity of the inversion result to varying noise levels of the data is determined by synthetic tests. Finally, the methods are applied to real data.

THE FORWARD PROBLEM

Ray tracing. The forward solution in seismic tomography experiments is determined by tracing rays between the sources and receivers. Numerous methods have been proposed for accurate tracing of rays in laterally varying media (e.g., Langan et al., 1985; Cassel, 1982; Whittall and Clowes, 1979). For these experiments, the tracing of rays are performed in a grid of triangles, each of a constant velocity gradient (e.g., Weber, 1988). The pattern of triangles are defined according to Olsen (1988b).

Amplitudes. The amplitude of the wave represented by the i'th ray can be expressed as

$$A_i = A_i^0 G_i(l) R_i L_i e^{\int_q(l) \, dl}$$  \hspace{1cm} (1)

where $A_i^0$ is the initial amplitude associated with the i'th ray, $G_i(l)$ is the geometrical spreading factor, $R_i$ is the radiation correction for the take-off angle of the i'th ray, $L_i$ is the response of the receiver corrected for the angle of incidence of the i'th ray, and $q(l)$ is the attenuation coefficient along the travel path of the i'th ray of length $L$ (e.g., Albright et al., 1988).

The coefficient of attenuation is defined by

$$q = \frac{\pi \cdot f}{\bar{Q} \cdot \nu}$$  \hspace{1cm} (2)
where \( f \) is the dominant frequency of the wave in a medium of seismic velocity \( v \) and quality factor \( Q \).

The geometrical spreading factor in equation (1), \( G_i(t) \), is conveniently approximated by the inverse of the ray path length

\[
G_i(t) = \frac{1}{L_i}.
\]  

(3)

This approach is valid for \( L_i \gg 1 \) and is invalid for caustics. If the attenuation coefficients are considered constant in each block, the integral in equation (1) reduces to

\[
- \pi \cdot f \cdot \sum_{j=1}^{NC} \frac{T_{ij}}{Q_j} \]

(4)

where \( Q_j \) and \( T_{ij} \) are, respectively, the quality factor and the travel time of the \( i \)'th ray in the \( j \)'th cell; there are \( NC \) cells. The correction term for the radiation pattern \( (R_i) \) is calculated as

\[
R_i = 2 - \cos^2 \gamma_i
\]

(5)

where \( \gamma_i \) is the ray path take-off angle measured from the horizontal (Fehler and Pearson, 1981).

The receiver response is normally obtained from calibration of the instruments involved in the specific survey. Thus, for the expressions derived in the following sections the receiver response is conveniently considered radial symmetric.

With these simplifications equation (1) reduces to

\[
A_i = \frac{A_i^0 \cdot (2 - \cos^2 \gamma_i)}{L_i} e^{-\pi f \sum_{j=1}^{NC} \frac{T_{ij}}{Q_j}}
\]

(6)

**Wavelet broadening.** A measure of the width of the wavelet representing the first arrival \( (\tau) \) can be expressed as

\[
\tau_i = \tau_i^0 + C \cdot \int \frac{dt}{Q}
\]

(7)

where \( \tau_i^0 \) is the initial width at \( t = 0 \), \( t \) is the travel time and \( C \) is a constant (Kjartansson, 1979). For a model discretized into blocks, each of constant \( Q_j \), equation (7) becomes

\[
\tau_i = \tau_i^0 + C \cdot \sum_{j=1}^{NC} \frac{T_{ij}}{Q_j}
\]

(8)

where \( T_{ij} \) and \( Q_j \) is the travel time and quality factor of the \( j \)'th block, respectively.
THE INVERSE PROBLEM

If the model response \( g_i \) is considered a linear function of the distribution of \( Q_j^{-1} \), we have

\[
g_i = g_i^0 + \sum_{j=1}^{NC} \frac{\partial g_i}{\partial Q_j^{-1}} \Delta Q_j^{-1}, \quad i = 1, \ldots, N
\]  

(9)

where \( g_i^0 \) is the response of the initial model and \( Q_j \) is the quality factor in the j'th cell. If the coefficients of attenuation are constant in each cell, the value of \( \frac{\partial g_i}{\partial Q_j^{-1}} \) is simply the travel time in each cell, \( T_{ij} \). For \( g_i^0 = 0 \), a cost function to be minimized is now selected as

\[
\sum_{i=1}^{N} \sum_{j=1}^{NC} d_i - \frac{\partial g_i}{\partial Q_j^{-1}} \Delta Q_j^{-1}
\]

(10)

where \( d_i \) is the i'th data value.

Amplitudes. The log of equation (6) gives

\[
\log A_i = -\pi \cdot f \cdot \sum_{j=1}^{NC} \frac{T_{ij}}{Q_j} - \log L_i + \log A_i^0 + \log(2 - \cos^2 \gamma_i)
\]

(11)

For \( N \) observations equation (11) can be cast in matrix notation as

\[
G \cdot x = d
\]

(12)

where

\[
G = \begin{bmatrix}
T_{11} & T_{21} & T_{31} & \cdots & T_{NC1} \\
T_{12} & T_{22} & T_{32} & \cdots & T_{NC2} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
T_{1N} & T_{2N} & T_{3N} & \cdots & T_{NCN}
\end{bmatrix}
\]

(13)

is the sensitivity matrix,

\[
x = (Q_1^{-1}, Q_2^{-1}, \cdots, Q_{NC}^{-1})^T
\]

is the parameter vector and

\[
d = (\log A_1', \log A_2', \cdots, \log A_N')^T,
\]

(15)

where
\[
\log A_i' = \frac{-1}{\pi \cdot f} \cdot \left[ \log A_i + \log L_i - \log A_i^0 - \log(2 - \cos^2 \gamma_i) \right]
\]

(16)

is the data vector and \( A_i' \) denotes the i'th corrected amplitude measurement. A significant problem is that the value of \( A_i^0 \) in equation (16) is not known. Instead, a mean value for all the rays associated with \( (A^0) \) is estimated by the expression

\[
\log \frac{L_i \cdot A_i}{2 - \cos^2 \gamma_i} = -\frac{\pi \cdot f \cdot L_i}{Q \cdot \bar{v}} + \log A^0
\]

(17)

where \( \bar{v} \) is the mean P-wave velocity (Albright et al., 1988). A mean value of \( f \) can be estimated by FFT-analysis or measurements of the predominant period of the first arrival in the time-domain. If the variation of \( f \) is limited, the error of the estimate of the frequency is not expected to blur the relative values of \( Q_i \).

**Wavelet broadening.** In matrix notation, equation (8) can be expressed as equation (12), and the sensitivity matrix and parameter vector are equal to equations (13) and (14), respectively. The data vector \( (d) \) is given by

\[
d = (\tau_1', \tau_2', \ldots, \tau_N')^T
\]

(18)

where

\[
\tau_i' = \frac{\tau_i - \tau_i^0}{C}
\]

(19)

and \( \tau_i' \) denotes the i'th corrected wavelet width measurement.

Similar to the amplitude inversion a mean value of the initial width \( \tau^0 \) is estimated as the intercept of a straight line fitting the data as a function of the propagation path length. An estimate of \( C \) can be found from the approximate value of \( Q \) determined by equation (17). The error of the estimated \( C \) is expected to effect the absolute values of the parameters only, retaining the relative distribution of \( Q \).

**Inversion technique.** A damped least squares technique is applied to obtain the values of \( Q_i^{-1} \),

\[
Q^{-1} = [G^T G + \theta^2 I]^{-1} G^T d
\]

(20)

where \( \theta^2 \) is a damping factor. In addition to the damping in equation (20), the model parameter vector is smoothed by incorporating the flatness matrix \( D \) as \( \lambda = \varepsilon^2 D \), where

\[
D = \begin{bmatrix}
-1 & 1 & & \\
& -1 & 1 & \\
& & \ddots & \ddots & \\
& \\
& & & \ddots & 1
\end{bmatrix}
\]

(21)

and \( \varepsilon^2 \) is a weighting factor, in the cost function to be minimized.
In order to analyze the the eigenvalues of the problem, The singular Value Decomposition (SVD) has been involved. The method implies a factoring of $G$ in equation (12) into 3 matrices,

$$ G = U \Lambda V^T $$

(22)

where $U$ is an $N \times N$ matrix spanning the data space, $V$ is a $NC \times NC$ matrix spanning the model parameter space and $\Lambda$ is a $N \times NC$ diagonal matrix containing the eigenvalues. A more detailed description of the SVD technique can be found in Menke (1984).

Elimination of $A^0$ and $\tau^0$. Elimination of the unknown values of $A^0$ and $\tau^0$ can be achieved by taking differences between adjacent data values, so that equations (16) and (19) become, respectively,

$$ \log \frac{A'_i}{A_{i-1}} = \frac{-1}{\pi \cdot f} \cdot [ \log \frac{A_i}{A_{i-1}} + \log \frac{L_i}{L_{i-1}} - \log \frac{2 - \cos^2 \gamma_i}{2 - \cos^2 \gamma_{i-1}} ] $$

(23)

and

$$ \tau'_i - \tau_{i-1}' = \frac{\tau_i - \tau_{i-1}}{C} $$

(24)

and the sensitivity matrix becomes

$$
\begin{bmatrix}
T_{11} - T_{11}' & T_{21} - T_{21}' & T_{31} - T_{31}' & \ldots & T_{NC1} - T_{NC1}' \\
T_{12} - T_{12}' & T_{22} - T_{22}' & T_{32} - T_{32}' & \ldots & T_{NC2} - T_{NC2}' \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
T_{1N} - T_{1N}' & T_{2N} - T_{2N}' & T_{3N} - T_{3N}' & \ldots & T_{NCN} - T_{NCN}'
\end{bmatrix}
$$

(25)

where $T_{ij}$ and $T_{ij}'$ denote the travel time of two adjacent rays in the $ij$'th cell.

However, the elimination of the initial parameters might introduce undesired secondary effects in the reconstructed model. From Radon transform theory an artificial layering of the inversion parameters is introduced. This problem is discussed in the appendix.

**MODELLING OF SYNTHETIC DATA**

Synthetic experiments are performed for a model defined by parameters from a typical Gulf of Mexico environment. Consider a vertical plane between 2 wells where the model is gridded into 25 square cells of area $20 \times 20 \text{ ft}^2$. A vertical velocity gradient of 1.5 $\text{sec}^{-1}$ and a constant $Q$-distribution of 100 is modelled outside an attenuating zone in the central block. An anomaly of 10% for the velocity and 50% for the quality factor is specified for the central area of the model. The sources and receivers are located along the left and right boundary of the model. The spacing of both the sources and receivers are 10 ft. For the synthetic experiments shown in this section, the *-symbol and .-symbol have been used to represent the results from the modelling of amplitudes and wavelet widths, respectively.
Figure 1 shows the ray paths for the synthetic model, and Figure 2 is an image of the assumed 'real' distribution of $Q$. Figure 3 shows the RMS residual between the 'real' and reconstructed values of $Q$, for varying levels of noise added to the synthetic data using equations (13), (16) and (19). These reconstructions assumed that $C$, $\tau^0$ and $A^\theta$ were known exactly. The critical noise level for recovering the model parameters is approximately 10% of the variation of the data.

Figure 4 shows the sensitivity of the inversion result to varying amounts of noise added to $A^\theta$ and $\tau^0$. For an error greater than $5 \cdot 10^{-2}$% of the initial parameters, no reliable reconstruction of the model can be expected. This is a very discouraging result.

To overcome the solution sensitivity to $\tau^0$ and $A^\theta$ we use the subtracted data formulation of equations (23) and (24) to invert for $Q$. Figure 5 shows the resulting RMS residual of the parameter distribution for varying noise levels added to the synthetic data. Due to the evaluation of the differences of the data rather than the absolute values, the noise level of the data critical for the reconstruction of the parameters is seen to be slightly lower than the result obtained from the experiment shown in Figure 3.

Figure 6 shows the reconstruction of $Q$ from the amplitudes, where a noise level of 10% of the maximal variation of the data has been added. The reconstruction of $Q$ from noisy data using wavelet widths are similar to the result shown in Figure 6. Moreover, the characteristics of the reconstructed distribution of $Q$ using equations (13), (16) and (19) are similar to the results obtained using the subtracted data formulation. Thus, the artifacts expected from the differencing method as discussed in the appendix are not encountered. Figure 7 shows the eigenvalues of the problem. Both the vertical characteristics of the eigenvectors and the number of near-zero singular values (= number of columns of cells - 1) are expected from the geometry of this crosswell experiment (Calnan, 1989).

INVERSION OF REAL DATA

Acquisition parameters. The $Q$-inversion technique using amplitude and wavelet broadening data is applied to field data from a crosswell experiment in Texas. The well spacing is 600 ft and the shot and receiver spacing are 10 ft. A total of 98 shots were recorded by 96 channels. The uppermost shot and receiver were located at a depth of 30 ft and 10 ft, respectively. A sampling interval and time of .25 ms and 1 sec, respectively, were applied in the survey. No information about hydrophone calibration was available.

Picking technique. Wavelet width and amplitude data were picked by the method described by Olsen (1988a). Amplitudes were extracted as the maximum value among the samples in an interval of length equal to half a wavelength adjacent to the estimated first arrival. A parabolic interpolation was used to increase the precision of the picking. The width of the wavelet was estimated as the zero-crossings of the waveform containing the amplitude picked from the seismogram. A linear interpolation was applied to samples at the zero-crossing.

Zero-offset analysis. Figure 8 shows the 94 seismograms representing the zero-offset traces in the survey. Each seismogram is normalized with the associated maximum amplitude. Figure 9 shows the first arrival times picked from the zero-offset traces. The average velocity distribution in Figure 10 is found by dividing the well-offset distance by the zero-offset travel times. Finally, Figure 11 shows the seismic data of Figure 8, scaled by a constant value for all the samples involved.

Figure 10 indicates a mean velocity gradient of 1.5 sec⁻¹. Velocity anomalies of approximately 500 ft/s seem to be located at depths 500-600 ft and 800 ft. Figure 11
indicates that attenuation could be associated with the two low velocity zones. The
analysis of the sonic log of the survey gives no positive indication of lithologic variation
associated with the expected anomalous zones (Calnan, 1989).

Model geometry. The cell area is estimated from the dominant wavelet frequency and the
source-receiver spacing of the experiment. From the average period of the first arrival
observed in the records and a FFT-analysis, a mean value of the seismic frequency is
found to be 250 Hz, implying a mean wavelength of 25 ft. Hence a cell width of 40 ft
was selected leading to a 15×25 = 375 cell model.

Mispick detection and smoothing of data. In order to carry out the inversion of real
data the standard deviation of the measured amplitude and wavelet widths have to be
reduced to the critical level, estimated from the synthetic experiments. Elimination of
significant outliers is obtained by a median filter that is shown to be efficient for travel
time data (Olsen 1988a). Finally, the data is smoothed by a running mean (box-car) filter.

Figures 12-14 show, respectively, an example of the extracted, median filtered and
smoothed log. A(t) for the shot gather located at a depth of 760 ft. Figures 15-17 represent
a similar process for the wavelet widths picked from the same shot gather.

