

# Theory of Daylight/Interferometric Imaging: Tutorial

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## Summary

I present the general theory of interferometric imaging (II). Interferometric imaging is any algorithm that inverts crosscorrelated data for the reflectivity or source distribution. As examples, I show that II can be used to image arbitrary reflectivity distributions by migrating ghost reflections in passive seismic data, and II generalizes the receiver-function imaging method used by seismologists, II can be used to migrate free-surface and peg-leg multiples in CDP data. I also show how source distributions can be imaged from passive seismic data.

## Interferometric Imaging

A working definition of *daylight data* might be defined as seismic data recorded on the earth's surface but generated by randomly distributed sources in the earth; and the source time history of each daylight source is a random time series. Seismic *daylight* implies that there are many seismic sources so that their combined intensity is as bright as *daylight*. The seismic "daylight imaging" method, pioneered by Claerbout (1968), Cole (1995), and Rickett and Claerbout (1999), is mostly based on Claerbout's conjecture: crosscorrelation of two *daylight* traces at surface locations A and B yields an entangled trace *equivalent* to a reflection trace at B generated by a source at A. Equivalent means that the kinematics of some correlated reflections are the same as those of primary reflections generated by a virtual source at A and received at B. Until now, the applications and theory for their methodology appeared to be restricted to a random dense distribution of deeply buried sources and to layered media. The sources also can be uniformly distributed on the surface.

Here I present the mathematical framework for imaging crosscorrelated data, i.e., interferometric imaging, for arbitrary reflectivity or source distributions. I show that interferometric imaging extends the daylight imaging concept to any number or distribution of sources and to arbitrary reflectivity distributions. Moreover, it offers new imaging opportunities, such as a very simple means to migrate multiples in data, migrate transmitted waves, or locate unknown source locations from daylight data. Simply put, interferometric imaging can be described as crosscorrelation migration (Schuster, 1999; Schuster and Rickett, 2000), or an extended form of autocorrelation migration (Schuster et al., 1997).

Instead of exploiting the entire phase of arrivals, interferometric imaging exploits the phase difference between different arrivals. These phase differences can reveal subtle variations between the arrivals, which can be indicative of subtle changes in the medium properties. For example, sunlight on an oil slick at sea will produce a rainbow of interference patterns: reflections from the top of the oil slick interfere with those from its bottom to reinforce at certain light colors and thicknesses of the oil slick. The common raypath of the top and bottom reflections have equal and opposite phase that cancel one another, and the phase difference we see accounts for just the transit path in the oil. Similarly, seismologists can construct interferometric data by crosscorrelating trace A with trace B. In this way we can exploit the phase difference between a certain arrival in trace A with certain arrivals in trace B. We will now generalize this interferometric imaging idea so that it extends the daylight imaging idea of Claerbout to arbitrary reflectivity and source distributions. I also show how II can be used to image free-surface multiples or peg-leg multiples from CDP data, generalize the receiver-function imaging of PS transmitted waves used by seismologists, and image source locations of unknown sources at depth.

**Ghost Reflection Imaging with Buried Sources.** The heuristic description of interferometric imaging is described below for the example of a ghost reflection and an unknown buried source at  $s$ . Here the ghost is defined to be a reflection from the earth's free surface. Other ghost reflection imaging conditions such as for a PS converted ghost can be constructed in a similar fashion.

1. A ghost reflection is shown in Figure 1a, where the total traveltime time is a composite of traveltimes

for each leg of the journey. The frequency-domain traces at positions B and A are given by

$$d_B = -RS(\omega)e^{i\omega(\tau_{sA}+\tau_{Ar}+\tau_{rB})} + o.t.; \quad d_A = S(\omega)e^{i\omega\tau_{sA}} + o.t., \quad (1)$$

where the ghost-reflection term is explicitly given for trace B and the direct-wave term is explicitly given for A; the reflection coefficient at the layer interface is taken to be  $R$ . Other arrivals such as the primary reflections or direct waves are included in the *other terms* denoted *o.t.*. The source spectrum is given by  $S(\omega)$ , and both the source spectrum and source location are assumed to be unknown. Note, that the trace at position A is also located at the specular reflection point for the ghost reflection.

