Redatuming CDP Data below Salt with VSP Green’s Function
Xiang Xiao and Gerard T. Schuster, University of Utah

Summary

Common depth point (CDP) data can be naturally redatumed below salt with a vertical seismic profile (VSP) Green’s function. In this case only VSP and CDP data are required and no velocity model is needed. Here both the Kirchhoff single-arrival datuming and multi-arrival (asymptotic) datuming (KMDA) methodologies are applied to SEG/EAGE salt model data and acceptable redatuming results are achieved. It shows that both of these methods can be used to redatum CDP data beneath the salt without knowing the velocity model of the salt body or overburden. However, the multi-arrival datuming method provides a more accurate datuming result than the single-arrival datuming method. The benefit of this method is that redatumed CDP data can be used, in principle, to more clearly image reflectors beneath the salt.

Theory

The idea of migrating CDP data with VSP Green’s function is addressed by Schuster (2002) in the form of a generalized diffraction stack migration (GDSM) algorithm. He suggested that migrating CDP data with VSP Green’s functions could overcome the problem of defocusing of migration images due to incorrect salt velocity models. A related method is virtual source imaging by Calvert et al. (2004) that naturally extrapolate VSP data so that sources are redatumed from the surface to the well. Schuster (2002) demonstrated the exact reverse-time migration operator obtained purely from VSP field data. A related development is by Wapeanar et. al (2005) who presented an acoustic reciprocity theorem as follows:

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\[
\text{A, B} \in V_0; \ 2Im[G(A|B)] = \int_{D_0} \frac{\partial G(x|B)}{\partial n_x} G(x|A) - G(x|B) \frac{\partial G(x|A)}{\partial n_x} dS_x, (1)
\]

where the integration is along the (earth) surface \(\partial D_0\) that surrounds a 2D volume \(V_0\) and \(G(x|A)\) is the Green’s function for this \(V_0\) media, which solves the Helmholtz equation \((\nabla^2 + k^2)G(x|A) = \delta(x - A)\). Here \(Im\) denotes the imaginary part. The \(G(A|B)\) can be interpreted as single well imaging (SWI) data redatumed from VSP data \(G(x|A)^{VSP}\) and \(G(x|B)^{VSP}\), where \(x\) are the sources and \(A, B\) are the receivers. This formula can be extended for redatuming surface CDP data to become virtual reverse vertical seismic profile (RVSP) data, by allowing the CDP receiver at \(A\) approach the surface \(\partial D_0 = S_{01} + S_{02}\) (see Figures 1 and 2). Assume that the infinitesimal semi-sphere \(S_{02}\) in the Figure 1 is centered about the point \(A\) and forms part of the deformed integration surface \(\partial D_0\); let the spherical coordinate system be centered at \(A\), and shrink the radius to zero. In this case the semi-sphere is so small that \(G(x|B) \rightarrow G(A|B)\) is a constant around the sphere and the non-singular part of the dipole Green’s function \(\frac{\partial G(x|B)}{\partial n_x}\) can be neglected because it will not contribute as the semi-sphere’s radius shrinks to zero. Recalling that the singular part of the Green’s function that solves the Helmholtz equation above is \(G(x|y) = e^{-ikr}/|r|\), the dipole contribution

\[
- \lim_{r \to 0} \int_{S_{02}} G(x|B) \frac{\partial G(x|A)}{\partial n_x} dS_x,
\]

\[= \frac{G(A|B)^*}{4\pi} \lim_{r \to 0} \int_0^{2\pi} \int_0^{\pi/2} \frac{\partial}{\partial r} \frac{1}{r^2} \sin \theta d\phi,
\]

\[= -\frac{G(A|B)^*}{2}.
\]

This dipole contribution can be inserted into equation 1 and leads to the formula for redatuming surface CDP data to become virtual RVSP data:

\[
A \in \partial D_0, B \in V_0; \ 2Im[G(A|B)] + \frac{G(A|B)^*}{2} = \int_{D_0} \left( \frac{\partial G(x|B)}{\partial n_x} G(x|A) - G(x|B) \frac{\partial G(x|A)}{\partial n_x} \right) dS_x, (3)
\]

where \(G(A|B)^{RVSP}\) is the RVSP data redatumed from CDP data \(G(x|A)^{CDP}\) with the VSP Green’s function \(G(x|B)^{VSP}\). This VSP formulation assumes a 2D medium because the well boundary is a line, and so the surface integral should be along a line as well. If the Green’s function is exact, then equation 3 contains the VSP Green’s functions that accounts for all multi-arrival events, which we denote as the multi-arrival dipole datuming method. Following the derivation in Wapeanar (2005), in the high frequency regime, the formula for the multi-arrival monopole datuming is approximated as:

\[
A \in \partial D_0, B \in V_0; \ 2Re[G(A|B)] + \frac{G(A|B)^*}{2},
\]

\[\frac{\partial G(x|B)}{\partial n_x} G(x|A) - G(x|B) \frac{\partial G(x|A)}{\partial n_x} dS_x.
\]

