Wave-Equation Wavefront Migration
Weiping Cao, University of Utah

Summary

The wave-equation wavefront migration (WWM) is a potentially efficient form of wave equation migration, where the finite-difference stencil is only applied around the leading portion of the wavefront. WWM has the accuracy of wave-equation migration, fewer aliasing and migration artifacts, and can potentially be much faster than the standard reverse-time migration. The potential drawbacks are the memory intensive operation of storing the Green’s function at each grid point, and that only a few of the earliest arrivals are used in downward imaging condition. In this report, we tested WWM on the 2-D SEG/EAGE salt model, a 2-D line in the Gulf of Mexico, and a 3-D synthetic model. Migration images show that the 2-D wavefront reverse-time migration gives results that are comparable in accuracy to the standard reverse time migration. The preliminary result for 3-D wavefront reverse time migration is encouraging.

Introduction

Among the various migration methods there are two extremes, Kirchhoff migration and reverse-time migration. Kirchhoff migration is efficient because it employs a high-frequency approximation so that it can only resolve the velocity variations which vary more slowly than the scale of the source wavelength. On the other hand, reverse-time migration is accurate but the calculations are intensive because there are no high-frequency assumptions about the data and the method considers all arrivals.

The wave equation wavefront migration method (Schuster, 2001) has nearly the same accuracy of the reverse-time migration, but can, in principle, be faster than the reverse-time migration because it applies the finite-difference stencil only along the leading portion of the wavefront. Considerable computation time can be saved because the finite-difference computations are implemented in a smaller region than the standard reverse time migration (RTM). Furthermore, some migration artifacts can be avoided since we only use the waveform part of the forward wavefield to image the medium with the backpropagated wavefield.

In this report, we test the WWM scheme with a 2-D SEG/EAGE salt model and a 2-D line from the Gulf of Mexico. Results show the possibility of reducing the cost of reverse-time migration by applying the FD stencil along each part of the model where the waveform is included. The 3-D WWM is also implemented with a simple 3-D model, which indicates the application to 3-D data sets.

Methodology

The theory of WWM was described in Schuster (2001). The WWM migration can be expressed as:

\[ m_{\text{mig}}(x') = \int_{\text{data space}} [g(x_s, t|\mathbf{x}', 0) \ast g(x_s, t|\mathbf{x}', 0)] \otimes d(x_s, t)|_{t=0} dx_s dx'_s. \tag{1} \]

where \( \ast \) denotes temporal convolution and \( \otimes \), together with \( |_{t=0} \), represents the correlation at zero-lag time which is equivalent to a 3-D dot-product of the backward and forward propagated wavefields. The \( d(x_s, t) \) term represents the trace at \( x_s \), while the \( g(x_s, t|\mathbf{x}', 0) \) and \( g(x_s, t|\mathbf{x}', 0) \) terms represent the scattered Green’s functions which propagate the energy from the source point at subsurface \( \mathbf{x}' \) to surface points at \( x_s \) and \( x_s' \), respectively. The bracketed term \( F(x, x', \mathbf{x}'; t) = [g(x_s, t|\mathbf{x}', 0) \ast g(x_s, t|\mathbf{x}', 0)] \) represents the focusing kernel obtained by convolving two Green’s functions computed using a finite difference solution of the wave equation. For the wavefront reverse time migration scheme, the Green’s functions are computed by finite differencing along the leading portion of the wavefronts instead of the whole model, and the bandlimited Green’s functions are stored in memory to form focusing kernels and migrate the recorded seismic data. Therefore, considerable computation is saved in the wavefront finite difference process, and fewer migration artifacts are introduced since only the waveform portion of the wavefield is involved in imaging. The potential drawback with WWM is the considerable memory requirements for storing the Green’s functions for all grid points.

The typical RTM code rearranges equation 1 so that it becomes

\[ m_{\text{mig}}(x') = \int_{\text{Forwardwavefield}} g(x_s, t|\mathbf{x}', 0) \cdot \int_{\text{Backpropagateddata}} g(x_s, t|\mathbf{x}', 0) d(x_s, t) dt. \tag{2} \]

In the typical RTM scheme the finite difference stencil visits each point in the entire model per time step. This is computationally wasteful compared to WWM, for which the \( g(x_s, t|\mathbf{x}', 0) \) is computed only over a small portion of the waveform per time step.
Implementation of WWM

To implement WWM according to equation 1, we need to compute the Green’s functions for all shot and receiver positions and model grid points using a wavefront finite difference procedure; hence we can form the focusing kernels and migrate the recorded seismograms. However, it is still difficult to store all these Green’s functions in the computer memory. To circumvent this difficulty, we implement WWM in the way similar to a typical RTM shown in equation 2. That is, we compute the forward wavefield from the shot position through wavefront finite differencing and store the Green’s functions from the shot to all model grids; then with these Green’s functions we can reconstruct the wavefront wavefield for each time step (Figure 1 shows an example of the reconstructed wavefront wavefield); we backpropagate the reversed seismograms at receiver positions using finite difference in the whole medium as standard RTM does at each time step; then the summation of the dot products between the forward wavefront wavefield with the backpropagated wavefield for all time steps gives the migration image for one shot gather. Finally we sum the migration images for all shots and obtain the migration results for all the data.

