Interferometric Traveltime Tomography with Closure Phase

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ABSTRACT

The Interferometric Traveltime Tomography (ITT) method was previously shown to be both effective and stable in eliminating either shot- or receiver-static errors using synthetic and field data example. In this report, we apply the ”closure phase” theorem to traveltime tomography and test its effectiveness in eliminating both the shot- and receiver-static errors simultaneously with synthetic crosswell data. Results show that the ITT method with ”closure phase” theorem, denoted as the CP method, can successfully eliminate both the shot- and receiver-timing shifts. However, the cost is a loss of resolution that sometimes is untolerable.

INTRODUCTION

The ITT method was previously tested (Schuster and Zhou, 1999; Zhou, 1999) on synthetic crosswell data, VSP+crosswell data, surface refraction data, and field data with either random or systematic timing shifts. Results showed that the ITT method can eliminate either large random/systematic source or receiver timing shifts that are unacceptable by the standard method, provided the master trace for each shot gather is associated with the shortest raypath within the shot gather, and the initial velocity model contains the DC component of the slowness field.

However, this ITT method can only deal with cases where there are only shot or receiver static errors, but not both. In contrast, field seismic data contain both shot and receiver static errors which should be considered together. The possible remedy is to apply the so-called ”closure phase” theorem to traveltime tomography which was proposed in earlier reports (Schuster and Zhou, 1999; Zhou, 1999).

In this report, I will develop the CP method, discuss the validation of the inversion operator in the CP method, and then test the CP method on synthetic crosswell data with both shot and receiver timeshifts.
METHODOLOGY

We have compared the performance of the standard tomography method to that of the ITT method in the previous report (Zhou, 1999). To combine the "closure phase" theorem with the ITT method, we need to modify the formulas for the misfit function and gradient.

In the CP method, traveltimes are picked and 6 rays from 3 shots and 3 receivers are properly chosen to form closure, and the time difference of these 6 rays are then calculated (Zhou and Schuster, 2000). This time difference is denoted as the closure time. These time differences between observed and calculated closure times are then smeared along the corresponding rays. So the CP misfit function is given by

$$\epsilon = \sum_i (\tilde{t}_{i}^{\text{obs}} - \tilde{t}_{i}^{\text{cal}})^2,$$

(1)

and the gradient is given by

$$\partial \epsilon / \partial s_k = -2 \sum_i (\tilde{t}_{i}^{\text{obs}} - \tilde{t}_{i}^{\text{cal}}) \partial \tilde{t}_{i}^{\text{cal}} / \partial s_k,$$

(2)

where $\tilde{t}_{i}^{\text{obs}}$ and $\tilde{t}_{i}^{\text{cal}}$ represent the observed and synthetic closure times for the $i$th closure, respectively; $s_k$ denotes the $k$th cell of a tomogram. Though equation 1 and equation 2 appear to be same as the standard misfit function and gradient, respectively (Zhou, 1999), the definition of $\tilde{t}_i$ is different. Here, the term $\tilde{t}_i$ represents the closure time for the $i$th closure in the CP method, as shown in Figure 1. Please note that for each closure time, traveltimes from 6 different rays are involved in addition and subtraction.

Next, we will compare the salient differences between standard and CP traveltime tomography. In general, the forward problem of the traveltime tomography can be expressed as

$$A m = b,$$

(3)

where $b$ is the data vector of the picked traveltimes, and vector $m$ represents the slowness. And each row of the matrix $A$ represents the segment lengths of one ray in standard traveltime tomography and one set of rays (6 rays) in the CP method. So the element $A_{ij}$ of the matrix $A$ represents the segment length of the $i$th ray (in the standard method) or the $i$th closure (CP method) in the $j$th model cell. The element $A_{ij}$ is non-negative in the standard method, but due to the subtraction of the segment lengths, for the CP method, $A_{ij}$ could be negative.

