Progress Report on DMO Deconvolution

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ABSTRACT

The DMO operation transforms non-zero offset data to equivalent zero-offset data. It is implemented by smearing the samples in non-zero offset traces to the ellipses associated with a specified source and receiver. With coarse sampling, summation of these ellipses has incomplete cancellation and so introduces artifacts in the DMO processed data. Applying the principle of migration deconvolution to the DMO data, we propose an algorithm to deconvolve the DMO artifacts from the DMO image. Preliminary results with 2-D synthetics show that DMO deconvolution can noticeably improve the quality of a DMO image, decrease DMO artifacts and improve the resolution of the poststack KM migration image.

INTRODUCTION

DMO transforms non-zero offset traces to zero-offset traces, i.e., it smears the reflection energy in a trace along the DMO ellipse associated with a specified source and receiver. Usually DMO will introduce alias-like artifacts into the processed data which include ellipse trails and waveform distortion. Attenuating DMO artifacts and improving the DMO image quality is the object of our research.

In principle, the DMO ellipse trails in the DMO’ed data should cancel one another by the stacking of different DMO offset data. However the number of offset traces is usually not large enough to completely cancel ellipses, and therefore artifacts appear in the data. Based on the principle of migration deconvolution, it is not difficult to devise a similar deconvolution method for DMO’ed data. Instead of deconvolving the migration Green’s function, we can find a function for the DMO deconvolution filter.

PRINCIPLE OF DMO DECONVOLUTION

The seismic data can be expressed as \( d(\mathbf{x}', h, t') \), i.e. a function of midpoint coordinate \( \mathbf{x}' \), half offset \( h \), and time \( t' \).

The procedure for dip moveout (DMO) and stacking yield an estimate for the zero-offset section:

\[
DMO(\mathbf{x}, t) = \int dh \int d\mathbf{x}' \int dt' s(\mathbf{x}, t | \mathbf{x}', h, t') d(\mathbf{x}', h, t'),
\]  

(1)
where \( s(\mathbf{x}, t \mid \mathbf{x}', h, t') \) is the DMO operator which has the form (Ronen and Liner, 1987):

\[
s(\mathbf{x}, t \mid \mathbf{x}', h, t') = A(\mathbf{x}, t \mid \mathbf{x}', h, t')\delta[t - t'\sqrt{1 - \left(\frac{\mathbf{x}' - \mathbf{x}}{h}\right)^2}],
\]

(2)

where \( A(\mathbf{x}, t \mid \mathbf{x}', h, t') \) is a slowly varying amplitude term; \( s(\mathbf{x}, t \mid \mathbf{x}', h, t') \) can be interpreted as the equivalent zero-offset section if the data are a single spike with normal move out time \( t' \), midpoint \( \mathbf{x}' \), and half offset \( h \). The expression \( \delta[t - t'\sqrt{1 - \left(\frac{\mathbf{x}' - \mathbf{x}}{h}\right)^2}] \) has support along a DMO ellipse (DMO smile) for a trace in the COG section with half offset \( h \). The ellipse is defined by:

\[
\frac{t^2}{t'^2} + \frac{(\mathbf{x}' - \mathbf{x})^2}{h^2} = 1.
\]

(3)

If we denote the ideal zero-offset section (or DMO section without artifacts) as \( d_0(\mathbf{x}_0, t_0) \), we can estimate the corresponding COG section with any offset \( 2h \) by the inverse DMO operator:

\[
d(\mathbf{x}', h, t') = \int dt_0 \int d\mathbf{x}_0 f(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0)d_0(\mathbf{x}_0, t_0),
\]

(4)

where \( f(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0) \) denotes the inverse DMO operator, which can be interpreted as the data after NMO if the zero-offset section contains a single spike at \( (\mathbf{x}_0, t_0) \). It is equivalent to a single trace generated by reflection from a semicircular reflector (poststack migration image of this zero-offset spike). According to (Ronen and Liner, 1987), the inverse DMO operator has the form:

\[
f(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0) = B(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0)\delta[t_0 - t'\sqrt{1 - \left(\frac{\mathbf{x}' - \mathbf{x}_0}{h}\right)^2}],
\]

(5)

where \( B(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0) \) is a smooth amplitude factor.