Direct evaluation of data. Figure 18 shows log. A(t) picked from the crosswell records as
a function of propagation path length. Figure 19 shows the mean of the distribution
shown in Figure 18, calculated at each value of the path length. With the correction with
the path length and take-off angle from equation (17), log. A(t) is found to be 12.6 ± 0.6, and
applying $T_i = L_i/v_i$, $v = 6000$ ft/sec and $f = 250$ Hz in equation (18), $Q$ is found to be 145.

Figure 20 shows the zero-crossing wavelet widths picked from the crosswell records as
a function of the propagation path length. Figure 21 shows the mean values of the
wavelet widths similar to the amplitudes in Figure 20. The intercept at $t = 0$
represents the mean of the initial wavelet widths ($r^0$) and is found to be $1.9 ± 0.4$ ms.
Applying $r^0$, $T_i = L_i/v_i$ and $v = 6000$ ft/sec in equation (8), the value of $C$ is estimated to be
0.2.

Inversion of amplitudes and wavelet widths. As expected from the synthetical modelling and the noise level of the estimated initial parameters no information was extracted from the amplitudes and wavelet widths by the method involving the estimate of the initial parameters. Moreover, inversion of subtracted amplitude data failed to provide meaningful results. It is conjectured that this was due to the lack of hydrophone calibration.

Figure 22 shows the distribution of $Q$ from the inversion of wavelet widths after elimina-
tion of $r^0$ from the system of equations (left), compared to the velocity distribution
determined without the surface data (right) (Calnan, 1989). Figure 23 is a comparison
between the tomograms of quality factors (left) and velocities (right) including 22 shots,
equally distributed along the surface and registered in the right well. As expected,
attenuating zones are generally found in areas of low velocity in the tomograms. The
attenuating zones expected from zero-offset analysis (Figures 9-11) at depths 500-600 ft
and 800 ft are very distinct from the tomograms. However, the layering of the recon-
structions might partially be due to the subtracted data technique and the coupling effect
of the relative constraints. Moreover, an expected attenuating zone associated with the
near-surface low velocity layer was not recovered.
DISCUSSION AND CONCLUSIONS

Two different time-domain properties of the seismic wavelet, amplitude and width, have been evaluated in a linear tomographic inversion for $Q$-values. The equations concerning the amplitudes are well established from empirical experiments. The tomographic inversion using linear expressions for the wavelet broadening represents a new and unexplored technique for crosswell seismology.

Synthetic experiments have been carried out for varying noise level added to the data. The inverse reconstruction of synthetic data is found to be extremely sensitive to the estimated error of the initial amplitude and initial wavelet. For the expected noise level of real data no reliable tomograms are expected to be achieved for the methods dependent on the initial parameters. The estimated error of the frequency in equation (23) and the proportionality factor $C$ in equation (24) is expected to bias the absolute values of $Q_I$ only, leaving the possibility of recovering a reliable relative distribution of the parameters.

In order to eliminate the initial parameters a method using the differences of adjacent data values is developed. From the theory of Radon transform the coupling of adjacent parameters involved in this technique is expected to introduce an artificial layering into the reconstructed model. However, no artifacts from the differencing operation are seen on the synthetic reconstructions. The critical noise level for the reconstruction using the differencing technique is found to be approximately 10% of the maximal variation of the data values. This is slightly lower than the value determined by the method which used the initial parameters.

The inversion methods have been tested on real data. The picking technique used for extracting the amplitude and wavelet width data is proved to be rather accurate and stable for travel time data. However, due to noise and superposition of later arrivals a considerable amount of mispick detection and smoothing was necessary. The unavoidable result of the filtering operations is a biased shaping of the model. Due to the considerable noise level of the data, the relative variation of the parameters had to be tightly constrained. The smoothing and coupling effects of the relative constraints tend to emphasize the horizontally directed eigenvectors as well as smearing potential local anomalous zones.

Preliminary results from the wavelet width inversion on real data show promising results. A general correspondence between the quality factors and the velocity field is evident from the tomograms. However, the significant layered structure of the $Q_I$-distribution might partially be due to the differencing technique and smoothing effects. Moreover, the lack of an expected near-surface attenuating zone is not explained. Unfortunately, no information on the lithology of the target area is available to support the estimates.

The results of the real data experiments suggest that it may not be possible to extract useful attenuation information from the subtracted amplitude inversion. The negative result is mainly due to a considerable sensitivity of the method to the variation of the receiver response and initial amplitudes.

The positive inversion results obtained from this work is based on a linear relation between the wavelet width and the quality factor. Future work will have to support this assumption.
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APPENDIX

It is important to consider how the subtraction of measured periods (equations 23-24) affects the propagation of data errors to model reconstruction errors. It can be demonstrated that subtracting the adjacent periods measured along the z-direction of the well will map slowly varying data noise into false horizontal layers. To show this, note that the Fourier transform of the data, \( t(z, \theta) \), in the \( z \)-direction is given by \( t(k_z, \theta) \), where \( \theta \) is the projection angle of the straight ray data. The projection angle is measured with respect to the vertical axis in the \((x,z)\) domain and with respect to the horizontal axis in the \((k_x,k_z)\) domain (see Figure A1). By the projection-slice theorem (chapter 6 in Deans, 1983), the data and model, \( Q(x,z)^{-1} \) denoted by \( Q'(x,z) \), are related by

\[
t(k_z, \theta) \approx \hat{Q}'( -k_z \tan \theta, k_z ).
\]  
(A1)

If the period data is subtracted at neighboring positions in \( z \), then this is akin to taking the \( z \)-derivative of \( t(z, \theta) \), i.e., \( \partial t(z, \theta) / \partial z \). Under a Fourier transform in \( z \), this spatial derivative transforms into a multiplication of \( t(k_z, \theta) \) by \( ik_z \). Hence, the subtracted data and model in transform space are related by

\[
\hat{Q}'( -k_z \tan \theta, k_z ) = \frac{t(k_z, \theta)}{ik_z}
\]  
(A2)

where \( \hat{Q}'(k_z, \theta) \) represents the transform of the derivative of the period data. Equation (A2) suggests that the model in transform space is reconstructed by a low-pass \( \hat{f}_z \) filtering of the transformed data. This means that data noise at \( k_z \approx 0 \) will transform into model artifacts with Q-gradients that are slowly varying in the vertical direction. Using Radon transform considerations, it can also be shown that smoothing the data \( t(z, \theta) \) by convolving with a narrow window is equivalent to admitting only lower frequency components of the model. In summary, smoothing followed by subtraction of noisy data may tend to 1) limit the ability to reconstruct highly oscillatory Q gradients in the \( z \)-direction and 2) induce false Q layers slowly varying in the \( z \)-direction.
LIST OF CAPTIONS

Figure 1 Ray paths traced in the synthetical model. A vertical gradient of velocity of 1.5 sec\(^{-1}\) is modelled outside a 10% low velocity anomaly in the central block.

Figure 2 Assumed 'real' distribution of \(Q_i\) for the synthetical modelling. A constant value of \(Q_i\) is specified for each block. A 50% low \(Q\)-anomaly is modelled in the central block.

Figure 3 RMS residual between the 'real' and reconstructed values of \(Q_i\) for varying noise level of the data for the inversion technique involving the initial parameters, \(A^0\) and \(\tau^0\). The *-symbol and .-symbol represent the results of amplitude and wavelet width inversion, respectively.

Figure 4 RMS residual between the 'real' and reconstructed values of \(Q_i\) for varying level of noise added to the initial parameters, \(A^0\) and \(\tau^0\). The *-symbol and .-symbol represent the results of amplitude and wavelet width inversion, respectively.

Figure 5 RMS residual between the 'real' and reconstructed values of \(Q_i\) for varying noise level of the data for the inversion technique eliminating the initial parameters, \(A^0\) and \(\tau^0\). The *-symbol and .-symbol represent the results of amplitude and wavelet width inversion, respectively.

Figure 6 Reconstruction of the distribution of \(Q\). A noise level of 10% of the maximal variation of the data has been added.

Figure 7 The eigenvalues of the sensitivity matrix involved in the synthetical modelling.

Figure 8 Zero-offset configuration plot covering the first arrivals. Each seismogram is normalized with the associated maximum amplitude.

Figure 9 First arrivals picked from the zero-offset traces.

Figure 10 Vertical variation of velocity from the zero-offset traces.

Figure 11 Zero-offset configuration plot covering the first arrivals, scaled by a constant value for all the samples involved.

Figure 12 Values of \(\log_a A_i\) extracted from the shot gather located at a depth of 760 ft.

Figure 13 Values of \(\log_a A_i\) from the shot gather located at a depth of 750 ft after median filtering.

Figure 14 Values of \(\log_a A_i\) from the shot gather located at a depth of 760 ft after smoothing.

Figure 15 Values of the wavelet widths extracted from the shot gather located at a depth of 760 ft.

Figure 16 Values of the wavelet widths from the shot gather located at a depth of 760 ft after median filtering.

Figure 17 Values of the wavelet widths from the shot gather located at a depth of 760 ft.
after smoothing.

Figure 18  Values of $\log_2 A_i$ as a function of propagation path length.

Figure 19  The mean of the distribution of $\log_2 A_i$ shown in Figure 18, calculated at each value of the propagation path length.

Figure 20  Wavelet widths as a function of propagation path length.

Figure 21  The mean of the distribution of wavelet widths shown in Figure 20, calculated at each value of the propagation path length.

Figure 22  Distribution of $Q_i$ in the target area from the inversion of the wavelet widths for the crosswell data (left), compared to the velocity distribution involved (right).

Figure 23  Distribution of $Q_i$ in the target area from the inversion of the wavelet widths for the crosswell and surface data (left), compared to the velocity distribution involved (right).

Figure A1  (a) Sources are located at the surface with x-intercept $\Delta x$ and the receivers are located along the horizontal line buried at depth $\Delta z_0$. The z-intercept of the ray is $\Delta z$ and it is assumed that data consists of direct wave travel time measurements. (b) The travel time data with common incidence angle $\theta$ is mapped under a Fourier transform to a slice of the model spectrum.
Fig. 1

Fig. 2
Fig. 3

Fig. 4
Fig. 5

Fig. 6
Fig. 13

Fig. 14
Fig. 15

Fig. 16
Fig. A1
3-Dimensional Modelling of Acoustic and Elastic Wave Propagation on Parallel Computers

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Abstract

We survey our recent experience with using parallel computers for modeling seismic wave propagation in the Salt Lake Basin. The computers under consideration are:

1. Distributed memory transputer array (we experimented with 16 to 96 processor machines, having from 0.25 to 2 Megabytes of memory per node) and

2. The Stellar GS1000 graphics super workstation.

We have implemented a second-order finite difference algorithm with first-order absorbing boundary conditions for the acoustic case and a staggered grid fourth-order finite difference algorithm for the elastic simulations. Our results suggest a linear speed-up with the increase in the number of nodes.
1 Introduction

The overall theme of our research is the application of parallel (distributed) processing to accelerate existing geophysical and computer science research in 3-dimensional forward modeling of the wave equation, geophysical imaging, scientific visualization techniques, and parallel programming.

The goal is to determine and exploit the intrinsic parallelism of these problems and to obtain algorithms appropriate for rapid interactive, visually supported solutions with programs efficiently running on a low cost parallel architecture computer. The primary significance of this research is that it establishes the foundation for implementing geophysical software on parallel computers. As a prototype example we compute the 3-dimensional acoustic and elastic response of the Salt Lake basin and examine resonance as a function of 3-D source directivity and incidence angle. This research leads to new insights into the characteristics of basin resonance; this may someday mitigate the major loss of life and property resulting from a large earthquake.

In Section 2 we justify using parallel computers for geophysical simulation; in Section 3 we summarize our experience with the acoustic simulations; in Section 4 we discuss the results from elastic simulations.

2 Why Parallel Computation?

Seismic wave simulation and geophysical imaging require significant amounts (0.5 - 1 Gbyte) of RAM memory to be efficiently implemented on a computer. Very fast computational speeds (1 Gflop) and interactive graphics capabilities are essential for solving real size problems. These requirements basically eliminate remotely located supercomputers (e.g., the San Diego Cray) as possible sources of computation for real size problems. Therefore, our attention turned to non-expensive parallel computers with distributed memory (transputer arrays) and graphics super-workstations as possible computational engines. It turns out that problems involving the solution of PDE equations (like the wave equation) enjoy excellent parallel subdivision easily implementable on a distributed memory machine.

Such problems can be distributed in the memory of several nodes by decomposing the 3-dimensional domain on which we solve the problem. This indicates that a distributed machine is very well suited to this type of problem. Moreover, the transputer array is one of the best computers in terms of performance/cost ratio, which is about 1.5 Mflop/$1,000 (with 1 Megabyte of RAM).
The transputer based machines are scalable and very flexible architectures. We can change the topology of connections in software and hardware, thus creating a configuration required by the specific needs of problems being solved.

3 3-Dimensional Simulation of the Acoustic Wave Equation

For this simulation we used a transputer based machine connected as a linear array (see Fig. 1).

We have been testing a second-order finite difference algorithm on 16, 24, 48 and 96 processor arrays, obtaining linear speed-up as the number of processors increased. For example, each time step takes 1.01 seconds on a 24 array, 0.53 second on a 48 array and 0.28 on a 96 array. The acoustic wave equation is given by

\[ \nabla^2 P - \frac{1}{C^2} \frac{\partial^2}{\partial t^2} P = F, \]

where \( P \) is the sound pressure, \( C \) is the velocity model, and \( F \) is the source function.

By using a 96 x 96 x 40 \((x, y, z)\) discretization of the domain and substituting symmetric second-order finite difference approximations for second derivatives a time marching (explicit) algorithm is obtained:

\[ P_{i,j,k,t+1} := (\Delta t)^2 C^2 \left[ D^2 P_{i,j,k,t} - F_{i,j,k,t} \right] + 2P_{i,j,k,t} - P_{i,j,k,t}, \]  

where \( D^2 P_{ijk} \) is the discrete approximation to \( \nabla^2 P \), and \( P_{ijk} \) is an approximation to \( P(i\Delta x, j\Delta y, k\Delta z, t\Delta t) \), \( \Delta x, \Delta y, \Delta z, \Delta t \) being the discretization intervals.

The absorbing boundary conditions were approximated using first-order paraxial approximations to the wave equation. Equation 1 implies that two 3-dimensional arrays are needed to implement this algorithm.

In our parallel decomposition we have uniformly divided the domain along the \( x \)-direction into the number of rectangular slices equal to the number of processors (Fig. 2).

After completing each time step of the computation, the consecutive processors (neighbors) exchange currently computed boundary values (on their assigned subdomains). The computation in the next time step then continues.
During execution the cumulative energy of the pressure near the surface of our basin is summed. This has been accomplished by transferring several surface pictures from each node to the server (PC-AT) machine. Graphical interpretation is then obtained by using the PC graphics board. The cumulative energy on the surface is illustrated for the plane wave incoming from the south (Provo) in Fig. 3a and for the plane wave incoming from the west (Nevada) in Fig. 3b.

