Anti-aliasing Condition and Filter for Reciprocity Equations of Correlation Type

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ABSTRACT
An anti-aliasing formula is derived for interferometric redatuming of seismic data. More generally, this formula is valid for numerical implementation of the reciprocity equations of correlation type, which is used for redatuming, extrapolation, interpolation, and migration. The anti-aliasing condition can be, surprisingly, more tolerant of a coarser trace sampling compared to the standard anti-aliasing condition. Numerical results with synthetic data show that interferometry artifacts are effectively reduced when the anti-aliasing condition is applied to interferometric redatuming.

INTRODUCTION
Redatuming wavefields using Green’s theorem, also known as backward extrapolation of wavefields, has long been used in the oil industry to remove elevation statics or to mitigate the defocusing effects of certain geologic features, such as weathering zones or salt bodies (Berryhill, 1979; Wapenaar and Berkhout, 1989; Bevc, 1997; Schneider, 1978). The key idea is to multiply the shot gather data \( D(A|\mathbf{x}) \) by the conjugate of a weighted Green’s function \( G(B|\mathbf{x}) \) and sum along the source position \( \mathbf{x} \) to estimate the wavefield for a source along a deeper datum level defined by the position vector \( \mathbf{B} \) (see Figure 1). Mathematically this operation is performed in the frequency domain as

\[
\mathbf{A}, \mathbf{B} \in V_0 \quad D(A|\mathbf{B}) \approx \int_{S_0} w(A, B, x) G(B|x) D(A|x) d^2x \quad (1)
\]

where \( \mathbf{A} \) is the trace position, \( S_0 \) is the surface area along which the shots are located, \( w(A, B, x) \) represents the weighting function that depends on the source-receiver coordinates and frequency \( \omega \), and \( D(A|\mathbf{B}) \) is the redatumed data at the new datum surface with new source position at \( \mathbf{B} \). Here, the Green’s function \( G(B|x) \) is model-based and is typically computed by using a ray-based method or a numerical solution to the acoustic wave equation.

The rigorous mathematical foundation for this redatuming equation was initially developed by Porter (1970) and Bojarski (1983) and is also known as the reciprocity equation of correlation type (de Hoop, 1995). A generalization of the reciprocity equations to acoustic, electromagnetic, elastodynamic, poroelastic, and electroseismic waves is described by Wapenaar (2007) who used a unified matrix-vector wave equation. Artifacts associated with redatuming wavefields measured on a coarse or irregular grid can be mitigated by a least squares approach (Liu and Sacchi, 2004; Ferguson, 2007).

Redatuming wavefields is one of two steps\(^1\) in seismic migration (Schneider, 1978; Claerbout, 1992) which estimates the subsurface reflectivity distribution from reflection data. Similar to the integrand in equation 1, the migration kernel consists of the product of the data and the conjugated Green’s function computed for an assumed velocity model.

In the latter part of the 20th century, a new type of redatuming was introduced such that the model-based Green’s function in equation 1 is replaced by a data-based Green’s function (Claerbout, 1968). Applications for redatuming wavefields included helioseismology (Duvall et al., 1993; Rickett and Claerbout, 1999) , earthquake seismology (Schérbann, 1987a,b; Shapiro and Campillo, 2004; Gerstoft et al., 2006; Larose et al., 2006), physical acoustics (Fink, 1993, 1997, 2006; Lobkis and Weaver, 2001), and exploration geophysics (Schuster et al., 2004; Calvert et al., 2004). These redatuming procedures were denoted by a variety of names such as reverse time acoustics, day-light imaging, interferometric imaging, and virtual source imaging, but they were eventually unified under the name

\(^1\)Migration is defined as extrapolating the wavefield recorded at the surface to a deeper depth level followed by the application of an imaging condition (Claerbout, 1992).
unrecoverable errors in estimating aliasing (Maeland, 2004); equivalently, sampling the in-the-phase variation of $\Gamma(G|x)$ spatially varies more slowly than the phase spectrum $\phi(B,A,x)$ and $x_{i-1}$ denotes the $x_{i-1}$ source position adjacent to the $x_i$ source position. This criterion is similar to the Nyquist sampling criterion for properly sampling a sinusoidal function $\sin(\phi(B,A,x))$, and prevents aliasing artifacts in the summation.