Because the raypath is dependent on the slowness field, equation 3 is non-linear and an iterative procedure is needed in the inversion. We apply the simultaneous iterative reconstruction techniques (SIRT) to determine the gradient (steepest-descent direction) of the slowness field for each model cell, and then use a line search scheme to determine the optimal step for the slowness update.
Starting with some initial approximation \( m^{(0)} \), we define the residual \( r^{(q)} \) for the approximation \( m^{(q)} \) after \( q \) iterations by

\[
r^{(q)} = b - Am^{(q)}. \tag{4}
\]

We determine a correction \( \Delta m^{(q)} \) to \( m^{(q)} \) which honors \( i^{th} \) equation

\[
r_i^{(q+1)} = 0. \tag{5}
\]

The least square solution of this problem is

\[
\Delta m_j^{(q)} = \frac{A_{ij}r_i^{(q)}}{\sum_k(A_{ik}^2)}, \tag{6}
\]

where \( \sum_k(A_{ik}^2) \) can be approximated by the square of the total length of the 6 rays in the \( i^{th} \) closure.

Similarly, we can determine the corrections for all other rows of the equation 4. And by averaging all these corrections, we get the correction \( \Delta m^{(q)} \) for the SIRT method

\[
\Delta m_j^{(q)} = \frac{1}{M_j} \sum_i A_{ij}r_i^{(q)} \sum_k(A_{ik}^2), \tag{7}
\]

where \( M_j \) denotes the number of non-zero elements in the \( j^{th} \) column of \( A \), i.e., the number of the rays passing through the cell \( j \). This correction also represents the steepest-descent direction or gradient for the slowness updating. The updated slowness field can then be expressed as

\[
m_j^{(q+1)} = m_j^{(q)} + \frac{\alpha}{M_j} \sum_i A_{ij}r_i^{(q)} \sum_k(A_{ik}^2), \tag{8}
\]

where \( \alpha \) is the step length determined by a line search scheme which can improve the convergence rate.

**Closure Phase Construction**

To construct the closure phase time which involves 6 rays, we typically need rays associated with 3 different shots and 3 different geophones. In general, there are several ways to construct the closures, however, for data in the common shot gather (CSG) or common receiver gather (CRG), we choose a way which is most convenient for programming. Figure 1 shows the closures we used in this study. The first 2 shots and 2 geophones with numbers 1 and 2 are fixed, and \( i \) and \( j \) represents the other shots and geophones. Closure A and closure B share 3 common rays (solid line) and contain 3 different rays (dashed lines). All 9 rays from these 3 shots and 3 receivers are both used in this case. Since each common ray is used twice in the closure A and B, its residual is calculated by averaging the two residuals from these two closures.
However, since the first 2 shots and 2 geophones are fixed, the rays associated with these shots and geophones are used much more times than those with other shots and geophones if the number of shots and geophones is larger than 3. This unbalanced ray coverage results in the unbalanced model resolution. For example, if the fixed shots and geophones are near the top of the velocity model, the tomogram will have better resolution at the top because of the better ray coverage; and if these shots and geophones are at the bottom, the tomogram will provide better resolution at the bottom part of the velocity model. We will demonstrate this phenomenon in the following numerical tests.

**NUMERICAL TESTS**

I now test the CP method on synthetic crosswell data associated with the 2-layer velocity model in Figure 2a. The fault-like model has an interface between 500 m and 700 m, with upper- and lower-layer velocities of 2000 m/s and 3000 m/s, respectively. Shot and receiver intervals are 10 meters, and the gridpoint spacing 5 m. A +50 milliseconds systematic shot timing shift, which is about 9% of the largest traveltime, is added to the synthetic data.

Figure 2b shows that the CP tomogram, 2 shots and 2 geophones for the closures are kept on the top of the model so that the upper part of the model is visited by more rays than the lower part of the model. In this case, the tomogram shows better resolution in the upper part of the model. Similarly, Figure 2c demonstrates better resolution for the lower part of the model because now the fixed shots and geophones are located at the bottom of the model. We prefer the relatively even resolution of the model, so we invert for tomograms where the two fixed shots and geophones are distributed from the top to the bottom of the model along 10 evenly-spaced intervals. As expected, Figure 2d provides more balanced resolution of the model than either Figure 2b or Figure 2c. But this is at the cost of 10 times more computation and the loss of some model resolution.

Figure 3 shows the true model in Figure 3a, as well as the inversion results from standard traveltime tomography (Figure 3b), the ITT method (Figure 3c), and the CP method (Figure 3d). The standard method can barely resolve the subsurface structure, while the ITT and CP methods provide a more accurate velocity model. The tomogram from the CP method has worse resolution than that from the ITT method because the addition and the subtraction of 6 rays involved in the closure phase theorem not only eliminates the shot and receiver timeshifts, but also sacrifices the useful information such as the DC component of the slowness field, and thus enlarges the null space of the inversion operator. Such drawbacks result in lower model resolution.