This experiment exhibits a very different basin response pattern compared to waves incoming from orthogonal directions. In both cases, however, most of the energy (red spots) is concentrated near the Wasatch Fault and in the area near the University of Utah. Blue colors indicate relative low seismic energy accumulation.

Several open research problems remain to be solved:

1. Which is a more practical parallel algorithm, second-order or fourth order correct finite differencing scheme? Is a second-order wave equation more convenient than two first-order coupled equations?

2. What is the optimal topology of the nodes? In our experiment we used a linear array interconnection. Another natural way of subdividing the domain would be to slice each of the $x$ and $y$ or each of $x$, $y$ and $z$ directions. This would lead to a 2-dimensional or a 3-dimensional mesh of processors. The amount of transferred data would definitely be smaller; however, more smaller messages would have to be sent, and the complexity of software would significantly increase.

3. Investigate parallel staggered grid methods, which have been implemented on the Stellar machine (see section 4). Here we give an estimate of predicted performance of a 512 node transputer array for the fourth-order staggered grid method in elastic simulation. For a 2km x 2km x 2km basin with a 0.1 km distance between nodes (corresponding to the frequency $f = 30$Hz and $V_p$ velocity 1.5 km/sec) we obtain 200 x 200 x 200 grid size. This model requires 576 megabytes of memory (512 nodes each with 2 Megabytes of memory would be sufficient). We have estimated that approximately 200 floating operations per grid point per unknown are needed. This adds to about $3 \times 10^{10}$ floating operations per time step. A typical simulation requires about 1000 time steps, which would require a total of $3 \times 10^{13}$ floating point operations. A 512 node machine (about 1 Gflop performance) would take approximately $3 \times 10^4$ seconds ($\approx 8.3$ hours) to complete this computation. We have recently applied to NSF to build a 512 node transputer machine at the University of Utah.
4 Three-dimensional Elastic Wave Simulation

A staggered grid finite difference scheme, fourth-order in space and second-order in time, is used to model the three-dimensional elastic response of the Salt Lake basin. This method solves for both velocity and stress at each grid point using two coupled first-order equations. This is to be compared to the previous second-order differencing scheme which solves for displacement at each grid point using the second-order wave equation. The main advantage of the staggered grid method is that it is stable over a wide range of Poisson's ratio and fault sources are easily implemented; the disadvantage is that it requires about 1.5 times more physical memory than an equivalent displacement scheme. However, the staggered grid scheme uses first-order partial differential equations so that the width of the spatial differencing star is less than that for an equivalent scheme based on second-order partial differential equations. For parallel computers it is important to keep the spatial differencing star as compact as possible so that the swapping of data between adjacent CPU nodes is minimized. The greater the overlap of differencing stars between adjacent CPU nodes, the greater the memory requirement at each CPU node.

The objective of the three-dimensional modeling exercise is to assess the three-dimensional nature of resonance and focusing in a basin. As a preliminary result, three-dimensional elastic simulations are performed for a point double-couple source located in the southwestern part of the basin. The basin model is correct in depth but is about 1/3 the areal extent of the actual Salt Lake Basin.

The basin model, depicted in Figure 2, is gridded into a 110 x 110 x 40 point model, i.e., a model 17km x 17km in areal extent and 6 km in depth. A fourth-order differencing scheme is used to discretize the model at five nodes/wavelength, where the minimum shear velocity (1.1 km/s) and maximum source frequency (1.1 Hz) is used to compute the node spacing of 0.15 km. The source is a Ricker wavelet with a center frequency of 0.7 Hz, the compressional wave velocities are approximately those given in Figure 2, and the $V_s/V_p$ ratio is taken to be 0.6. Although the actual basin model is about 40km x 35km x 3km in dimension, this scaled down basin can serve as a pilot model to understand some of the important basin features which control seismic amplification.

As an example, Figures 4 and 5 depict the results from simulating a dip-slip fault (represented by a double couple point source) rupturing in the southwest part of the basin model at a depth of 3 km. These simulations are an attempt to model earthquakes which are observed to erupt along the western part of the basin; such events have the potential for inducing strong ground shaking within the valley.

Figure 4 depicts the cumulative seismic energy for the horizontal particle velocity along the valley floor. The total seismic energy at the elapsed time of 1.5 seconds and 30 seconds
is given, respectively, in Figures a and b. These figures indicate that the low frequency seismic energy accumulates in the deeper part of the basin. Figure 5 depicts snapshots of the wave propagation for an E-W cross-section in the middle of the basin, which viewed in a rapid movie like manner illuminates the important physics of the wave propagation. Figure 6 depicts the z-component seismograms for a line of surface receivers trending E-W and centered in the middle of the basin. It is apparent that there is a dominant phase reflecting from the fault surface and traveling from East to West. It is interesting to note that simulations for a source located just south of the basin (Provo earthquake) produced a cumulative energy pattern (not shown) quite different than that depicted in Figure 4. In this case, most of the energy accumulated in the northern, not the eastern, part of the basin.

The elastic finite difference results were computed on a GS1000 Stellar vector computer, consisting of 4 processors and 32 Mbytes of memory. About 4 CPU seconds were required for each time step, and the program memory required over 40 Mbytes. Only the vector compiler was invoked since the parallel compiler did not seem to produce a noticeable speed improvement. Perhaps with the judicious use of directives, the parallelizer could be optimized to increase the speed. For a two-dimensional finite difference code, the Stellar achieved speeds between 16-20 Mflops.
FIGURE 1.

FIGURE 2. Salt Lake Basin model
Subdivision into 16 slices.

C1 = 2.0 km / sec
C2 = 2.3 km / sec
C3 = 5.2 km / sec
C4 = 6.0 km / sec
Figure 4  The cumulative horizontal seismic energy on the valley floor of the Figure 2 Model for a double couple point source located in the southwest portion of the basin at a depth of 2.25 km. The left portion depicts the energy distribution at the time of 1.5 seconds and the right depicts energy distribution at 30 seconds. The top (bottom) of each figure is west (east); red (green) color indicates greatest (least) seismic energy.
Figure 5. Vertical component snapshots for E-W cross-sections centered in the middle of the basin model. Time increases from left to right and from top to bottom, with about 1.5 seconds of time between snapshots. The upper leftmost snapshot is at the time of 1.5 seconds from the initiation of the earthquake.
Figure 6. Vertical component seismograms measured on the free-surface for a W-E seismic line centered in the middle of the Figure 2 basin model.
Static Corrections Using
Surface Wave Inversion

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ABSTRACT

The purpose of this work is to examine the suitability of using surface wave information to provide independent constraints on near surface velocities. Several approaches to surface wave inversion are examined; 1) sequential Rayleigh wave inversion i.e. inversion for S-wave velocities followed by inversion for P-wave velocities repeated iteratively, 2) simultaneous S- and P-wave inversion of Rayleigh waves with 2 different sized data sets, and 3) repeated Love wave inversion for the S-wave velocity model which is then used for Rayleigh wave inversion for the P-wave velocity model. Preliminary results show that S-wave velocity models from Love wave inversion appear to be much more reliable than those from Rayleigh wave inversion. Density recovery appears impractical using surface wave inversion. Synthetic data inversion seems to indicate that a combination of both Love and Rayleigh wave inversions will provide the most favorable convergence. This is especially true when the S-wave velocities first recovered in the Love wave inversion are applied to the Rayleigh wave inversion for P-wave formation velocities.
INTRODUCTION

In many geophysical data sets, poor delineation of near surface velocities often cause a variety of processing problems including data busts, cycle skips, and false structures. Quite often, residual statics, refraction statics, and hand statics fail to result in a seismic section that will satisfy everyone. It is therefore desirable to develop an improved method which could provide independent constraints on the velocity structure above the first good reflector would be welcomed. If this new procedure could be carried out using the existing data, all the better; but it would still be of great value if the Geophysicist and Geologist could return to the field long after the original shooting and, in a few days using portable seismic equipment, recover reasonable estimates of the velocity structure across troublesome geology.

It may be possible to estimate S- and P-wave velocities and gradients beneath and between seismic data shot holes by studying the behavior of the surface waves passing through the formations of interest. By inverting Love and Rayleigh wave dispersion curves recorded on closely spaced seismic traces, one could, in principle, recover the velocity distribution which caused the dispersion curves. Since the Love and Rayleigh waves only penetrate the surface to a depth of several wavelengths, one can assume that any changes in their character at any given frequency can be attributed to near surface velocity structures within a depth of several wavelengths. In no way do we mean to suggest that the resulting velocity model will be as accurate as one could measure in a shot hole, but we do believe that the resulting model could be useful in estimating velocity changes in areas which were not sampled with uphole times.

There are several advantages inherent to this technique. First, the fact that surface waves penetrate the earth to a depth of one to two wavelengths allows for modelling to depths much greater than those measured with uphole timings. Second, since the input information comes from adjacent geophone stations, the subsurface sampling rate can be far more dense than that measured only at the shot holes. Thirdly, the resulting velocity model may reveal detail in areas of complex layered geology such as seen in areas with lava flows, clinker beds, or other anomalous layered formations. And finally, the results are completely independent of other statics methods such as uphole and refraction, thus providing an additional source of information about the critical shallow velocities.

The main disadvantages of this method are that it is pseudo 1-D, although it can be developed for 2-D inversion; and that it may also require much effort to extract phase velocities from the CDP data.

SURFACE WAVES

Figures 1 and 2 show several examples of shot records with strong surface waves. Figure 1 has two real data examples. The first, from a program
shot by the department in the summer of 1988 at Hill A.F.B., and the second, from a program shot by Barrows (1987) in Nevada. Station spacing for the Hill and Nevada data are 1 meter and 10 feet respectively. Figure 2 is a vertical component synthetic shot record also exhibiting strong Rayleigh waves.

The synthetic was generated with a reflectivity program running on a Cray XMP supercomputer in San Diego. Particle motion in Rayleigh waves is elliptical and retrograde. The waves propagate at approximately 0.9 times that of the shear wave velocity. Although Rayleigh wave velocities are strongly associated with the S-wave velocities in a given media, there is also a dependence upon the P-wave velocity. Love waves are transverse shear waves and apparent only on the SH component of the geophone. There is no relationship between Love wave and P-wave velocities.

THEORY AND APPLICATION

We intend to extract Rayleigh waves from closely spaced vertical component geophones, and Love waves from similarly spaced SH component geophones. Transforming the surface waves into the frequency domain, one can then determine the phase velocity of each frequency in the window of interest by examining the phase delay at that frequency. The resulting function of phase velocity vs. frequency will form the dispersion curve data input into the inversion process. Starting with a model of a constant velocity gradient, and the dispersion curve from the data, the program will then iterate toward the best fit of the data by varying the model parameters and forward modelling using new parameters. Figure 3 depicts the procedure in a flow chart.

Theory of Surface Wave Inversion

For an N layer model, we wish to invert for individual layer velocities

$$V = (V_1, V_2, V_3, \ldots, V_N)^T$$

from the measured phase velocity data $C(\omega)$, where $\omega$ is the angular frequency. The first step is to expand $C(\omega)$ in a Taylor Series about an initial model $V_0$:

$$C(\omega, V) = C(\omega, V_0) + \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \cdot (V_i - V_i) + \text{higher orders of } V_i - V_i$$

or

$$C(\omega, V) - C(\omega, V_0) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \cdot \Delta V_i + \cdots$$
or

\[ \Delta C (\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \cdot \Delta V_i + \cdots \]

Steps:

Step 2: To simplify the calculations, we want to linearize (1) by assuming that the initial guess is close enough to the actual model so that the higher orders of \( \Delta V_i \) approach 0. Therefore:

\[ \Delta C (\omega, V) = \sum_{i=1}^{N} \frac{\partial C}{\partial V_i} \cdot \Delta V_i \]

Step 3: Now, from the linear equation (2), we have \( N \) unknown values of velocity \((V_1, V_2, V_3, \cdots, V_N)^T = V\). \( V \) is determined from the \( M \) data samples.

\[ \Delta C (\omega_1, V) = \sum_{i=1}^{N} \frac{\partial C (\omega_1, V)}{\partial V_i} \cdot \Delta V_i \]

\[ \Delta C (\omega_2, V) = \sum_{i=1}^{N} \frac{\partial C (\omega_2, V)}{\partial V_i} \cdot \Delta V_i \]

\[ \Delta C (\omega_3, V) = \sum_{i=1}^{N} \frac{\partial C (\omega_3, V)}{\partial V_i} \cdot \Delta V_i \]

\[ \vdots \]

\[ \Delta C (\omega_m, V) = \sum_{i=1}^{N} \frac{\partial C (\omega_m, V)}{\partial V_i} \cdot \Delta V_i \]

(3) \[ \Delta \vec{C} = G \Delta \vec{V} \]

where \( \Delta \vec{C} \) is the column vector of the data, \( \Delta \vec{V} \) is the column vector of the model parameters, and \( G \) is an \( M \times N \) matrix where:

\[ G_{ik} = \frac{\partial C (\omega_i, V)}{\partial V_k} \]
Step 4: Find $\Delta \vec{V} = (G^T G)^{-1} G^T \Delta \vec{C}$.

Step 5: Update the velocity model by $\Delta \vec{V}$ and iterate.

To determine $G_{ik}$, one perturbs the $k^{th}$ element of $\vec{V}$, recalculates the dispersion curve, computes the difference from the original dispersion curve, and divides the result by the perturbation.

It is advisable to normalize the columns of $G$ prior to the inversion to avoid computational round off errors. The original values can be recovered after the inversion by multiplying by the row and column normalization matrices.

DISPERSION CURVES

Two Layer Model

Figure 4a represents the phase velocity of a Love wave as a function of frequency. These curves were generated using a simple 2 layer model of crustal scale using an algorithm described in Schwab and Knopoff (1972). The shallow layer has an S-wave velocity of 3.5 km/sec., a P-wave velocity of 6.1 km/sec., and a density of 2.7 gm/cc. The bottom layer has an S-wave velocity of 4.5 km/sec., a P-wave velocity of 8.1 km/sec., and a density of 3.36 gm/cc. The depth of the interface is at 35 km. The longest curve is that of the fundamental mode, while subsequent curves to the upper right represent the higher modes.

As one would expect, when the frequency approaches 0 Hz. and the wavelength increases, the Love wave is influenced by deeper and deeper formations resulting in a phase velocity approaching that of the deeper layer. As the frequency increases, the deeper layer becomes invisible, and the phase velocity approaches that of the shallow layer S-waves.

Figure 4b uses an identical model calculated for Rayleigh waves. The most significant difference is that the dispersion curves asymptotically approaches the S-wave velocity of the shallow layer times 0.9. The Love wave computation is a much simpler and straightforward procedure resulting in more logical and predictable behavior.

The next sets of figures show what happens to the dispersion curves when the defining parameters are varied. They clearly demonstrate which geologic variables are significant and in general, for which frequencies they are most significant. Each parameter in the two layer demonstration model above has been varied by plus and minus 20%.

Figure 5 shows the effect of variation in the S-wave velocity for the upper layer. The substantial separation of the dispersion curves demonstrates that this is a significant variable. The lower most curve in each mode represents a velocity of 2.8 km/sec., while the upper most curve of each
mode represents a velocity of 4.2 km/sec., with six intermediate values between. The Love wave graph shows that at the lowest frequencies, a variation in the shallow layer can not be distinguished, but the Rayleigh wave dispersion curves do diverge and can give some additional information. At the higher frequencies, or shorter wavelengths, variations in shallow velocities are clearly distinguished.