Physical Meaning
The physical meaning of the aliasing condition in equation 5 is determined by decomposing the Green’s functions into a summation of direct waves, primary reflections, and other events:

$$G(B|A) = G(B|A)_{{\text{direct}}} + G(B|A)_{{\text{primary}}} + ...$$

Figure 1a illustrates the direct and ghost reflection rays for these Green’s functions, which can be asymptotically represented as

$$G(B|x)_{{\text{direct}}} \approx \beta(B,x)e^{i\omega\tau_{{\text{direct}}}};$$

$$G(x|A)_{{\text{ghost}}} \approx \alpha(A,x)e^{i\omega\tau_{{\text{ghost}}}}.$$  

Here, $\tau_{{\text{direct}}}$ is the specular traveltimes for a direct arrival from $x$ to $x'$, and $\tau_{{\text{ghost}}}$ is the specular ghost reflection for a source at $x$ and receiver at $A$. The terms $\alpha(A,x)$ and $\beta(B,x)$ account for geometrical spreading in the reflection and direct waves, respectively.

Redatuming the receiver from $x$ to $B$ (see Figure 1) is accomplished by temporally correlating the direct arrival with the ghost reflection and summing the result over the source coordinates as described in equation 3. Mathematically this is accomplished in the frequency domain by the replacements: $G(B|x) \rightarrow G(B|x)_{{\text{direct}}}$ and $G(x|A) \rightarrow G(x|A)_{{\text{ghost}}}$ in equation 4 to give

$$\Gamma(B,A,x) \approx k\Delta x^2\alpha(A,x)\beta(B,x)e^{i\omega(\tau_{{\text{ghost}}} - \tau_{{\text{direct}}})},$$

where the phase is denoted as $\phi(A,B,x) = \omega(\tau_{{\text{ghost}}} - \tau_{{\text{direct}}})$. Plugging this phase into the anti-aliasing condition of equation 5 yields

$$|\phi(B,A,x_i) - \phi(B,A,x_{i-1})| < \pi,$$

where it is assumed that the amplitude term $|\Gamma(B,A,x)|$ spatially varies more slowly than the phase spectrum $\phi(B,A,x)$ and $x_{i-1}$ denotes the $x_{i-1}$ source position adjacent to the $x_i$ source position. This criterion is similar to the Nyquist sampling criterion for properly sampling a sinusoidal function $\sin(\phi(B,A,x))$, and prevents aliasing artifacts in the summation.

**Anti-Aliasing Condition for Interferometric Redatuming**

For simplicity of exposition, we will use the far-field approximation to the reciprocity equation of correlation type (Wapenaar, 2004; Schuster, 2009) such that the Green’s function $G(A|B)$ is given by

$$A,B \in V_0 \quad Im[G(B|A)] = k \int_{S_0} G(B|x)G(x|A)d^2x,$$

where $k$ is approximated by the wavenumber at the receiver location, the integration is over the surface denoted by $S_0$ and the points $A$ and $B$ are within the volume $V_0$ bounded by $S_0$ and the lower half-space. Here we assume that the source signature has been deconvolved and the above equation can be estimated from equation 1 by replacing $D(A|x)$ by $G(A|x)$, $D(A|B)$ by $Im[D(A|B)]$, and $w(A,B,x)$ by $k$. Note, reciprocity allows for the interchange of the source and receiver positions in the Green’s functions so $G(A|B) = G(B|A)$.