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\[ \approx \int_{\partial D_0} \frac{C_{DP}}{V_{SP}} G(x|A)G(x|B^*) dS_x, \]  
\[ \approx \int_{\partial D_0} \frac{C_{DP}}{4\pi|x-B|^2} G(x|A)e^{-iw_{t_B}} dS_x, \]

where \( \theta \) is the angle between the ray \( xA \) and the normal of surface \( \partial D_0 \). If we asymptotically approximate the 2D VSP Green’s function by \( G(x|B)^{V_{SP}} \approx \frac{1}{4\pi|x-B|^2} e^{iw_{t_B}} \), this formula can be approximated by the single-arrival (asymptotic monopole) approximation:

\[ A \in \partial D_0, B \in V_{01}; \quad \text{Re}[G(A|B)] + \frac{G(A|B)}{2}, \]
\[ \approx \int_{\partial D_0} \frac{C_{DP}}{4\pi|x-B|^2} G(x|A)e^{-iw_{t_B}} dS_x, \]

where \( t_B \) is the natural direct \( P \) traveltime from \( x \) to \( B \). In this report, both equations 5 and 4 are applied to synthetic CDP data associated with the 2D SEG/EAGE salt model. These methods require that the VSP data are available, but not velocity model is needed. The merits and shortcomings of these methods are discussed in the conclusions.

**Numerical Test**

The SEG/EAGE salt model shown in Figure 3 is used to test the fidelity of naturally transforming CDP data to be RVSP data at the well using the VSP data as extrapolators. For the CDP geometry, there are 322 shots evenly distributed at 49 m (160 ft) intervals and 645 receivers at 24 m (80 ft) intervals on the model’s surface. The VSP data share the same sources but only two receivers are buried at the depth of 1.44 km (4720 ft) and 2.42 km (7920 ft), respectively. Our goal is to naturally redatum the CDP sources to the well and transform all surface sources/receivers to RVSP receivers with the VSP Green’s function. Natural redatuming does not require a velocity model and only uses the “natural” data for extrapolation. A 4th-order finite-difference solution to the 2D acoustic wave equation, with a perfectly matched layer (PML) boundary, is used to compute the seismogram with a 10-Hz peak frequency Ricker wavelet as the source wavelet. The VSP common receiver gathers (CRG) with receivers at depths of 1.44 km and 2.42 km are shown in Figures 4 and 5, respectively.

Both single-arrival and multi-arrival datuming methods are applied to the data, and the results are shown in Figures 6, 7, 8 and 9. Head wave traveltimes are picked from the VSP gathers and the datumed traces are muted prior to the picked traveltimes of head waves. Results show that the multi-arrival datumed data contains more subsalt reflections than those by the single-arrival datuming method. Also, there is less noise before the first arrival and at the far offset traces compared to the RVSP CSG by the single-arrival datuming method. There is a tolerable error between the RVSP redatumed data and actual data, which is a result of the monopole approximation and the finite and discrete aperture of the data. The comparison between the synthetic RVSP trace and redatumed one in Figures 10, 11 and 12 show the superiority of multi-arrival datuming methods over traditional single-arrival methods.

**Conclusion**

Numerical test results suggest that both single-arrival and multi-arrival datuming by VSP Green’s function can successfully redatum the surface CDP data to be RVSP data below salt. The superiority over the traditional model-based redatuming method is that no traveltime error will be introduced, no statics at the surface are accounted for and no overburden or salt model is needed for datuming. These can be useful for 3D CDP datuming where a certain number of VSP profiles are provided and there is difficulty in estimating the overburden and salt velocity model (Figure 13).

**Acknowledgements**

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**References**


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Figure 2: Model with VSP geophones B along the well ∂D₁ and CDP sources x and geophones A along the free surface ∂D₀.

Figure 3: SEG/EAGE salt model used in the CDP to RVSP datuming. There are 322 surface shots and 645 surface receivers for the CDP geometry. Only two VSP geophones at the depth of 1.44 km (4720 ft) and 2.42 km (7920 ft) are placed at the offset of 0 km.

Figure 4: A synthetic RVSP common shot gather (CSG) with the shot at the depth of 1.44 km, and 645 receivers evenly distributed along the free surface.

Figure 5: A synthetic RVSP common shot gather (CSG) with the shot at the depth of 2.42 km, and 645 receivers evenly distributed at the free surface. The single-arrival calculated by the eikonal solver is denoted as a dash-dot line, and it doesn’t agree with the synthetic one.

Figure 6: The redatumed RVSP gather by the single-arrival asymptotic monopole datuming method. The shot is located at the depth of 1.44 km. The gather is muted using the picked traveltimes of RVSP head waves in Figure 4 as a reference to the limit of the mute window.

Figure 7: The redatumed RVSP gather by the multi-arrival asymptotic monopole datuming method. The shot is located at the depth of 1.44 km. The same gain and muting are applied here and in Figure 6.
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Figure 8: The redatumed RVSP gather by the single-arrival asymptotic monopole datuming method. The shot is located at the depth of 2.42 km. The gather is muted using the picked traveltimes of RVSP head waves in Figure 5 as a reference to the limit of the mute window.

Figure 9: The redatumed RVSP gather by multi-arrival asymptotic monopole datuming method. The shot is located at the depth of 2.42 km. The same gain and muting are applied here and in Figure 8.

Figure 10: A comparison between the synthetic RVSP trace and the redatumed one for shot at 1.44 km.

Figure 11: A comparison between the synthetic RVSP trace and the redatumed one for shot at 2.42 km.

Figure 12: A zoom view for Figure 11 between 4 s to 6 s.

Figure 13: Dense 3D surface CDP and low fold 3D VSP can be combined together and transformed to high fold 3D reversed VSP. Here we assume the CDP and VSP share the same sources.
EDITED REFERENCES
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REFERENCES