The model in Figure 6 incorporates the +/- 20% shear wave velocity variation in the deeper layer. It is apparent that nearly all resulting divergence appears in the lower frequencies of each mode as one would expect. S-wave velocity again demonstrates a significant contribution.

Figure 7 is associated with variation in the shallow layer’s P-wave velocity. Since the Love wave is a shear wave, one would not expect any variation in its dispersion curve, and none is exhibited. The variation seen in the fundamental mode of the Rayleigh curve implies that P-wave velocities can be modeled and measured to some extent with surface waves.

Figure 8 is associated with a +/- 20% variation in the P-wave velocity of the deeper layer while the upper layer remains constant. Once again, the Love wave dispersion curves show no effect. The Rayleigh wave curves show only a slight deviation mostly at the low end of the fundamental. The last two examples demonstrate that most of the P-wave information will be contained in the fundamental mode of the Rayleigh wave, and that this is the place to look if one is interested in inverting for P-wave as well as for S-wave velocities.

The next two figures demonstrate the minimal significance of even a +/- 20% variation in density. Figure 9 has a shallow density range of 2.16 to 3.24 gm/cc., and Figure 10 has a deep density range of 2.69 to 4.03 gm/cc. Since variations of this magnitude would not be observed in our work, and since there is such a small contribution even for these excessive values, we shall not consider modelling for densities.

Velocity Gradient Models

Figures 11-16 represent three 10 layer models each with a total depth of 100 meters. They are intended to show the variation in the dispersion curves as a result of varying the character of the velocity gradient, but not the endpoint velocities. The S-wave velocities are plotted on the left, the P-wave on the right. The minimum and maximum S-wave velocities were taken from Suyama, et. al. (1984) for Alluvial and Tertiary deposits and represent a conservative ratio in their magnitudes. The endpoint P-wave velocities represent a conservative range taken from uphole timings in the western Williston Basin. In the results, one can see a significant and encouraging difference between the different gradient models.
SURFACE WAVE EXTRACTION

A number of phase velocity extraction methods have been published and are available to us. P-K filtering may be a feasible candidate, since it holds the promise of separating some of the P-wave and air blast noise from the surface wave data.

SYNTHETIC INVERSION RESULTS

Figure 17 displays the convergence results of 4 different inversion techniques. The graphed values represent the RMS error between the input synthetic data and forward modelled data using the inverted velocity results. Figures 18 and 19 list the intermediate layer velocity results of each technique. The layer thicknesses used in the inversion from top to bottom are: 10 m., 10 m., 10 m., 20 m., 20 m., and 30 m. The starting model velocities for each of the 6 layers are listed on the left hand side of the table, the desired results on the right. The first 6 values represent S-wave velocities while the next 6 represent the P-wave velocities.

Technique 1 uses 10 dispersion curve data points for the observed data. The frequency range sampled is 0 - 16 Hz. The procedure first inverts the Rayleigh waves for the S-wave velocities which are then input into the Rayleigh wave inversion for the P-wave velocities. The process is then repeated. The intermediate results show poor convergence generally, and in some cases, divergence.

Technique 2 uses the same Rayleigh model, but inverts for all 12 velocities simultaneously. The general convergence is quicker, but many layer velocities are still diverging from the desired results. Technique 3 is similar to Technique 2 except that twice as many data points covering the same frequency window are used. The simultaneous inversion now converges much quicker for many of the model velocities, especially for the S-wave values. Some layer velocities, however, are still wandering.

Inversion Technique 4 also uses 20 dispersion curve data points, but uses Love wave inversion to first recover the S-wave values which are then held constant in the Rayleigh wave inversion for the P-wave values. This technique is the only one tried so far that has had such a positive effect on P-wave convergence rates. Further iterations would almost certainly result in their convergence.
LIMITATIONS

It is hoped that the technique can be applied directly to CDP data. Problems may arise in two areas. First, the use of geophone arrays may selectively attenuate desired frequencies within the data window. Since the attenuation of such strong signals may be incomplete, there could still be enough energy from all important frequencies to allow for inversion.

Second, the frequencies of interest may straddle the dominant frequencies of the geophone, or may lie completely below the dominant frequency. In this case, there may be no useful data for the geologic formations of interest. If the technique can be shown to be useful outside geophone and array limitations, then one may want to consider geophone frequencies and array patterns when setting up a program so as to take advantage of the possible use of this technique. Later localized acquisition using small crews and portable equipment could easily overcome both of these limitations.

DISCUSSION

Surface wave inversion holds the promise of recovering near surface velocities from Love and Rayleigh waves. Preliminary inversion of synthetic data suggests that the simultaneous inversion of Love and Rayleigh waves will provide the most accurate reconstruction of S- and P-wave velocities. Additionally, the inversion of Love waves for S-wave velocities which can then be used to constrain the Rayleigh wave inversion for P-wave velocities appears to be quite promising. Inclusion of higher mode data, when available, could also aid in the inversion process.

Future work will include further synthetic tests, development of phase velocity extraction algorithms, and application of the technique to real data.
REFERENCES


FIGURES

Figure 1. Rayleigh wave data examples. First example from recent department acquisition at Hill A.F.B. Second example from E.P.A. shallow seismic survey by Barrows, 1987.

Figure 2. Synthetic seismic data generated with reflectivity program showing strong Rayleigh waves.

Figure 3. Flow chart for inversion process.

Figures 4 - 10 based on two layer model with interface depth at 35 km.

Figure 4. Love and Rayleigh wave Dispersion Curves. Layer 1 S-wave velocity: 3.5 km/sec, P-wave velocity: 8.1 km/sec, density: 2.70 g/cc. Layer 2 S-wave velocity: 4.5 km/sec, P-wave velocity: 8.1 km/sec, density: 3.36 g/cc.

Figure 5. Same model as in Figure 4 calculated through a variation in the shallow layer S-wave velocity of + and - 20%.

Figure 6. Variation in this case is + and - 20% of the Layer 2 S-wave velocity.

Figure 7. Same model calculated with a + and - 20% variation in the shallow layer P-wave velocity.

Figure 8. Variation in this case is + and - 20% of the Layer 2 P-wave velocity.

Figure 9. Density variation of + and - 20% in the shallow layer density.

Figure 10. Density variation of + and - 20% in the deeper layer’s density.

Figures 11, 12. Ten layer model with linear velocity gradient. End point velocities fixed using measured S-wave values from Suyama et. al. (1984), and P wave values from western Williston Basin uphole times.

Figures 13, 14. Same model using a different velocity gradient having its greatest rate of change deep in the section.
Figures 15, 16. Same model with the highest velocity gradient in the shallow layers.

Figure 17. Convergence rates and model layer velocities for 4 sample inversion runs.

Figure 18. Table of Inversion Techniques 1 - 3. Technique 1 alternates inversions for S-wave velocities with inversions for P-wave model velocities using Rayleigh waves and 10 dispersion curve data points. Technique 2 simultaneously inverts Rayleigh waves for S- and P-wave model velocities. Technique 3 is similar to 2 except that 20 dispersion curve data points covering the same frequency window are used.

Figure 19. Table of Inversion Technique 4. Similar to Technique 3 in the number and range of data points, this technique uses Love wave inversion to reconstruct the S-wave model which is then used in the Rayleigh wave inversion for the P-wave model.
40 Hertz Phones, Filter Out

P Wave First Arrivals

Fundamental Mode

trace number

0 20 40 60 80 100 120 140 160 180 200
time in milliseconds

Figure 1
10 Layer Synthetic Seismogram Model

\[
\begin{align*}
V_{s1} &= 0.200 \text{ km/sec} & V_{s10} &= 0.500 \text{ km/sec} \\
V_{p1} &= 0.990 \text{ km/sec} & V_{p10} &= 1.980 \text{ km/sec} \\
\rho &= 2.45 \text{ gm/cc} & Z_{10} &= 0.333 \text{ km}
\end{align*}
\]

Figure 2
DETERMINING VELOCITY STRUCTURE USING SURFACE WAVE INVERSION

Extract Surface Waves From Shot Records

Fourier Transform 2 Adjacent Traces

Calculate Dispersion Curve (Phase Velocity vs. Frequency) From Phase Delay vs. Frequency

Pick Initial Model $V_0$

Calculate Dispersion Curve $C(f, V)$

Calculate Difference Between Measured and Modelled Dispersion Curves

$\Delta C(f) = DATA(f) - C(f)$

$\Delta C = G \ast \Delta V$

$G_{ik} = \frac{\partial C(f_i, V)}{\partial V_k}$

$\Delta V = (G^T G)^{-1} G^T \Delta C$

Add $\Delta V$ To $V$

Is $\Delta V <$ Threshold

No

Yes

Output Velocity Model

Figure 3
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model

 +/- 20% Variation in Vs2

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure 6
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model

+/- 20% Variation in Vp1

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure 7
LOVE WAVE

Rayleigh Wave

Omega (radians/sec)

Phase Velocity (km/sec)

2 Layer Model

+/- 20% Variation in Vp2

Figure 8
LOVE WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

2 Layer Model

+/- 20% Variation in rho1

RAYLEIGH WAVE

Phase Velocity (km/sec)

Omega (radians/sec)

Figure 9
LOVE WAVE

2 Layer Model

+/− 20% Variation in rho2

Figure 10
10 Layer Gradient Model

\[ \begin{align*}
V_{s1} &= 0.200 \text{ km/sec} & V_{s10} &= 0.500 \text{ km/sec} \\
V_{p1} &= 0.990 \text{ km/sec} & V_{p10} &= 1.980 \text{ km/sec} \\
\rho &= 2.45 \text{ gm/cc}
\end{align*} \]
Figure 12
10 Layer Gradient Model

\[ V_{s1} = 0.200 \text{ km/sec} \quad V_{s10} = 0.500 \text{ km/sec} \]
\[ V_{p1} = 0.990 \text{ km/sec} \quad V_{p10} = 1.980 \text{ km/sec} \]
\[ \rho = 2.45 \text{ gm/cc} \]
10 Layer Gradient Model

$V_{s1} = 0.200 \text{ km/sec}$  $V_{s10} = 0.500 \text{ km/sec}$
$V_{p1} = 0.990 \text{ km/sec}$  $V_{p10} = 1.980 \text{ km/sec}$
$\rho = 2.45 \text{ gm/cc}$

Figure 15
Figure 16
Inversion Technique:

1. Alternating S and P-wave inversion using Rayleigh waves and 10 dispersion curve data points.
2. Simultaneous S and P-wave inversion using Rayleigh waves and 10 dispersion curve data points.
4. S-wave velocity inversion using Love waves used in subsequent Rayleigh wave inversion for P-wave velocity model, 20 dispersion curve data points.
### Inversion Technique 1

<table>
<thead>
<tr>
<th>Start (km/sec)</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Dispersion curve data points, Alternating solution for S and P Wave Velocities using Rayleigh Wave:</td>
<td></td>
</tr>
<tr>
<td>0.350 0.300 .... 0.260 .... 0.256 .... 0.250 .... 0.247 .... 0.200</td>
<td></td>
</tr>
<tr>
<td>0.350 0.298 .... 0.378 .... 0.334 .... 0.318 .... 0.319 .... 0.233</td>
<td></td>
</tr>
<tr>
<td>0.350 0.348 .... 0.391 .... 0.398 .... 0.405 .... 0.395 .... 0.267</td>
<td></td>
</tr>
<tr>
<td>0.350 0.393 .... 0.351 .... 0.397 .... 0.425 .... 0.438 .... 0.333</td>
<td></td>
</tr>
<tr>
<td>0.350 0.386 .... 0.343 .... 0.363 .... 0.388 .... 0.413 .... 0.400</td>
<td></td>
</tr>
<tr>
<td>0.350 0.484 .... 0.501 .... 0.488 .... 0.489 .... 0.489 .... 0.500</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.460 .... 1.399 .... 1.394 .... 1.379 .... 1.372 .... 0.970</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.580 .... 1.667 .... 1.625 .... 1.650 .... 1.666 .... 1.91</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.543 .... 1.543 .... 1.550 .... 1.567 .... 1.581 .... 1.92</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.468 .... 1.442 .... 1.487 .... 1.476 .... 1.467 .... 1.414</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.457 .... 1.436 .... 1.457 .... 1.444 .... 1.431 .... 1.536</td>
<td></td>
</tr>
<tr>
<td>1.500 .... 1.517 .... 1.512 .... 1.503 .... 1.513 .... 1.525 .... 1.970</td>
<td></td>
</tr>
</tbody>
</table>

Data RMS Error: 0.786 0.346 0.336 0.278 0.269 0.227 0.221 0.157 0.151 0.125 \( \times 10^{-1} \)

### Inversion Technique 2

<table>
<thead>
<tr>
<th>Start (km/sec)</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Dispersion curve data points, Concurrent solution for S and P Wave Velocities using Rayleigh Wave:</td>
<td></td>
</tr>
<tr>
<td>0.350 0.322 0.279 0.241 0.234 0.240 0.248 0.240 0.246 0.239 .... 0.200</td>
<td></td>
</tr>
<tr>
<td>0.350 0.307 0.304 0.322 0.326 0.334 0.309 0.314 0.330 0.335 .... 0.233</td>
<td></td>
</tr>
<tr>
<td>0.350 0.333 0.368 0.374 0.361 0.357 0.377 0.366 0.369 0.365 .... 0.267</td>
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</tr>
<tr>
<td>0.350 0.392 0.416 0.419 0.430 0.431 0.405 0.409 0.399 0.407 0.413 .... 0.333</td>
<td></td>
</tr>
<tr>
<td>0.350 0.395 0.392 0.445 0.423 0.501 0.535 0.546 0.568 0.583 .... 0.400</td>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>1.500 1.478 1.471 1.475 1.490 1.495 1.511 1.523 1.523 1.528 .... 0.970</td>
<td></td>
</tr>
<tr>
<td>1.500 1.525 1.546 1.549 1.551 1.552 1.565 1.577 1.573 1.573 .... 1.081</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>1.500 1.481 1.478 1.462 1.443 1.419 1.389 1.355 1.331 1.308 .... 1.536</td>
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</tr>
<tr>
<td>1.500 1.571 1.610 1.631 1.647 1.661 1.674 1.692 1.710 1.725 .... 1.970</td>
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</tr>
</tbody>
</table>

Data RMS Error: 0.786 0.471 0.279 0.236 0.200 0.174 0.143 0.119 0.091 \( \times 10^{-1} \)