2. Form the correlated data. Crosscorrelating trace A with trace B gives

$$d_A^* \cdot d_B = -R|S(\omega)|^2 e^{i\omega(\tau_{Ar}+\tau_{rB})} + other\ stuff, \quad (2)$$

where the correlation between the direct wave at A and the ghost at B is explicitly given, and the *other stuff* represents the other correlations. Note, the unknown phase term  $\omega\tau_{sA}$  has been canceled because the direct raypath at A is the same as the first-leg raypath of the free-surface multiple at B.

3. The goal of migration is to image the reflectivity structure, so the crosscorrelation migration operator tuned to the ghost reflection is  $m(x) = e^{-i\omega(\tau_{Ax}+\tau_{xB})}$ , where  $x$  is the trial image point. Multiplying this migration operator  $m(x)$  by the correlated data in equation 2 will give the migration image at  $x$ . When  $x$  is coincident with the actual specular reflection point at  $r$  then there will be annihilation of the phases to give maximum migration amplitude at  $x = r$  for all  $\omega$ . Thus, the ghost reflection is used to image the reflector, even though we do not know the source location or the source time history! The *other stuff* will not hopefully contribute because the migration operator is tuned to only cancel the phase of ghost reflections at  $x = r$ .

For buried sources with unknown time histories or locations, daylight or interferometric imaging requires the recording of passive seismic data over a wide array of geophones. Assume a master trace at location A, crosscorrelate this trace with all the other traces in the data set, and migrate the resulting pseudo-shot gather with the migration operator. Repeat this procedure for all different locations A and stack the migrated sections together to get the final reflectivity image. Note the above methodology is applicable to any number of sources, any depth of burial, and can image an arbitrary reflectivity distribution. If the sources are contemporaneously excited, then the success of this method demands that the source time histories be uncorrelated, e.g., random time series. However, caution should be exercised because stationary phase analysis says that other events can be accidentally tuned to the migration operator (Schuster and Rickett, 2000).

**Free-Surface Multiple Imaging with CDP Data.** Now we will show how interferometric imaging can be used to migrate multiples in CDP data. In comparison to the Delft method of *autoconvolving* traces and *subtracting* the computed multiples from the original traces, we will *crosscorrelate* the data to generate shifted pseudo-replicas of multiples and migrate these multiples. The multiple migration image is then combined with the primary reflection migration section to determine the common reflector locations.

Placing the source at the surface gives rise to the diagram in Figure 1b. Here the free-surface multiple recorded at B and the primary reflection recorded at A are explicitly represented by

$$d_B = R^2S(\omega)e^{i\omega(\tau_{sr'}+\tau_{r'A}+\tau_{Ar}+\tau_{rB})} + o.t.; \quad d_A = -RS(\omega)e^{i\omega(\tau_{sr'}+\tau_{r'A})} + o.t. \quad (3)$$

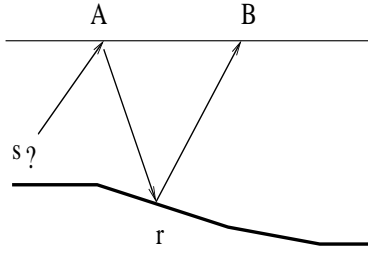
The crosscorrelation of these two traces annihilates the common phase terms in the exponents to give

$$d_A^* \cdot d_B = R^3|S(\omega)|^2 e^{i\omega(\tau_{Ar}+\tau_{rB})} + other\ stuff, \quad (4)$$

where the phase term in this equation suggest kinematics equivalent to a primary reflection generated by a source at A and a receiver at B. As in the oil-slick example, the common path of the primary reflection recorded at A and the ghost reflection recorded at B cancel one another, to leave their phase difference, namely that of a virtual primary reflection. That is, this virtual reflection is kinematically equivalent to a reflection generated by a source at A and received at B, a verification of Claerbout's conjecture.