Approximating the integral in equation 2 by a discrete sum over evenly spaced grid points along the source line $S_0$ yields the following equation:

$$A,B \in V_0 \quad Im[G(B|A)] = \sum_i \Gamma(B,A,x_i),$$

where

$$\Gamma(B,A,x_i) = kG(B|x_i)G(x_i|A)\Delta x^2,$$

$$= |\Gamma(B,A,x_i)|e^{i\phi(B,A,x_i)}.$$  

Here, $\Delta x$ is the spatial sampling interval along the source line denoted by $S_0$, $x_i$ is the $ith$ position of the source along $S_0$ and $\phi(B,A,x_i)$ represents the phase of the integrand in equation 2.

Sampling a signal too sparsely in space causes spatial aliasing (Maeland, 2004); equivalently, sampling the integrand in equation 2 too coarsely in space will result in unrecoverable errors in estimating $G(A|B)$. Therefore, the general anti-aliasing condition for equation 3 is that the phase variation of $\Gamma(B,A,x_i)$ between adjacent geophones is less than $\pi$. For equation 4 this means

$$|\phi(B,A,x_i) - \phi(B,A,x_{i-1})| < \pi,$$

where $\Delta x = |x_i - x_{i-1}|$. Inserting these terms into equa-
tion 9 and assuming a linear moveout in the events between adjacent geophones (i.e., neglect 2nd-order terms in the Taylor expansion) yields

$$\Delta x < \frac{T}{2 \left| \frac{\partial \tau_{ghost}}{\partial x} \right|}$$

which compares to the anti-aliasing condition for properly sampling either a direct wave or a primary reflection:

$$\Delta x < \frac{T}{2 \left| \frac{\partial \tau_{direct}}{\partial x} \right|}$$

For positive moveout velocities, the denominator in equation 11 is less than those in equation 11, so we conclude that the anti-aliasing condition described by equation 11 is more tolerant of coarse sampling intervals than the conditions described by equations 12. Note events with moveout velocities opposite in sign require more stringent geophone sampling criterion in equation 11. In contrast, the reciprocity equations of convolution type (Wapenaar, 2007) results in the anti-aliasing condition of

$$\Delta x < \frac{T}{2 \left( \left| \frac{\partial \tau_{ghost}}{\partial x} \right| + \left| \frac{\partial \tau_{direct}}{\partial x} \right| \right)}$$

which is less tolerant of coarse geophone sampling for moveout velocities with the same polarity.

The physical meaning of the interferometric anti-aliasing condition in equation 11 is discovered by recognizing its similarity to applying a local linear moveout (LLM) to reflection data. In this case the linear moveout velocity is that of the direct arrival with apparent velocity $\frac{\partial \tau_{direct}}{\partial x}$, where the associated time shift lessens the likelihood of aliasing the reflection event with a coarse source spacing.

The product of two Green’s functions described by equation 6 yields many combinations of events that lead to, e.g., a primary event with the receiver redatumed to position B. For example a source side ghost in Figure 2a correlated with the related ghost reflection in Figure 2b yields the redatumed primary reflection in Figure 2c. Therefore the anti-aliasing condition in equation 11 can be generalized to

$$\Delta x < \frac{T}{2 \left| \frac{\partial \tau_{event1}}{\partial x} - \frac{\partial \tau_{event2}}{\partial x} \right|}$$

where event1 at A and event2 at B correspond to the two distinguished events in the recorded data that are transformed into a redatumed event under correlation and summation of trace pairs at A and B.

### NUMERICAL TESTS

We test the interferometric anti-aliasing condition on synthetic traces generated for an Vertical Seismic Profile (VSP) experiment where the VSP gathers are transformed into virtual surface seismic profile (SSP) data. Figure 4 depicts the recorded VSP data generated by a first-order solution to the acoustic wave equation for the 3-layer model shown in Figure 3. 100 receivers are deployed in the well with a recording spacing of 20 m, and 200 shots are excited on the surface with an interval of 20 m. VSP traces are transformed into virtual SSP gathers using equation 2. Similar to the scheme of interferometric migration of VSP ghost in He et al. (2007), only the direct arrival of $G(B|x)$ is included in the correlation redatuming.