### Inversion Technique 3

<table>
<thead>
<tr>
<th>Start (km/sec)</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 Dispersion curve data points, Concurrent solution for S and P Wave Velocities using Rayleigh Wave:</td>
<td></td>
</tr>
<tr>
<td>0.350 0.311 0.282 0.250 0.229 0.216 0.209 0.205 0.204 0.204 .... 0.200</td>
<td></td>
</tr>
<tr>
<td>0.350 0.304 0.268 0.248 0.237 0.229 0.225 0.224 0.223 0.221 0.221 .... 0.233</td>
<td></td>
</tr>
<tr>
<td>0.350 0.321 0.306 0.304 0.302 0.300 0.297 0.295 0.292 0.291 0.290 .... 0.267</td>
<td></td>
</tr>
<tr>
<td>0.350 0.338 0.341 0.341 0.339 0.339 0.337 0.334 0.332 0.332 0.333 .... 0.333</td>
<td></td>
</tr>
<tr>
<td>0.350 0.348 0.355 0.356 0.357 0.358 0.360 0.360 0.361 0.362 0.363 .... 0.400</td>
<td></td>
</tr>
<tr>
<td>0.350 0.422 0.462 0.482 0.491 0.491 0.497 0.498 0.498 0.499 0.500 0.500 .... 0.500</td>
<td></td>
</tr>
<tr>
<td>1.500 1.452 1.421 1.396 1.380 1.371 1.366 1.364 1.362 1.362 1.361 .... 0.970</td>
<td></td>
</tr>
<tr>
<td>1.500 1.503 1.512 1.515 1.516 1.517 1.516 1.515 1.515 1.515 1.516 .... 1.081</td>
<td></td>
</tr>
<tr>
<td>1.500 1.517 1.528 1.534 1.538 1.541 1.541 1.544 1.545 1.547 1.549 1.551 .... 1.192</td>
<td></td>
</tr>
<tr>
<td>1.500 1.523 1.534 1.543 1.551 1.556 1.551 1.565 1.569 1.572 1.575 .... 1.414</td>
<td></td>
</tr>
<tr>
<td>1.500 1.522 1.533 1.542 1.551 1.556 1.561 1.566 1.569 1.573 1.575 .... 1.636</td>
<td></td>
</tr>
<tr>
<td>1.500 1.566 1.600 1.614 1.619 1.622 1.622 1.622 1.622 1.622 1.621 1.620 1.619 .... 1.970</td>
<td></td>
</tr>
</tbody>
</table>

Data RMS Error: 1.160 0.804 0.544 0.339 0.197 0.109 0.064 0.044 0.036 0.032 \( \times 10^{-1} \)
### Inversion Technique 4

20 Dispersion curve data points. Solution for S Wave velocity using Love Waves followed by solution for P Wave Velocities using Rayleigh Wave:

<table>
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<th>Start (km/sec)</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>0.350</td>
<td>0.250</td>
<td>0.214</td>
<td>0.203</td>
<td>0.201</td>
<td>0.200</td>
<td>0.233</td>
<td>0.275</td>
<td>0.287</td>
<td>0.333</td>
<td>0.400</td>
<td>0.500</td>
</tr>
<tr>
<td>0.350</td>
<td>0.243</td>
<td>0.238</td>
<td>0.233</td>
<td>0.271</td>
<td>0.281</td>
<td>0.289</td>
<td>0.332</td>
<td>0.400</td>
<td>0.500</td>
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<td></td>
</tr>
<tr>
<td>1.500</td>
<td>1.413</td>
<td>1.335</td>
<td>1.265</td>
<td>1.204</td>
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<td>1.073</td>
<td></td>
<td></td>
<td>0.970</td>
<td></td>
</tr>
<tr>
<td>1.500</td>
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<td>1.300</td>
<td>1.250</td>
<td>1.209</td>
<td>1.176</td>
<td>1.150</td>
<td></td>
<td></td>
<td>1.097</td>
<td></td>
</tr>
<tr>
<td>1.500</td>
<td>1.440</td>
<td>1.388</td>
<td>1.345</td>
<td>1.310</td>
<td>1.281</td>
<td>1.259</td>
<td>1.241</td>
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</tr>
<tr>
<td>1.500</td>
<td>1.474</td>
<td>1.460</td>
<td>1.449</td>
<td>1.442</td>
<td>1.437</td>
<td>1.433</td>
<td>1.431</td>
<td></td>
<td></td>
<td>1.414</td>
<td></td>
</tr>
<tr>
<td>1.500</td>
<td>1.513</td>
<td>1.527</td>
<td>1.540</td>
<td>1.551</td>
<td>1.561</td>
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<td>1.577</td>
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<td>1.536</td>
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</tr>
<tr>
<td>1.500</td>
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<td>1.872</td>
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<td>1.933</td>
<td></td>
<td></td>
<td>1.970</td>
<td></td>
</tr>
</tbody>
</table>

Data RMS Error:

| 1.144 | 0.614 | 0.305 | 0.107 | 0.031 | 0.010 | 0.008 | 0.007 | 0.006 | 0.005 | 0.004 |

*10^{-1}*

### Typical Eigenvalues For Model Parameters (20 point, simultaneous solution):

**S Waves:**

- 5.143
- 3.736
- 2.857
- 1.4694
- 0.8551
- 0.3564

**P Waves:**

- 0.1527
- 0.0820
- 0.0441
- 0.0182
- 0.0029
- 0.0009

---

*Figure 19*
Poro–Elastic Modeling in a Crosswell Environment

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Poro–Elastic Modeling in a Crosswell Environment

ABSTRACT

This paper attempts to determine some of the significant relationships between seismic data and the parameters of a poro-elastic medium, such as porosities, pore fluid velocities, etc.. The finite difference method is used to solve Biot’s (1956) equations for wave propagation in poroelastic media. The influence of porosity and properties of pore fluid on the velocities, attenuation and dispersion of seismic waves are examined for three kinds of seismic waves (i.e. fast p wave, slow p wave, and s wave). Our results suggest the following:

1. The S to P wave energy ratio radiated from the source depends not only on the mechanism of the seismic source, but also on the property of the pore fluid surrounding the source. This might be used in hydrocarbon recovery to estimate the content of rock fluids.

2. The porosity will affect velocities of S and P waves, but for different pore fluids, the effects differ significantly. For example, the P velocity of water-saturated rock is greater than that of gas-saturated rock. However, the S velocity of water-saturated rock is smaller than that of gas-saturated rock. This confirms the well known observation that the Vp/Vs ratio can be used to estimate both the porosity and the composition of pore fluids.

3. The dissipation and dispersion caused by fluid viscosity and fluid-solid interaction are almost indistinguishable. This may indicate the need to refine Biot’s model to account for the actual attenuation observed in actual rocks.
DYNAMIC EQUATIONS OF WAVE PROPAGATION

IN POROELASTIC MEDIUM

The dynamic equation governing the propagation of seismic waves in fluid-saturated porous medium are (Biot 1956)

\[ \sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i + b(\dot{u}_i - \dot{U}_i) \]  \hspace{1cm} (1a)

\[ S_{,i} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i - b(\dot{u}_i - \dot{U}_i) \]  \hspace{1cm} (1b)

where \( u_i \) and \( U_i \) are the averaged \( i \)th displacement components of the solid skeleton and the interstitial fluid, respectively.

For a statistically homogeneous and isotropic solid-fluid system, the constitutive relations are (Biot, 1956)

\[ \sigma_{ij} = A u_{k,k} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + QU_{k,k} \delta_{ij} \]  \hspace{1cm} (2a)

\[ S = QU_{k,k} + RU_{k,k} \]  \hspace{1cm} (2b)

By defining \( d_i = \dot{u}_i \) and \( D_i = \dot{U}_i \), eqs (1) and eqs (2) can be written as first order derivative equations,
\[
\rho \frac{\partial d_x}{\partial t} = -\rho_{12} \frac{\partial S}{\partial x} + \rho_{22} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \right) + b (\rho_{22} - \rho_{12})(D_x - d_x) \tag{3a}
\]

\[
\rho \frac{\partial d_z}{\partial t} = -\rho_{12} \frac{\partial S}{\partial z} + \rho_{22} \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right) + b (\rho_{22} - \rho_{12})(D_z - d_z) \tag{3b}
\]

\[
\rho \frac{\partial D_x}{\partial t} = \rho_{11} \frac{\partial S}{\partial x} - \rho_{12} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \right) + b (\rho_{11} - \rho_{12})(D_x - d_x) \tag{3c}
\]

\[
\rho \frac{\partial D_z}{\partial t} = \rho_{11} \frac{\partial S}{\partial z} - \rho_{12} \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} \right) + b (\rho_{11} - \rho_{12})(D_z - d_z) \tag{3d}
\]

\[
\rho \frac{\partial \sigma_{xx}}{\partial t} = 2\mu \frac{\partial d_x}{\partial x} + A \left( \frac{\partial d_x}{\partial x} + \frac{\partial d_z}{\partial z} \right) + Q \left( \frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} \right) \tag{3e}
\]

\[
\rho \frac{\partial \sigma_{xz}}{\partial t} = 2\mu \frac{\partial d_z}{\partial z} + A \left( \frac{\partial d_x}{\partial x} + \frac{\partial d_z}{\partial z} \right) + Q \left( \frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} \right) \tag{3f}
\]

\[
\rho \frac{\partial \sigma_{zz}}{\partial t} = \mu \left( \frac{\partial d_x}{\partial x} + \frac{\partial d_z}{\partial z} \right) \tag{3g}
\]

\[
\rho \frac{\partial S}{\partial t} = Q \left( \frac{\partial d_x}{\partial x} + \frac{\partial d_z}{\partial z} \right) + R \left( \frac{\partial D_x}{\partial x} + \frac{\partial D_z}{\partial z} \right) \tag{3h}
\]

A second-order staggered grid method (Virieux, 1986) is used to solve this system of equations. The merits of solving the first order equations by staggered grids are simple computer coding, sources can be naturally incorporated, and stability is present for a wide range of large Vp/Vs ratios. The coefficients in eqs (3), \( \rho , \rho_{11}, \rho_{12}, \rho_{22}, b, A, Q, R, \mu \), can be expressed in terms of more familiar parameters (Hassanzadeh, 1988),

\[
\rho = \rho_{11} \rho_{22} - \rho_{12} \tag{4}
\]

\[
\rho_{12} = \phi \rho_f (1-\tau)
\]

\[
\rho_{11} = \rho_s (1-\phi) - \rho_{12}
\]

\[
\rho_{22} = \tau \phi \rho_f
\]
\[
\tau = \frac{1}{2}(1 + \frac{1}{\phi})
\]

\[
b = \frac{\phi^2 \eta}{\kappa}
\]

\[
A = [(1-\phi)(\alpha-\phi)K_s + \phi \frac{K_s K_b}{K_f}] \frac{1}{D} - \frac{2}{3}\mu
\]

\[
R = \phi^2 K_s / D
\]

\[
D = \alpha - \phi + \phi K_s / K_f
\]

\[
\alpha = 1 - K_b / K_s
\]

In the above relations, shear modulus \(\mu\) and bulk modulus \(K_b\) of the frame are derived by Thomson (1985). Figure 1 depicts the behaviour of \(K_b\) as a function of \(\mu\). All other parameters need to be determined experimentally, namely: solid grain density \(\rho_s\), fluid density \(\rho_f\), porosity \(\phi\), viscosity \(\eta\), permeability \(\kappa\), bulk modulus of solid grain \(K_s\), and the bulk modulus of fluid \(K_f\). Equivalently, we assume eight parameters are known, i.e., density of solid grain \(\rho_s\), density of fluid \(\rho_f\), porosity \(\phi\), viscosity \(\eta\), permeability \(\kappa\), \(P\) velocity of the solid grain \(\alpha_s\), \(S\) velocity of the solid grain \(\beta_s\), and the \(P\) velocity of the fluid \(\alpha_f\).

**EQUATIONS CONCERNING VELOCITY AND DISSIPATION**

Insertion of the constitutive relations into eqs. (1) yields the wave equation

\[
(A + 2\mu)\nabla(\nabla \cdot u) + Q \nabla(\nabla \cdot U) - \mu \nabla \times \nabla \times d = \rho_{11}\ddot{u} + \rho_{12}\ddot{\dot{u}} + b(\dot{U} - \dot{u}) \tag{5a}
\]

\[
Q \nabla(\nabla \cdot u) + R \nabla(\nabla \cdot U) = \rho_{12}\ddot{u} + \rho_{22}\ddot{\dot{u}} - b(\dot{U} - \dot{u}) \tag{5b}
\]

Taking the divergence of eqs. (5) we have for constant coefficients

\[
(A + 2\mu)\nabla^2 u_{k,k} + Q \nabla U_{k,k} = \rho_{11}\ddot{u}_{k,k} + \rho_{12}\ddot{\dot{u}}_{k,k} + b(\dot{U}_{k,k} - \dot{u}_{k,k}) \tag{6a}
\]

\[
Q \nabla^2 u_{k,k} + R \nabla U_{k,k} = \rho_{12}\ddot{u}_{k,k} + \rho_{22}\ddot{\dot{u}}_{k,k} - b(\dot{U}_{k,k} - \dot{u}_{k,k}) \tag{6b}
\]
We now consider a plane dilatational wave described by

\begin{align*}
  u_{k,k} &= C_1 \exp (i (lx + \omega t)) \\
  U_{k,k} &= C_2 \exp (i (lx + \omega t)).
\end{align*} \tag{7a, 7b}

Substituting (7) into (6) and eliminating \( C_1 \) and \( C_2 \) yields

\begin{align*}
  (AR - Q^2) \frac{l^4}{\omega^4} - (A \rho_{11} + R \rho_{22} - 2Q \rho_{12}) \frac{l^2}{\omega^2} \\
  + (\rho_{11} \rho_{22} - \rho_{12}^2) + \frac{ib}{\alpha} [(A + R + 2Q) \frac{l^2}{\omega^2} - \rho] &= 0.
\end{align*} \tag{8}

Equation (8) has two solutions, denoted by \( l_1 \) and \( l_2 \), that are physically meaningful if their real parts are greater than zero. In terms of the two solutions, we get

\begin{align*}
  V_1 &= \frac{\omega}{\text{Real}(l_1)} \tag{9a} \\
  X_1 &= \frac{1}{\text{Imag}(l_1)} \tag{9b} \\
  V_2 &= \frac{\omega}{\text{Real}(l_2)} \tag{10a} \\
  X_2 &= \frac{1}{\text{Imag}(l_2)} \tag{10b}
\end{align*}

It is easy to see that when \( x = X_1 \)

\[ |\exp (i (lx + \omega t))| = \frac{1}{e} \]

So, \( V_1 \) and \( X_1 \) represent, respectively, the velocity and attenuation for the fast P waves, and \( V_2, X_2 \) for the slow P waves. Similarly, for S waves, we get the equation

\[(\mu \omega^2 \rho_{22} + i \mu \omega b)l^2 - [\omega^4(\rho_{11}^2 - \rho_{12}^2) + i \omega^3 b (\rho_{11} + \rho_{22} + 2\rho_{12})] = 0 \tag{11} \]
which is similar eq. (8). Moreover, we can calculate $V_z$ and $X_z$ from eqs. (11).

**NUMERICAL SIMULATIONS**

To observe the effects of various parameters, such as porosity and source frequency, on the three kinds of waves, equation (3) is solved by the finite difference method. The objective is to determine the dependence of velocity and dissipation on medium parameters by solving eqs. (8) and eqs (11). Three results will be illustrated: (1) effect of pore fluid on composition of source radiation energy, (2) velocity as a function of porosity $\phi$, viscosity of fluid $\eta$, source frequency, and density of fluid $\rho_f$.