The detailed implementation can be described as:

1. Calculate the traveltimes $\tau_{xs}$’s from the shot position to all model grids, where $s$ represents the shot position and $x$ denotes any model grid.

2. Define the wavefront region for the current time step from traveltimes or minimum and maximum velocities, apply finite difference stencil only in this wavefront region, and store the wavefields at grid points where the current time value is between $\tau_{xs}$ and $\tau_{xs} + 2T_d$ at the grid point, where $T_d$ denotes the period of the dominant frequency.

3. Iterate step 2 until the current time exceeds the maximum traveltime from the shot position to the model grids.

4. Reconstruct the forward wavefront wavefield from the stored wavefronts and take the dot product of it with the backpropagated data, and iterate this for all time steps. Sum all these dot product results to obtain the WWM image for this shot gather.

5. Loop 1, 2, 3 and 4 over all shot gathers and get the WWM image for all the data.

Data Examples

2-D SEG/EAGE Salt Model

I tested the WWM method on with the 2-D SEG/EAGE salt model shown in Figure 2. The data include 200 shots with 200 traces for each shot gather, and the shot and geophone intervals are both 14.6 m. There are 6000 time samples with a time interval of 0.5 ms in each trace. Figure 3 shows the wavefront reverse time migration image for the 200 shot gathers. The figure indicates a clear imaging of the structures for most of the subsalt reflectors.

2-D Real Data Test

A 2-D line from a 3-D data set from the Gulf of Mexico is used to test the WWM scheme. In this 2-D line there are 1001 shot gathers with a shot interval of 12.5 m. In the original data there 41 traces in each gather, and the offset ranges from 0 to 6000 m. We interpolate the traces so that there are 120 traces in each shot gather with an even receiver interval of 50 meters. Figure 4 shows an interpolated shot gather. Figure 5 shows the RMS velocity model, and 120 shot gathers are migrated using WWM and compare the resulting image with the Kirchhoff time migration result. Figure 6 show the Kirchhoff time migration image and the WWM image for the same region. The comparison shows that in the WWM image resolution is remarkably higher and there is more information about the deep structures.

3-D Synthetic Data Test

The implementation of 3-D WWM is similar to the 2-D method. We apply a finite difference method to only part of the model containing the leading portion of the wavefront and store the wavefront wavefield. Then the seismograms recorded at receivers are backpropagated and crosscorrelated with the forward wavefront wavefield to image the velocity model. The 3-D model shown in Figure 7 is designed and synthetic data are generated using a finite-difference solution to the wave equation. The model size is 200 by 200 grid points in the horizontal direction and 80 grid points in the depth direction, and the grid point interval is 12.2 m. There is a flat reflector at the depth of 854 m and a high velocity block in the low velocity layer. The velocities of the top layer, the bottom layer and the block are 2134 m/s, 2439 m/s and 2743 m/s respectively. There 361 shots with 122 m shot intervals in the $x$ and $y$ directions. There are 10000 receivers with a 24.4 m interval in both the $x$ and $y$ directions. Figure 8 shows a 2-D line from a common shot gather. Then the 3-D WWM algorithm is used to migrate the data. Figures 9 and 10 show the comparisons between two sections from the 3-D WWM image cube obtained from 9 shot gathers and the velocity section at the same positions. The comparison indicates that the positions of the reflector and high-velocity block are correctly imaged though there are considerable artifacts due to the insufficient stack.

Conclusions

WWM is proposed as a computationally efficient scheme to implement reverse time migration. Since a much
smaller region of the model is visited by WWM per time step, THEN WWM in principle can be faster than standard reverse time migration and still give desirable images. The main drawback is storage of the Green’s functions at each grid point of the model. 2-D tests with synthetic data and real data indicate that WWM gives promising results. The 3-D WWM method is also applied to a simple 3-D synthetic model. The result shows the possibility for further application. Future work includes a quantitative performance comparison with the standard reverse time migration and further testing on more complex synthetic data and real data.

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Figure 1: Snapshot of the wavefront wavefield obtained through finite differencing along the partial model that contains the leading portion of the wavefront.

Figure 2: 2-D SEG/EAGE salt model. There are 150 and 645 grid points in the z and x directions respectively.

Figure 3: The WWM image from 200 shot gathers.

Figure 4: An interpolated common shot gather from the 2-D real data. The shot is located at (4538 m, 0 m). There are 120 traces in the shot gather with an even receiver spacing of 50 meters.
Figure 5: The input RMS velocity model used in WWM.

Figure 6: (Left) Kirchhoff time migration image and (Right) WWM image for the same region from 120 shot gathers. In the WWM result the image resolution is remarkably higher and there are more information about the deep structures.

Figure 7: The 3-D velocity model with a flat reflector at the depth of 854 m and a high velocity block in the middle of the model.

Figure 8: A 2-D line from a synthetic 3-D shot gather generated using a finite difference code. The shot is placed at (122 m,122 m, 0 m)

Figure 9: (Left) WWM section at x=1220 m from 9 shot gathers and (Right) section of the 3-D velocity model at x=1220 m. The positions of the reflector and high velocity block are correctly imaged.

Figure 10: (Left) WWM section at y=1220 m from 9 shot gathers and (Right) section of the 3-D velocity model at y=1220 m. The positions of the reflector and high velocity block are correctly imaged.
EDITED REFERENCES
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REFERENCES