Figures 5a-c depict the virtual SSP traces redatumed with VSP gathers for different recording spacings. Here the minimum apparent wavelength $\lambda_{min}$ is estimated to be 164 m. Comparing the virtual SSP gathers with the true SSP gather in Figure 6 shows increasing artifacts for coarser recording spacing in the well. Similar to the anti-aliasing filter used in migration (Gray, 1992; Lumley et al., 1994), we can low-pass filter the aliased data to avoid violation of the anti-aliasing condition in equation 14. This procedure consists of following steps:

1. The LLM $\frac{\partial \tau_{direct}}{\partial x}$ of $G(B|x)$ is estimated from the local slope of the direct wave traveltime curve of the VSP shot gather.

2. For each time sample of $G(A|x)$, the LLM $\frac{\partial \tau_{ghost}}{\partial x}$ is calculated using local slant stack in the VSP shot gather, and equation 13 is applied to check if current sample is aliased. The value of aliased samples in the traces is set to zero, and a filtered trace $G_{f}(A|x)$ is obtained.

3. $G(B|x)$ and $G_{f}(A|x)$ are input to equation 2 and the redatuming result with the anti-aliasing condition is obtained.

Figures 5d-f depicts the results of this anti-aliasing operation where many of the aliasing artifacts are removed.

To reduce the interferometry artifacts, we also applied dip filtering to the VSP gathers so that only the down-going arrivals remain. Figure 7 shows the VSP gather after a dip filter, to give mainly the down-going events. Figures 8a-c depict the virtual SSP gathers obtained with the VSP gathers after dip filtering. Comparing Figures 8a-c with Figures 5a-c indicates fewer artifacts in the virtual SSP gathers when only the down going arrivals are employed in the interferometric redatuming. However, when the anti-aliasing condition is applied, many of the remaining artifacts are removed from the virtual SSP gathers as shown in Figures 8d-f.
CONCLUSIONS

An anti-aliasing condition is derived for interferometric redatuming of seismic data, and should be applicable to any numerical implementation of the reciprocity equation of correlation type. This formula can be used for interferometry, redatuming, extrapolation, interpolation, and migration.

The anti-aliasing formula is equivalent to the one obtained by locally time shifting the event of interest by a linear moveout correction, where the moveout velocity is that of an event with a shorter raypath. Honoring the anti-aliasing condition allows unaliased interferometric redatuming and the design of an anti-aliasing filter that mitigates aliasing artifacts.

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Figure 3: The velocity model and VSP recording geometry. Both the recording spacing in the well and the shooting interval on the surface are 20 m.

Figure 4: A VSP shot gather. The surface shot is deployed at (300 m, 10 m).

Figure 6: The true SSP gather simulated with finite difference modeling.

Figure 7: The downgoing arrivals from the VSP shot gather in Figure 4.
Figure 1: Goal of redatuming here is to extrapolate receiver positions from \( x \epsilon S_o \) along surface to the position at \( B \). Traces in the frequency domain are denoted by \( D(A|x) \) and recorded by geophones excited by a source at \( x \). a) and b) depict original rays, and c) depicts the rays of the ghost reflection arrival after datuming the receiver from \( x \) to \( B \).

Figure 2: Correlation of a a) source-side ghost with a b) related ghost reflection followed by summation over source locations at \( x \) yields the redatumed primary reflection in c).
Figure 5: SSP gathers redatumed using standard and anti-aliased interferometry from the VSP gathers with different recording spacings in the well. The comparison shows the anti-aliasing condition effectively removes many of the artifacts in the interferometry results.
Figure 8: SSP gathers redatumed using standard and anti-aliased interferometry from the down-going arrivals of the VSP gathers with different recording spacings in the well. The comparison shows the anti-aliasing condition effectively removes many of the artifacts in the interferometry results.
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