**Source Radiation Energy**

Figure 2 shows a layered model with the fluid parameters are listed in Table 1 and the solid grain parameters labeled in Figure 2. In the first and third layer of this model, the fluid is water, but in the middle layer, synthetic seismograms are calculated for both water (Figure 3) and oil (Fig 4). Figures 3 and 4 show similar responses, but as Figure 5 shows, the $z$-component amplitude of S waves shows a remarkable difference. When the source is located in water-saturated rocks, it apparently magnifies the S wave amplitude relative to a source in oil-saturated rocks. Can this phenomenon be used to detect oil-saturated layers in well logs or cross-well data?

**Velocity**

Figure 6a and Figure 6b illustrates $V_1$, $V_2$, $V_z$ as a function of porosity. Here $V_1$, $V_2$, and $V_z$ are, respectively, the fast P wave velocity, slow P wave velocity, and S wave velocity. All curves are normalized by solid grain S velocity $(\alpha_s = 2.7\text{km/s})$. In both Figures, there are three bundles of curves. From top to bottom, each curve bundle corresponds to $V_1$, $V_2$, and $V_z$ respectively. In each bundle, there are three curves which represent water-saturated rock (solid line), gas-saturated rock (dash line), and oil-saturated rock (dotted line). In Figure (6a), it is interesting to note that the $V_1(\text{gas-saturated}) < V_1(\text{water-saturated})$, however, the $V_z(\text{gas-saturated}) > V_z(\text{water-saturated})$. This figure might suggest clues to porosity could be estimated by a decrease of $V_1$, and the composition of pore fluid could be analyzed from the ratio of $V_1/V_z$.

In Figure 6b, all parameters are the same as in Figure 6a except no viscosity is assumed. For P- and S- waves, there are only slight differences between Figure 6a and Figure 6b velocities when $\eta = 0$. However, the velocity contrast between the slow waves do vary significantly. In addition, the slow P wave velocity of water saturated rock increases with porosity. Synthetic seismograms (Fig. 7) show the results to be identical with Figure 6b. In Figure 7a and 7b, all parameters are the same except $\phi = 0.2$ in Fig 7a, and $\phi = 0.4$ in Fig 7b. It is clear that the slow P wave velocity in Figure 7b is greater than that in Figure 7b.

Figure 8 shows that $V_1$ and $V_z$ does not change much with variation in frequency content of the Ricker wavelet. This indicates there is little dispersion if the wave propagates in the poroelastic medium according to Biot's theory.
The influence of fluid density on velocities are shown in Figure 9, in which we see that as the density decreases, the fast wave velocity decreases. Conversely, the S wave velocity increases. This may be partly the reason for $V_1/V_2$ decreases with the introduction of gas.

**Attenuation**

Figure 10 shows $X_2$ for the slow P wave (eqs. 10b), and Figure 12 shows $X_1$ and $X_2$ for fast P waves and S waves (eqs. 9b and 11). It must be emphasized that meters is the unit of the vertical coordinate in Figure 10, but kilometers in Figure 11. Both results are calculated for gas-saturated rock. Figure 10 is identical with the fact that slow waves attenuate very fast if some viscosity is introduced. The synthetic seismograms confirm such a result. Figure 11a is the x-component seismograms without viscosity, in which the slow P waves is prominent. The slow wave disappears in Figure 11b, in which 0.1 centipoise viscosity is introduced.

Figure 12 indicates that a pathlength of ten kilometers is needed to decrease the amplitude of fast P or S waves by a factor of $\frac{1}{e}$. In crosswell data, this attenuation is too small to be detected. Hence, the Biot model we have employed may be inadequate to explain actual attenuation observed in data.

**DISCUSSION**

Some preliminary numerical results are presented which simulate Biot poro-elastic wave propagation in a crosswell environment. If the Biot model is correct then modeling exercises such as these may provide valuable insights into the relationship between seismic data and poro-elastic media properties. Due to the underdetermined nature of this problem, these modeling exercises are most valuable when both log, VSP and CDP data are available. For example, an extensive study on lithology vs Vp/Vs ratios (see Gallagher, 1988) combined with a poro-elastic modeling study would enhance the reliability of seismic based hydrocarbon predictions.

**TABLE 1**

<table>
<thead>
<tr>
<th>Property of Fluid</th>
<th>P-wave velocity</th>
<th>Density</th>
<th>Viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1500 m/sec</td>
<td>1.0 g/cm$^3$</td>
<td>1 cp</td>
</tr>
<tr>
<td>Oil</td>
<td>500 m/sec</td>
<td>0.8 g/cm$^3$</td>
<td>1 cp</td>
</tr>
<tr>
<td>Gas</td>
<td>400 m/sec</td>
<td>0.1 g/cm$^3$</td>
<td>0.015 cp</td>
</tr>
</tbody>
</table>
FIGURES

Figure 1. Ratio of frame modulus to solid grain modulus. $\frac{K_b}{K_s}$ is for the solid lines, $\frac{\mu}{\mu_s}$ is for the dashed lines. Subscript, s, denotes the parameter of the solid grain.

Figure 2. Layer model for calculating crosswell data. The value displayed in this Figure represents the parameters of solid grain. The fluid parameters are listed in Table 1.

Figure 3. Synthetic seismograms for the Figure 2 model; the middle layer contains water. Figures 3a and 3b are the, respectively, x-component and z-component seismograms.

Figure 4. Synthetic seismograms for the Figure 2 model. The middle layer contains oil. Figures 4a and 4b are the x-component and z-component seismograms.

Figure 5. Comparison of $51_{st}$ trace in Figures 3 and 4. Figure 5a is the x-component and Figure 5b is the z-component.

Figure 6. Velocities of fast P wave, slow P wave, and S wave with porosity. Water-saturated rock corresponds to the real line, gas saturated rock corresponds to the dashed line, and oil-saturated corresponds to the dotted line. All nine curves are normalized by the solid grain P velocity. Figure 6a is with viscosity, while viscosity is absent in Figure 6b.

Figure 7. Synthetic seismograms for unbounded homogeneous medium. In Figure 7a $\phi = 0.2$ and $\phi = 0.4$ in 7b.

Figure 8. Velocities of fast P wave, slow P wave, and S wave as a function of frequency. The porosity is fixed at 0.25. Water-saturated rock corresponds to the solid line, gas saturated to the dashed line, and oil-saturated to the dotted line. All nine curves are normalized by the solid grain P velocity.

Figure 9. Velocities of fast P wave, slow P wave, and S wave with frequency. Here, the parameters of the skeleton are the same as in Figure 8, but only one case is considered. Porosity is set to be 0.25 and fluid velocity is set to be 1.5 km/s.

Figure 10. Dissipation of slow P waves.
Figure 11. Synthetic seismograms in unbounded homogeneous medium. In Figure 11a, the viscosity equals zero, in Figure 11b the viscosity equals 0.1 centipoise.

Figure 12 Dissipation of fast and shear waves.

REFERENCES


Figure 2 Model of cross well
All water $dx=0.001$ $dt=0.0002$ $vm=100$. $fi=0.05,0.25$

**FIGURE 3a**
FIGURE 4b
FIGURE 5a
FIGURE 5b
FIGURE 6a
FIGURE 6b
FIGURE 8
FIGURE 9
FIGURE 10
FIGURE 11a
FIGURE 11b
FIGURE 12
A flexible procedure for seismic modeling experiments

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Abstract

This paper presents a method for tracing rays in laterally varying media that is suitable for solving the forward problem in inverse experiments. The application of a model composed of triangles of constant gradients determines a continuous velocity field demanding no smoothing in order to retain stability in the ray tracing. The efficiency that is highly required in inverse seismic experiments is obtained by dealing with only second order discontinuities. An easy-to-use input scheme for the model parameters that allows several types of parameters to be associated to the nodes or elements of the grid is suggested. In order to limit the amount of input, a grid of trapezoid is specified by the user and the exact location of the triangles is left to the program. A fully-automated shooting and two-point iteration technique that converges in zones without caustics is presented to support the modelling procedure. Shots and receivers may be located in positions convenient for the modelling of surface data as well as VSP or crosswell data. By controlling the travel path during the ray tracing, transmitted rays are separated from rays reflected from interfaces of the model. Finally, the ability of and the CPU-time required on a STELLAR GS1000 for the technique to perform a two-point ray tracing in models of varying complexity are evaluated.
Introduction

Numerous methods have been proposed for tracing of seismic rays in an arbitrary grid of velocities (e.g., Chander 1977, Um & Thurber 1987, Cassel 1982). The methods differ considerably in efficiency, accuracy, flexibility and simplification in the geologic model. Calculation of the ray paths as circular arcs in cells of constant velocity gradients appear to be a more accurate and efficient method of ray tracing (e.g., Gebran 1976, Whittall and Clowes 1979). By the definition of the velocity gradients in rectangular areas, however, undesirable discontinuities occur at the edges of the cells, and some sort of smoothing has to be involved (Langan et al. 1985). This problem is avoided if ray tracing is performed in a grid of triangles where the velocity gradients are defined uniquely from the velocities specified at the edges (e.g., Weber 1988).

Grid and parameter specification

A direct user specification of a triangular grid pattern may be a cumbersome procedure. Therefore, an input scheme, defined by $N_v$ nodal points and associated velocities located on each of $N_v$ vertical lines, is suggested for the model discretization. The scheme is apparently composed of $N_v-1$ layers, each containing $N_v-1$ trapezoid-shaped areas. Each of these areas are automatically subdivided into a number of triangles.

The most simple and efficient way of subdividing the trapezes would be a definition of two triangles from the diagonals. However, this approach implies an undesirable spatial bias of the calculated ray paths due to correlation of the directions of diagonalization (Langan, pers. comm.). In order to avoid this problem, an approach of subdividing the trapezes automatically into four triangles with a common point and associated velocity, given by the geometrical mean of the values specified at the edges of the block, is proposed. Although involving twice as many cells, the bias of the triangle pattern on the travel paths is minimized.

In several occasions, varying tomography projects are performed in the same target area, requiring different kinds of modelling parameters. Dependent on the specific project, velocities of P-waves or S-waves, attenuation coefficients or quality factors, densities etc. are associated to the discretized model. The most convenient way of handling the parameters (initial guess, a priori knowledge or inversion results) seems to be the development of a file containing the model and the parameters specified for each node or element of the grid. Every time a modelling experiment is performed, the grid and parameter file of the specific target area is involved. Moreover, the file may be used for storage of parameters to be used in various kinds of geological/geophysical modelling projects.

Layer interfaces are simulated by a horizontal succession of trapezes, comprising large gradients of velocity (see Ray tracing). Anyway, the input scheme described is apparently applicable to define almost any geological model. Smooth depth and velocity variations are discretized by a small number of vertical grid lines. Complex geological structures, however, may require many closely spaced vertical grid lines where gradients are significant.

Shot and receiver positions may be specified on either of the vertical grid lines or at the upper horizontal boundary, allowing VSP as well as crosswell and surface seismic data to be used in the modelling (see Examples).
Ray tracing

Within a constant gradient of velocity the ray path is given by a circular arc, completely described by the radius and center of the circle (e.g., Telford et al. 1976). Given the location, velocity and ray direction of the point of entrance to the triangle, the location, velocity and ray direction of the first point of exit from the area as well as the travel time is given by analytical expressions (e.g., Gebran 1976). The most expensive part of forward modelling is normally the evaluation of a line integral along the ray path.

\[
\int_0^L \frac{\partial t_i}{\partial s} ds
\]  

(1)

This could be the computation of the travel time,

\[
t_i = \frac{1}{||G||^{1/2}} \ln \left[ \frac{G \times D_1}{G \times D_0} \frac{||G||^{1/2} + G \cdot D_0}{||G||^{1/2} + G \cdot D_1} \right]
\]

(2)

\[
t_i = \frac{1}{||G||^{1/2}} \ln \left[ \frac{\nu_1}{\nu_0} \right], \ G \times D_0 = 0
\]

\[
t_i = \frac{[(x_i-x_0)^2 + (z_i-z_0)^2]^{1/2}}{\nu_0}, \ G = 0
\]

where \(G\) and \(L\) are the gradient of velocity and length of the ray path in the current triangle \(t_i\), respectively, and \(D_0\) and \(G_1\) are the direction of the ray at the point of entrance \((x_0,x_0)\) with the velocity \(\nu_0\) and point of exit \((x_i,x_i)\) with the velocity \(\nu_1\), respectively. For a trial-and-error modelling technique, the evaluation of the line integral (1) is necessary only during the final iteration. When a two-point routine is involved, however, this computation must be carried out in order to ensure the concept of minimal travel time.

Layer interfaces defined as a discontinuous variation of the velocity field across specified curves (first order discontinuities, e.g., Langan et al. 1985) require a set of special conditions to be switched on when the ray is located near the discontinuity. This approach must necessarily decrease the efficiency of tracing of reflected rays. By simulating the interfaces by thin zones of considerable gradients of velocity (second order discontinuities), the ray tracing is performed equally all over the model. In the limit of infinitely thin boundary zones, moreover, travel times equal to the values calculated by the use of first order discontinuities are obtained.

To illustrate this, fig. 1 shows the error in travel time and point of emergence for a ray reflected at a second order interface. The source and receiver position were fixed above the equivalent horizontal first order discontinuity and upper boundary of the second order discontinuity. The velocities at either side of the boundary were 1500 m/s and 3000 m/s, respectively. The errors are less than the errors of the travel times and the tolerance of the point of emergence in the two-point ray tracing that is normally considered in the sort of model described.

In the exact expression for the point of exit of the ray, the square of the radius of curvature \(R\) for the ray path must be calculated. In order to retain numerical stability for small values of \(G\) (large values of \(R\)), the ray tracing should be carried out in double precision.
The definition of the model described excludes such phenomena as diffraction and conversion of waves. However, arrivals corresponding to these types of waves are difficult to pick on the seismic records and are normally not considered in tomography experiments.

The two-point problem

When the points of emergence of two rays \( E_1 \) and \( E_2 \) are encountered to each side of the seismic source, a simple and usually convergent method of estimating an improved initial angle of the ray touching a receiver of location \( x_r \) is given by

\[
\theta_N = \theta_1 + [x_r - E_1] \frac{\theta_2 - \theta_1}{E_2 - E_1}
\]

where \( \theta_1 \) and \( \theta_2 \) are the initial angles of the rays. Convergence using (3) is obtained only when the point of emergence (\( E_N \)) from the estimate \( \theta_1 < \theta_N < \theta_2 \) fulfils \( E_1 < E_N < E_2 \), that is outside caustic zones.

The probability of an improved estimate by (3), however, increases when \( \theta_2 - \theta_1 \) is small, and this concept is applied in a two-point iteration routine. Instead of using the conventional procedure of shooting a manually specified initial beam of rays (e.g., Langan et al. 1985), a scheme for an automatic initiation of the routine is presented.

\( N \) rays, given by initial angles of equidistant spacing \( (DX = N/360) \) are emitted from the source.