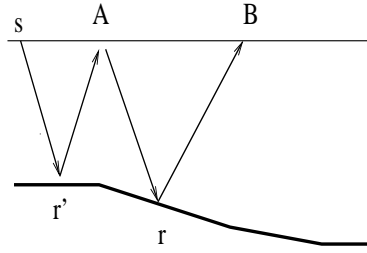
The obvious migration operator for the correlated data is given by  $m(x) = e^{i\omega(\tau_{Ax}+\tau_{xB})}$  where  $x$  is the migration image point. The procedure for migrating multiples is as before, form crosscorrelation pseudo-shot gathers from the master trace at A, and migrate this pseudo shot gather. Repeat this procedure for all

a). Passive Seismic Imaging



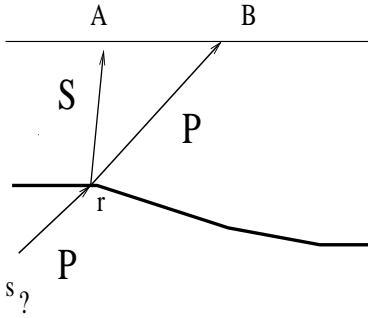
$$m(x) = e^{i\omega(\tau_{Ar} + \tau_{rB})}$$

b). CDP Multiple Imaging



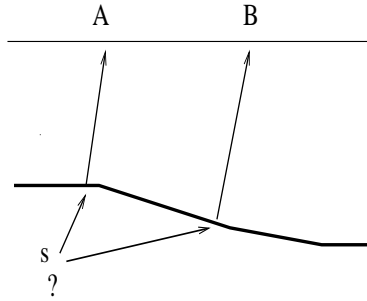
$$m(x) = e^{i\omega(\tau_{Ar} + \tau_{rB})}$$

c). Transmitted PS Imaging



$$m(x) = e^{i\omega(\tau_{rA}^S - \tau_{rB}^P)}$$

d). Source Location Imaging



$$m(x) = e^{i\omega(\tau_{sA} - \tau_{sB})}$$

Figure 1: Crosscorrelation migration operators  $m(x)|_{x=r}$  tuned to different correlated events. (a). Correlated event is  $d_A^* d_B$ , where the direct wave at A is given by  $d_A = e^{i\omega\tau_{sA}}$ , and the ghost reflection at B is  $d_B = -R e^{i\omega(\tau_{sA} + \tau_{Ar} + \tau_{rB})}$ . In this case the depicted migration operator will annihilate the phase of the common direct ray in  $d_A$  and  $d_B$ . Here the source location does not need to be known, such as in difficult VSP data or in passive seismic data. (b). Same as (a)., except the source is at the surface and its location is known, such as in CDP data. Here the migration operator images the 1st-order free-surface multiple reflection. Crosscorrelating the data twice will lead to the same migration operator, but for 2nd-order multiples. (c). Correlated event is  $d_A^* d_B$ , where the direct P-wave at B is given by  $d_B = e^{i\omega\tau_{sB}^P}$ , and the transmitted converted PS-wave at A is given by  $d_A = e^{i\omega\tau_{sA}^S}$ . This allows for imaging the layer interface from transmitted PS waves, as is done in the receiver-function method (Langston, 1977) or transmission PS migration (Sheley and Schuster, 2000). From the diagram in (a)., it is obvious from that ghost PS reflection imaging can now be implemented for receiver function imaging. (d). Same as (c), except now the source location is sought and the two correlated events are the direct P waves at A and B. The migration operators for (c) and (d) have less resolving capability than that for (a) and (b). But this problem can be partly remedied by including the apparent incidence angle as a constraint in the migration operator.