The most significant benefit of the closure phase theorem is that it can eliminate both shot- and receiver-timing shifts simultaneously. The ITT alone can only get rid of either shot- or receiver-timing shift, so that the timing shift from the other side could corrupt the tomogram. Figure 4 shows the inversion results from the ITT
Figure 1: The construction of the closure phases. Three shots and three geophones are involved in each closure. Shot 1, shot 2, geophone 1 and geophone 2 represent 2 fixed shots and 2 fixed geophones, while shot $i$ and geophone $j$ indicate the other shots and geophones in the data. $t_{ij}$ denotes the traveltime of the ray associated with the $i^{th}$ shot and $j^{th}$ geophone. $\tilde{t}_i$ represents the closure time associated with the corresponding closure configuration above it. Closure A and B represent the closures we used in this study where we use all 9 rays associated with these three shots and three geophones. Solid lines indicate the three common rays in these two closures, while the dashed lines represent the 6 different rays. For each common ray, the residuals from the closure A and B are averaged, and the averaged residual is used to update the velocity field during the inversion.
Figure 2: Synthetic model and inversion results for the crosswell refraction experiment: shot and receiver interval is 10 meters, and the grid size 5 m by 5 m; Velocities are 2000 m/s (upper layer) and 3000 m/s (lower layer). A +50 msec shot timing shift is added to the synthetic data. a) True Model, b) tomogram from the CP method, the 2 shots and 2 geophones are kept on the top of the model (near the surface), c) same as b) but the shots and geophones are at the bottom of the model, d) same as b) and c) but the fixed shots and geophones move down from the top to the bottom of the model with 10 equal steps as the iteration proceeds. A linear-gradient velocity model is used as the initial model with a surface velocity of 2000 m/s and a deep velocity of 3000 m/s.
Figure 3: Synthetic model and inversion results for crosswell experiment: the same synthetic model and data as Figure 2. a) True Model, b) tomogram from standard traveltime tomography, c) tomogram from the ITT method, d) CP tomogram with same source-receiver configuration used to generate Figure 2d.
and CP methods with the synthetic data (already with +50 msec systematic shot timeshifts) subjected to or free of the additional random receiver-timeshift normally distributed from -50 msec to +50 msec. Figure 4a indicates that the ITT method can reconstruct the true model quite well with the data free of the receiver timeshifts, while Figure 4b shows that the ITT method fails to reconstruct the true velocity model with both shot- and receiver-timeshifts in the data. It is the receiver timeshift that corrupts the tomogram because the ITT method in this test is designed to eliminate the shot timeshifts. Figure 4c shows the CP tomogram with only shot timeshifts in the data, and this tomogram is almost identical with that in Figure 4d, the tomogram from the CP method with both shot and receiver timeshifts in the data. It indicates that the CP method is effective in eliminating both the shot and receiver timeshifts. However, the CP tomogram has worse resolution than the tomograms in Figure 4a and Figure 4c.

**DISCUSSION**

The CP method is tested on synthetic crosswell data with both shot- and receiver-timing shifts. Results show that the CP method can simultaneously eliminate the shot and receiver timeshifts. However, the model resolution of the CP tomogram is worse than that of the ITT tomogram with the data free of the receiver timeshift. This further loss of resolution is due to the addition and subtraction of traveltimes for 6 rays in each closure, and perhaps not acceptable. As we have described in earlier reports (Zhou, 1999; Zhou and Schuster, 2000) about the ITT method, choosing the shortest ray in a shot gather as the master trace improves the model resolution. I conjecture that a proper choice of the closure rays will lead to further improvement in model resolution in the CP tomogram.

Future work will aim at improving the slowness resolution of CP tomograms and applying the CP method to field data.

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**REFERENCES**


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Figure 4: Tomograms from: a) ITT method with only +50 msec systematic shot timing shifts, b) same as a) but with random receiver timing shifts ranging from -50 msec to +50 msec, c) CP method with only +50 msec systematic shot timing shifts, d) same as c) but with random receiver timing shifts ranging from -50 msec to +50 msec. Dashed line indicates the true velocity boundary. For the data with both shot and receiver timeshifts, the tomogram from the CP method d) is the best.