Each time a ray is found to hit the receiver interval, it is evaluated as a candidate of an array of succesful rays. The criterion to be accepted in the array would be that the distance from the corresponding point of emergence to a receiver is within the specified tolerance and that the travel is minimal. The content of the array is replaced during the shooting so that only the two rays with points of emergence closest to each receiver is retained, leaving a maximum number of \( 2 \cdot \text{[no. of receivers]} \).

The succesful rays are then used to estimate the initial angles of rays, intended for the actual receiver positions by (3).

The space of angles currently considered (VR) is updated by the interval delivering the succesful rays, and N is doubled.

This process is continued until \( DX \) is less than a threshold value or the difference \( |E_n - x_r| \) is a acceptably low.

In order to be able to use the traced rays in an inversion process, a separation of the travel times is necessary. This is achieved by incorporating constraints of the possible ray paths. In this way, reflected transmitted rays are separated by the potential turning points in the zones defining the interfaces.

The rate of convergency of the technique depends on the complexity of the model.
Examples

In order to evaluate the scheme described in this paper, the application to three different seismic modelling experiments, is presented. In the examples, the travel time for each receiver is calculated in addition to the tracing of rays. A tolerance for the point of emergence of approximately 10% of the receiver spacing is considered for the modelling.

Fig. 2 shows a crosswell experiment with 12 x 12 blocks, 12 sources and 12 receivers a central anomalous zone, where the minimum velocity is 75% of the value expected from a vertical background gradient of velocity of 1.6 sec⁻¹. A total amount of 203 CPU-secs was spent in the tracing of 2898 rays on a STELLAR GS1000 computer. 119 rays were traced to the receivers.

Fig. 3 shows a VSP experiment. The cell geometry, receiver configuration and velocity distribution is equivalent to the crosswell model, except for the lack of the low velocity zone. 134 CPU-secs were needed for the tracing of 3017 rays on a STELLAR GS1000 computer. 144 rays were traced to the receivers.

Fig. 4 shows the modelling of a salt pierce in a surface seismic experiment. An end-on spread with 2 shots and 39 receivers were located at the upper boundary of the model. The background gradient of velocity varies between 0.3 sec⁻¹ and 18 sec⁻¹ and the constant velocity of the salt (shaded) is 1.5 times larger than the expected value in the middle of the pierce. 61 out of 748 emitted rays were traced to the receivers in 38 CPU-secs on a STELLAR GS1000 computer.

Conclusion

A flexible modelling scheme for grid definition, parameter storage and two-point ray tracing has been developed for seismic modelling experiments. The technique of using a file containing several types of parameters associated to the nodes or elements of the grid provides a convenient way of handling the velocities and elastic parameters needed in the modelling. A discretization of the model along vertical grid lines in a trapezoid pattern reduces the amount of input parameters. Ray tracing in a smooth velocity field of triangles, defined from the four edges of the trapezes, is shown to deliver the least biased calculation of the travel paths. Finally, an automatic two-point iteration technique developed for the scheme provides an efficient and stable solution of the forward problem in the modelling procedure.
References

List of captions

Fig. 1 Simulation of a second order discontinuity as a first order discontinuity. (a) The model. A constant velocity of 1500 m/s and 3000 m/s is considered above and below the horizontal discontinuity of finite thickness, respectively. (b) The error in percent for the travel time (full line) and the point of emergence of the ray (dashed line).

Fig. 2 Ray tracing in a crosswell experiment, modelling a low velocity zone. The minimum velocity is 75% of the value expected from the background gradient of velocity of 1.6 sec⁻¹. 119 rays were traced to the receivers.

Fig. 3 Ray tracing in a VSP experiment. A constant vertical gradient of velocity of 1.6 sec⁻¹ is modelled, and 144 rays are traced to the receivers.

Fig. 4 Ray tracing in a surface seismic experiment, modelling a salt piercement. The background gradient of velocity is varying, and the constant velocity of the salt is 1.5 times larger than the expected value in the middle of the piercement. 61 rays are traced to the receivers.
Fig. 1
PROLEGOMENON FOR A COMPARISON BETWEEN
SEISMIC AND ELECTRIC RESERVOIR DESCRIPTION AND MONITORING

by

Alan C. Tripp
Introduction

For seven years, Dr. J. W. Hohmann has directed an electromagnetics modelling program at the University of Utah. The research objectives of the consortium are the development of modelling software for electromagnetics and the use of such software in gaining insight into the nature and use of electromagnetics in geophysical exploration. The research is sponsored by a consortium of mining and oil companies.

Because of the recent increased emphasis on geophysical oil reservoir description and monitoring, the University of Utah EM research group has begun to consider the problem of describing and monitoring oil reservoirs using electromagnetics. One component of this research is the comparison of the electromagnetic and seismic methods in a reservoir environment. We undertook this investigation because it seemed reasonable that the two methods would be complementary in reservoir description and monitoring.

Dr. Schuster kindly offered to collaborate in this research. Since our collaborative efforts are only a few months old, our results are still modest and of a preliminary nature. However, we are encouraged that electromagnetic and seismic methods are complementary. This talk is intended to supply a brief justification for our optimism.

Electric and Seismic Rock Properties

Why did we feel that the two techniques would be complementary? In short, because in a reservoir environment seismic techniques measure the mechanical properties of the
reservoir while electromagnetic techniques measure the ionic properties of the reservoir. In some cases, a reservoir property, such as porosity, will influence the mechanical and the ionic properties of the reservoir. In other cases, a reservoir property will influence only the mechanical or the ionic nature of the reservoir.

Viewgraph 1 summarizes the complementarity between the electromagnetic and seismic methods as reflected in rock properties measurements. Reservoir porosity affects both electrical resistivity, as shown in Viewgraph 2, and seismic velocities, as shown in Viewgraph 3. It seems reasonable that given the proper model constraints, that both methods could resolve the porosity structure of a reservoir. Both resistivity and seismic velocities are also influenced by the presence of clay, as shown by Viewgraphs 4 and 5. However, the electric resistivity will only be influenced by clays having exchangeable cations. Since these clays are precisely the clays which are swellable and likely to affect reservoir permeability given a gross change in reservoir chemistry, such as occurs during a steam flood, the selective nature of the electric measurements may be important. In contrast, seismic velocities seem to be independent of the surface chemistry of the clays. Viewgraphs 6 and 7 show the influence that freezing of pore water has on the electric conductivity, compressional velocity, and the attenuation. Viewgraphs 8, 9 and 10 show the effects of steam on the seismic and electric properties of rock. These results indicate that both seismic and electric techniques could be used
successfully to delineate or monitor frozen zones or steam zones under suitable circumstances.

So far we have enumerated reservoir properties which have both a seismic and an electric expression. We cannot definitely state at this point which of the two techniques should be used to delineate these features. Indeed it is possible that in certain cases the two would complement each other. However there are reservoir properties which influence only seismic properties. Viewgraph 11 shows the influences of hydrocarbon temperature and type on velocity, while Viewgraph 12 shows the influence of pore pressure on velocity. The electric resistivity of rock will not vary with hydrocarbon type or pore pressure since these two parameters do not affect the ionic balance of the pore water. The resistivity will vary with hydrocarbon temperature only insofar as the hydrocarbon temperature tracks the pore water temperature.

Similarly, there are reservoir properties which predominately influence the electric resistivity. Viewgraph 13 shows the influence of pore water salinity and temperature on the resistivity of the power water and hence on rock resistivity. To the best of my knowledge pore water salinity has little effect on seismic parameters, while the pore water temperature would become significant only after boiling. Viewgraph 14 shows the effect of percent water saturation on rock resistivity, while viewgraph 15 shows the effects of percent water saturation on seismic parameters. It is obvious that the electric effects are profound
across the entire range of saturation coefficients, while the seismic effects, are limited to large percent water saturations.

A reservoir parameter which is of prime importance is the permeability. There is at present a great deal of work being done with the goal of relating permeability to seismic or electric parameters. Although rigorous relationships may yet be found, it is unclear whether practical determination of permeability using either seismic or electric techniques is eminent. My reason for saying this can be illustrated by viewgraph 16. This shows a crossplot of the permeability with respect to a power of the porosity for a particular suite of core samples. Since the power is 4.4, any error in determining porosity will be magnified into a much larger error in permeability. Thus the effects of any geological noise will be profound. For example, the presence of minor amounts of clay could introduce serious error into the determination of permeability by either technique.

**Electric Modeling**

We have considered which reservoir properties have electric and seismic expressions. We must now consider the ability of the electric method to resolve resistivity features on the scale encountered in the reservoir environment. There are several modelling packages which we can use to accomplish these tasks, as shown in Viewgraph 17. Basically, we can model the response of 1D, 2D, and 3D reservoirs to a number of different electric sources. Inverse algorithms are more limited in scope. Specific forward and inverse schemes will be considered below.
Viewgraphs 18, 19 and 20 illustrate possible source-receiver geometries for delineating or monitoring a reservoir. Each geometry has particular advantages and disadvantages which must be considered in regards to any experiment. In general, given the novelty of electromagnetic cross-borehole work at sub-radar frequencies, much work remains to be done to define the range of applicability of each geometry. I will discuss the response of several of these geometries to the model of an EOR case shown in Viewgraph 21. This model simulates a shallow oil field which is being produced by a steam flood or fire-flood. The earth above and below the oil zone has a resistivity of 10-Ωm, while the oil-zone has a resistivity of 50 Ω-m. The steam zone or burn zone has a resistivity of 500 Ω-m. All of these resistivities are based on actual well-logs or on laboratory data, but the model itself does not represent any particular oil field.

Viewgraphs 22 through 30 illustrate the results of a model study done by Craig Beasley at the University of Utah Research Institute under Department of Energy funding. The chapter of Craig’s thesis in which he describes his work is included as an appendix. Briefly, Craig approximated the model of Viewgraph 21 by a 2D model and then expanded the depleted zone step by step. At each step he inverted direct current grounded electrode measurements, contaminated by 10% random noise, to a 2D model. He found that he was successful in tracing the advance of the depleted zone if he took care in including enough resistivity model blocks. He was also able to invert to the true model in
the case in which the steam zone formed a cap on the hot water/oil bank, as shown in Viewgraph 28. If the true model cannot be represented by the model cells included in the inversion, the resulting reduced chi-square value is diagnostically high and the model resulting from the inversion is not a good representation of the true model, as one might expect. Craig also demonstrates, through the models shown in Viewgraphs 29 and 30 that the model cells must be such that they can represent resistivity variations included by the EOR process outside the oil reservoir.

Craig’s work demonstrates that hole to hole resistivity sounding is feasible theoretically in the case of oil fields which are candidates for steam flooding or fire-flooding. There are, of course, several practical problems with the grounded electrode resistivity technique. Perhaps the most obvious is that it is awkward or impossible in the presence of casing. The second is that it is sensitive to the immediate region around the electrodes and care must be exercised in adequately accounting for this region. Still when all aspects are considered, the technique seems to have promise for at least special limited applications.

Since the direct current resistivity method is controlled by Laplace’s equation, it is natural to expect that there is a wide range of models which will fit any particular data set. Although this need not be an overwhelming problem in reservoir work where the inversion model can be constrained to a certain extent by known stratigraphy, we still expect it to be troublesome.
Fortunately the same degree of inversion model non-uniqueness does not exist for every electromagnetic technique. As the frequency of the source is raised into the range traditionally used in electromagnetic exploration the response is governed by the diffusion equation. As the frequencies approach radar frequencies the response is governed by the wave equation. One of the purposes of electromagnetic research at this time is to determine the range of frequencies which give maximum resolution in the reservoir environment.

I will now discuss preliminary electromagnetic modelling for an ungrounded source and a magnetic receiver such as is shown in Viewgraph 20. This particular arrangement has the possible advantage of being executable in the presence of steel casing. Viewgraph 31 illustrates the modelling technique which we will use. The algorithm can model the response of an arbitrary 3D body in the presence of a layered earth. The algorithm uses the integral equations formulation in which scattering currents in the presence of the body are determined and are then used to calculate total fields outside the body. Various simplifications of the basic integral equation are possible in special cases. In particular, when the 3D body has some geometric symmetry, group representation theory can be used to simplify the resulting equations.

Viewgraph 32 shows the transmitter-receiver geometry used for our preliminary modelling. For purposes of illustration I will present the frequency-domain values calculated for a receiver at a depth of 540 m and x- and y-coordinates of (71,,
The transmitter will be at (0., 0., 480.) and is taken to be a small coil of moment 10,000. Viewgraph 33 shows the in-phase and quadrature of the secondary vertical magnetic field as a function of frequency for the cases where the depleted zone has a resistivity of 500 Ω-m and 10 Ω-m. The case of 10 Ω-m would correspond to a water-flood. In the 10 Ω-m case the maximum in both in-phase and quadrature occurs at 10,000 Hz and has a magnitude of approximately .01 γ. This field level should be measurable using a coil having a permeable core. In the case of the 500 Ω-m zone the maximum response also occurs at 10,000 Hz and has values of .002 γ and .005 γ for in-phase and quadrature components respectively. These field values might be on the border line of detectability. In either case the measurements should be done in the time-domain because the secondary fields tend to be small compared to the primary fields.