codes do not need to be changed, you simply change the input from raw traces to crosscorrelation traces. Similar to the Delft method, peg-leg multiples might be migrated by extrapolating data to the peg-leg generator and the crosscorrelation imaging procedure can be repeated. However, a major problem with II is that the migration operator is tuned to a correlation with a weak strength of  $R^3$ . This weak correlation competes with stronger correlations, such as primary with primary correlations that are  $R^2$  strength that can inadvertently be tuned to the multiple migration operator (see Sheng et al., 2001).

**Receiver Function Imaging.** Seismologists use converted PS transmission waves to image the geometry of a layer interface, often the Moho (Langston, 1977). For a recorded teleseism, they crosscorrelate the vertical component with the horizontal component, where the largest correlation amplitude is the converted PS transmitted wave at the Moho. The lag time of this PS correlation is related to the depth of the Moho if the P/S velocity ratio is known. Figure 1c shows the ray diagram for transmitted waves that are converted at the interface. It can be seen from this diagram that the crosscorrelation of trace A with B will annihilate the common phase term, so that the PS transmission migration operator is shown in the figure.

**Source Location Imaging.** Sometimes the seismic source location is needed but it is unknown, such as in the case of a hydro-fracing test where the induced fracture location is not known. In this case, the ray diagram in Figure 1d shows that the phase of the  $d_A^*d_B$  is annihilated by the migration operator given in the figure, provided the trial image point coincides with the source location.

## Summary

I presented the theory for migrating crosscorrelograms, which I define as interferometric imaging. The II method extends the daylight imaging concept of Rickett and Claerbout to arbitrary distributions of sources and arbitrary reflectivity distributions, and validates Claerbout's conjecture. A key step in the validation was the use of stationary phase analysis of the multi-fold Kirchhoff integrals. The II method can be used to migrate free-surface multiples and peg-leg multiples in CDP data, image unknown source locations, and generalizes the receiver-function imaging method used by seismologists. Autocorrelation migration is somewhat similar to zero-offset migration, while crosscorrelation migration is similar to prestack migration. It can be easily shown that ghost migration typically illuminates a wider area of the subsurface than can primary reflection migration for VSP data. A caveat: ghost reflections should always have less signal/noise than primary reflections because they traverse at least double the path length. Also, the migration operator can coherently migrate correlations other than the intended one, which is similar to standard migration where multiples are coherently migrated to the wrong location.

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## References

- Claerbout, J., 1968, Synthesis of a layered medium from its acoustic transmission response: *Geophysics*, 264-269.
- Cole, S., 1995, Passive seismic and drill-bit experiments using 2-D arrays: Ph.D. thesis, Stanford University.
- Langston, C.A., 1977. Corvallis, Oregon, crustal and upper mantle receiver structure from teleseismic P and S waves: *Bull. Seism. Soc. Am.*, 67, 713-724.
- Rickett, J., and Claerbout, J., 1999, Daylight imaging: in Abstracts of 69th Annual SEG Houston meeting.
- Sheley, D., and Schuster, G.T., 2000, Reduced-time migration of transmission PS waves: in Abstracts of 70th Annual SEG Calgary meeting, 473-476.
- Schuster, G. T., and Rickett, J., 2000, Daylight imaging in  $\mathbf{V}(x, y, z)$  media: in Midyear reports of Stanford Exploration Project and UTAM.
- Schuster, G.T., Liu, Z., and Followill, F., 1997, Migration of autocorrelograms: in Abstracts of 67th SEG meeting, 1893-1896.
- Schuster, G., 1999, Seismic interferometric imaging with waveforms: 1999 UTAM Midyear Rep., 121-130.
- Sheng, J., Sun, H., and Schuster, G.T., 2001, Interferometric imaging with multiples: (submitted to 2001