The magnitudes of the results discussed are critically dependent on the configuration of the transmitted magnetic fields and the moment of the transmitter. A larger moment could be achieved by replacing the magnetic field transmitter with a grounded electric field transmitter. The use of a downhole electric field transmitter would also lead to a different field geometry which could well increase the magnitude of the measured secondary magnetic fields. In subsequent model experiments we will test this hypothesis.
Conclusions

Preliminary research indicates that seismic and electromagnetic methods may well be complementary in delineating and monitoring oil reservoirs. Further work will concentrate on joint seismic and electromagnetic modelling of generic oil reservoir situations.
BIBLIOGRAPHY


COMPLEMENTARITY BETWEEN ELECTROMAGNETIC AND SEISMIC METHODS SUGGESTED BY ROCK PROPERTIES MEASUREMENTS

PARAMETERS INFLUENCING BOTH SEISMIC AND ELECTRIC DATA

- Porosity
- Presence of Clay
- Permafrost
- Presence of Steam

PARAMETERS INFLUENCING ONLY SEISMIC DATA

- Hydrocarbon type and temperature
- Pore pressure

PARAMETERS INFLUENCING ONLY ELECTRIC DATA

- Temperature and salinity of reservoir water
- Water saturation fraction
- Clay surface chemistry

PROBLEMATIC PARAMETER FOR SEISMIC AND ELECTRIC DATA

- Permeability
POROSITY EFFECTS

ELECTRIC

FIGURE 5-5 Formation factor of unconsolidated artificial samples. Key indicates trend of decreasing sphericity from A to E. Solid lines show Archie's equation for three values of m. (After Jackson et al., 1978.)
Figure 1. Measured (a) compressional ($V_p$) and (b) shear ($V_s$) velocities in 80 saturated sandstone samples at confining pressure of 40 MPa and pore pressure of 1 MPa. Straight lines are best linear fits to clay-free sandstones (10 samples) and clay-bearing ones (70 samples) (from Han et al., 1986).
CLAY EFFECTS

SEISMIC

\[ V_p = 5.99 - 6.93 \times \theta - 2.18 \times c \]

\[ V_s = 3.52 - 4.91 \times \theta - 1.89 \times c \]

**Figure 3** Deviation of measured \( V_p \) values in 80 sandstones from the best linear fit to the data \( V_p = a_0 - a_1 \theta - a_2 c \), where \( \theta \) is porosity and \( c \) is volume clay content. Deviations are shown vs. (a) porosity, and (b) clay content. Note that the measured clean sandstones (clay-free) are systematically higher than predicted (from Han et al., 1986).

\[ V_p = 5.99 - 6.93 \times \theta - 2.18 \times c \]

\[ V_s = 3.52 - 4.91 \times \theta - 1.89 \times c \]

**Figure 4** Deviations of measured \( V_s \) values in 80 sandstones from the best linear fit to the data \( V_s = b_0 - b_1 \theta - b_2 c \) where \( \theta \) is porosity and \( c \) is volume clay content. Deviations are shown vs. (a) porosity; and (b) clay content (from Han et al., 1986).
CLAY EFFECTS

ELECTRIC

FIGURE 5-12 Model for conduction in a shaly sand. The heavy line represents the trend of typical experimental data. (After Waxman and Smits, 1968.)
Figure 16 Effect of freezing on compressional (a) velocity, and (b) attenuation in $H_2O$. Similar changes are found in rocks saturated with $H_2O$ upon freezing (from [10 et al., 1979].)
FIGURE 4: Relationship between resistivity and temperature near the freezing point as a function of water content: 1 to 1.8%; 2 to 14.2%; 3 to 24.4%; 4 to 34.2%.
Figure 13 Compressional and shear wave (a) velocities and (b) amplitudes in rocks with steam and hot water in their pore space. The transition between hot water and steam and 150°C occurs at 4.7 bars. In general, velocities increase sharply upon the transition from steam to hot water in the pores, and attenuation shows a sharp decrease in shear, and a sharp minimum in compressional amplitudes.
Figure 4. Conductance vs. time at different grid stations for steam displacement of brine.
Figure 5. Conductance vs time at selected grid stations for steam displacement of heavy oil.
**EFFECTS OF HYDROCARBON TYPE AND TEMPERATURE**

**SEISMIC**

![Graph showing compressional velocities in heavy crude oil and tar vs. temperature.](image)

*Figure 19* Compressional velocities in heavy crude oil and in tar vs. temperature. Note the large temperature sensitivity of the velocity, which in this case is clearly not due to melting (from Wang and Nur, 1986).

![Graphs showing compressional velocities in Kern River Oil Sand and Venezuelan Oil Sand](image)

*Figure 23* Compressional velocities in heavy oil sands from California and Venezuela vs. temperature. The largest decreases in velocity of 20 percent and 43 percent respectively are found in samples saturated with oil. No temperature dependence remains in the clean, brine-saturated samples. Mixtures of 50 percent brine and 50 percent oil yield intermediate dependence on temperature. These results imply that the oil is responsible for the very large temperature dependence in these rocks (from Tosaya and Nur, 1982).
Figure 27 Schematic velocity profiles associated with (a) a gas-bearing zone; and (b) a geopressed zone. Note the diagnostic difference in shear velocity behavior.
FIGURE 5.5 Dependence of resistivity of NaCl solution on temperature and salinity. (Courtesy of Schlumberger, 1972.)
EFFECTS OF WATER SATURATION

ELECTRIC

Fig. 33: Relationship of the resistivity index, $R_n$, to the coefficient of water saturation, $k_w$: (1, 2, 3) sandstone rocks which are hydrophilic, moderately hydrophilic, and hydrophobic, respectively, (4) carbonate rocks (from Archie).
EFFECTS OF WATER SATURATION

SEISMIC

Figure 13: The dependence of compressional and shear velocities and their specific attenuation on partial saturation in sedimentary rock. Note the absence of velocity changes with saturation, except for $V_p$ when saturation is close to 100 percent. In contrast, $Q^{-1}$ data suggest that it might be possible to distinguish between low water saturation (low Poisson's ratio, modest $Q_p^{-1}$ and $Q_s^{-1}$), high water saturation (low Poisson's ratio, high $Q_p^{-1}$ and modest $Q_s^{-1}$), and very high water saturation (high Poisson's ratio, low $Q_p^{-1}$ and high $Q_s^{-1}$) (from Murphy, 1982).
PERMEABILITY EFFECTS

SEISMIC AND ELECTRIC

FIGURE 12.3 Fit of permeability to porosity and irreducible water saturation, based on 155 core samples from three fields. (From Timur, 1968.)
ELECTROMAGNETIC MODELLING CAPABILITIES
AT THE UNIVERSITY OF UTAH

FORWARD SOLUTIONS

- 1D, 2D, and 3D solutions for arbitrary sources and sub-radar frequencies.

INVERSE SOLUTIONS

- 1D and 2D for many sources and sub-radar frequencies
- Preliminary 3D
POSSIBLE SOURCE-RECEIVER GEOMETRIES

SURFACE OR BOREHOLE GROUNDED SOURCE-GROUNDED RECEIVER, DIRECT CURRENT

ADVANTAGES
- LARGE SIGNAL POSSIBLE
- RELATIVELY WELL-DEVELOPED INTERPRETATION SOFTWARE

DISADVANTAGES
- SENSITIVE TO SMALL FEATURES CLOSE TO THE BOREHOLE
- INVERSION HIGHLY NON-UNIQUE, NEEDS CONSTRAINTS
- PROBABLY IMPOSSIBLE TO USE WITH STEEL CASING
POSSIBLE SOURCE-RECEIVER GEOMETRIES
SURFACE OR BOREHOLE GROUNDED SOURCE-MAGNETIC FIELD RECEIVER, TIME DOMAIN

V V V

V

SOURCES

V

V

Receivers

ADVANTAGES

• LARGE SIGNAL POSSIBLE
• STEEL CASING PERMITTED IN RECEIVER BOREHOLE
• RECEIVER MISALIGNMENT NOT CRITICAL

DISADVANTAGES

• ONLY MAGNETIC FIELD PARALLEL TO STEEL CASING PROPAGATES THROUGH CASING. ONLY LOW FREQUENCY FIELDS PROPAGATE THOUGH CASING.

• PREFERENTIAL FIELD DIRECTION MAY CAUSE LACK OF RESOLUTION.
POSSIBLE SOURCE-RECEIVER GEOMETRIES

BOREHOLE MAGNETIC SOURCE-MAGNETIC FIELD RECEIVER, FREQUENCY DOMAIN

<table>
<thead>
<tr>
<th>Sources</th>
<th>Receivers</th>
</tr>
</thead>
</table>

ADVANTAGES
- STEEL CASING PERMITTED IN BOTH TRANSMITTER AND RECEIVER BOREHOLES

DISADVANTAGES
- ONLY MAGNETIC FIELD PARALLEL TO STEEL CASING PROPAGATES THROUGH CASING. ONLY LOW FREQUENCY FIELDS PROPAGATE THROUGH CASING.
- PREFERENTIAL FIELD DIRECTION MAY CAUSE LACK OF RESOLUTION
3D EOR FIELD GEOMETRY

PLAN

LONGITUDINAL SECTION

CROSS-SECTION

VIEWGRAPH 21
Figure 24. The generic reservoir model upon which all subsequent models are based. The transmitter-receiver configuration used for each of the inversions is shown. Although not shown here transmitters and receivers are actually located in both boreholes to yield a full cross-borehole data set.
Figure 25. (a) The true model of the oil reservoir before the onset of a hot water flood. (b) The computed model after 9 iterations given a starting model of a 10 Ω·m half-space. The computed resistivity and percent standard deviation of each parameter are given.
Figure 26. (a) The true model of the oil reservoir after the hot water flood has migrated 20 m toward the production well. (b) The computed model after 5 iterations given a starting model of a 10 Ω·m half-space. The computed resistivity and percent standard deviation of each parameter are given.
Figure 27. (a) The true model of the oil reservoir after the hot water flood has migrated 40 m toward the production well. (b) The computed model after 5 iterations given a starting model of a 10 Ω·m half-space. The computed resistivity and percent standard deviation of each.
Figure 28. (a) The true model of the oil reservoir after the hot water flood has migrated 50 m toward the production well. (b) The computed model after 7 iterations given a starting model of a 10 Ω·m half-space. The computed resistivity and percent standard deviation of each parameter are given.
Figure 29. (a) The true model of the oil reservoir after the steam flood has migrated 40 m toward the production well. The steam bank is lead by a 20 m wide hot water/oil bank. (b) The computed model after 12 iterations. The resistivities of the starting model are 10 Ω·m for the bounding strata and 500 Ω·m for all of the parameters contained within the reservoir zone. The computed resistivity and percent standard deviation of each parameter are given.
Steam Flood - Steam Front at $x = 50\text{m}$, Hot Water Front at $x = 60\text{m}$

True Model

- Host Medium - $\rho = 10\Omega\cdot\text{m}$
- Steam Bank - $\rho = 500\Omega\cdot\text{m}$
- Hot Water/Oil Bank - $\rho = 10\Omega\cdot\text{m}$
- Oil Bank - $\rho = 50\Omega\cdot\text{m}$

(a)

Computed Model After 12 Iterations

- $10 \pm 0.1\%$
- $501 \pm 0.4\%$
- $591 \pm 11\%$
- $10 \pm 1\%$
- $51 \pm 2\%$
- $50 \pm 2\%$

$\chi^2_R = 0.96$

(b)
Figure 30. (a) The true model of the oil reservoir after the nonvertical steam flood has migrated toward the production well. The steam bank is lead by a hot water/oil bank. (b) The computed model after 10 iterations. The resistivities of the starting model are 10 $\Omega \cdot m$ for the bounding strata and 500 $\Omega \cdot m$ for all of the parameters contained within the reservoir zone. The computed resistivity and percent standard deviation of each parameter are given.
Steam Flood - Discontinuous Front

True Model

100 m

500 m

20 m

Injection Well

Production Well

- Host Medium - $\rho = 10\Omega \cdot m$
- Steam Bank - $\rho = 500\Omega \cdot m$
- Hot Water/Oil Bank - $\rho = 10\Omega \cdot m$
- Oil Bank - $\rho = 50\Omega \cdot m$

(a)

Computed Model After 10 Iterations

10 ± 1%

501 ± 1 % 467 ± 13 % 609 ± 36 % 10 ± 4 % 51 ± 1 %

507 ± 1 % 10 ± 4 % 10 ± 3 % 49 ± 4 % 50 ± 1 %

(b)

$\chi^2_R = 0.96$
Figure 31. (a) The true model of the oil reservoir capped with a variable resistivity shale layer after the nonvertical steam flood has migrated toward the production well. The steam bank is lead by a hot water/oil bank. (b) The computed model after 9 iterations using an incorrect parameterization which does not account for the shale layer. The resistivities of the starting model are $10 \ \Omega \cdot m$ for the bounding strata and $500 \ \Omega \cdot m$ for all of the parameters contained within the reservoir zone. The computed resistivity and percent standard deviation of each parameter are given.
Steam Flood - Discontinuous Front - With Shale Layer
Incorrect Parameterization

True Model

Injection Well

Computation Model After 9 Iterations

\( \chi^2_R = 18.8 \)
Figure 32. (a) The true model of the oil reservoir capped with a variable resistivity shale layer after the norventical steam flood has migrated toward the production well. The steam bank is lead by a hot water/oil bank. (b) The computed model after 11 iterations using a correct parameterization which accounts for the shale layer. The initial resistivities of both the bounding strata and the shale layer are 10 Ω·m and the initial resistivities of all parameters within the reservoir zone are 500 Ω·m. The computed resistivity and percent standard deviation of each parameter are given.
Steam Flood - Discontinuous Front - With Shale Layer Correct Parameterization

True Model

- $\rho = 2.5 \, \Omega \cdot m$
- $\rho = 3 \, \Omega \cdot m$
- $\rho = 10 \, \Omega \cdot m$

Host Medium - $\rho = 10 \, \Omega \cdot m$
Steam Bank - $\rho = 500 \, \Omega \cdot m$
Hot Water/Oil Bank - $\rho = 10 \, \Omega \cdot m$
Oil Bank - $\rho = 50 \, \Omega \cdot m$

Injection Well | Production Well

Computed Model After 11 Iterations

- $10 \pm 1\%$
- $2.5 \pm 1\%$
- $2.4 \pm 2\%$
- $3.0 \pm 5\%$
- $2.5 \pm 8\%$
- $10 \pm 4\%$
- $497 \pm 0.03\%$
- $650 \pm 24\%$
- $276 \pm 33\%$
- $11 \pm 9\%$
- $51 \pm 1\%$
- $501 \pm 2\%$
- $9 \pm 5\%$
- $11 \pm 5\%$
- $48 \pm 5\%$
- $51 \pm 1\%$

$\chi^2_R = 0.96$

Injection Well | Production Well

(b)
INTRODUCTION TO 3D EM MODELLING

BASIC GEOMETRY

\[ \begin{align*} \sigma = 0 & \quad \sigma_1 \\
\sigma_2 & \quad \sigma_3 \\
\bar{j}_p / \bar{m}_p & \quad \bar{\sigma}_b \\
\end{align*} \]

Fig. 1. General model for formulation of a numerical solution.

INTEGRAL EQUATIONS FORMULATION

\[ \bar{E}(\bar{r}) = \bar{E}_p(\bar{r}) + \int_{\sigma_b} \bar{G}(\bar{r}, \bar{r}') \cdot \bar{\sigma}_b(\bar{r}') \bar{E}(\bar{r}') \, dv' \]
\[ \bar{\sigma} \cdot \bar{J}_s = \bar{E}_p \]
3D EOR TRANSMITTER-RECEIVER GEOMETRY

PLAN

CROSS-SECTION

CROSS-SECTION

Transmitter positions (x)  0.480  0.490  0.480  0.490
Receiver positions (z)  0.510  0.510
0.530  0.530
0.540  0.540

VIEWGRAPH 32
### 10 ohm-m case

<table>
<thead>
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<th>In-phase (gammas)</th>
<th>Quadrature (gammas)</th>
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### 500 ohm-m case

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<td>10,000.</td>
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Correction To Polarization Imaging Principle

Jerry Schuster

Bradley (Prestack Migration using a Polarization Imaging Principle, 1987 Utah Annual Tomography Report) presented a migration method which could distinguish crack reflections from layer interface reflections. The key idea is that reflections recorded by source-receiver pairs under a crack reflector will have the same polarity as recordings from source-receiver pairs above the crack reflector. However, if the reflector is a layer interface, then the recordings will be opposite in phase. Thus, a crosswell experiment can distinguish crack reflections from layer reflections by the absence or presence of polarity reversal in recordings above and below the reflector. This part is true.

The part that is incorrect is that pre-stack Kirchhoff migrated sections will also exhibit this polarity phenomena. This is not correct. Only a migration method such as ray map migration will allow you the option of preserving the seismic section's polarity in the migrated section. The normal Kirchhoff migration method does not allow this flexibility. Numerical results in Bradley's paper suggested this to be true, but it isn't.

I would like to thank Dr. John Sherwood for pointing out my errors in formulating this principle.