Substituting equations 4 and 5 into equation 1 yields:

\[
DMO(\mathbf{x}, t) = \int dh \int d\mathbf{x}' \int dt's(\mathbf{x}, t \mid \mathbf{x}', h, t') \int dt_0 \int d\mathbf{x}_0 f(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0)d_0(\mathbf{x}_0, t_0) = \int dt_0 \int d\mathbf{x}_0 E(\mathbf{x}, t \mid \mathbf{x}_0, t_0)d_0(\mathbf{x}_0, t_0),
\]

(6)

where \( E(\mathbf{x}, t \mid \mathbf{x}_0, t_0) \) is called DMO impulse response function, and it can be interpreted as the DMO stacking image for one time sample of raw input data for the true zero-offset section (or DMO image without artifacts and amplitude distortion). According to equations 2 and 5, it can be written as:
\[ E(\mathbf{x}, t \mid \mathbf{x}_0, t_0) = \int dh \int d\mathbf{x}' \int dt' s(\mathbf{x}, t \mid \mathbf{x}', h, t') f(\mathbf{x}', h, t' \mid \mathbf{x}_0, t_0) \]
\[ = \int dh \int d\mathbf{x}' \int dt' AB \delta[t - t' \sqrt{1 - (\frac{x' - \mathbf{x}_0}{h})^2}] \delta[t_0 - t' \sqrt{1 - (\frac{x' - \mathbf{x}_0}{h})^2}] \]
\[ = \int dh \int d\mathbf{x}' C \delta[t - \frac{1 - (\frac{x' - \mathbf{x}_0}{h})^2}{\sqrt{1 - (\frac{x' - \mathbf{x}_0}{h})^2}}] t_0. \tag{7} \]

where \( C = AB \) is, again, a smoothly varying amplitude term; and the integration with respect to \( \mathbf{x}' \) has the constraint of \(|\mathbf{x}' - \mathbf{x}_0| < h\). For a given offset \( 2h \), the integration on the RHS means a group of DMO smiles passing through \((\mathbf{x}_0, t_0)\) as shown in Figure 1. So the DMO impulse response function \( E(\mathbf{x}, t \mid \mathbf{x}_0, t_0) \) also represents the stacked DMO smiles passing through \((\mathbf{x}_0, t_0)\) for all offsets.

For the discrete data, we can rewrite equation 6 as:
\[ DMO(\mathbf{x}, t) = \int_{S_1} E(\mathbf{x}, t \mid \mathbf{x}_0, t_{s1}) \delta_0(\mathbf{x}_0, t_{s1}) d\mathbf{x}_0 + \int_{S_2} E(\mathbf{x}, t \mid \mathbf{x}_0, t_{s2}) \delta_0(\mathbf{x}_0, t_{s2}) d\mathbf{x}_0 
+ \ldots + \int_{S_N} E(\mathbf{x}, t \mid \mathbf{x}_0, t_{sn}) \delta_0(\mathbf{x}_0, t_{sn}) d\mathbf{x}_0, \tag{8} \]

where each layer integral is over the area of the horizontal plane in the DMO image space associated with each time sample denoted by \( S_i \) for \( i = 1, 2, \ldots, N \); \( DMO(\mathbf{x}, t) \) is the DMO image obtained by the DMO operation; and \( \delta_0(\mathbf{x}_0, t_{si}) \) is the DMO image without artifacts and amplitude distortion. Note the time integral in equation 5 has been approximated by a summation after time indices.

For the common offset data set, the DMO ellipse is laterally invariant and is the DMO impulse response function for an infinite aperture. Invoking shift invariance in the x-coordinate for equation 8, yields
\[ DMO(\mathbf{x}, t) = \int_{S_1} E(\mathbf{x} - \mathbf{x}_0, t \mid t_{s1}) D(\mathbf{x}_0, t_{s1}) d\mathbf{x}_0 + \int_{S_2} E(\mathbf{x} - \mathbf{x}_0, t \mid t_{s2}) \delta_0(\mathbf{x}_0, t_{s2}) d\mathbf{x}_0 
+ \ldots + \int_{S_N} E(\mathbf{x} - \mathbf{x}_0, t \mid t_{sn}) \delta_0(\mathbf{x}_0, t_{sn}) d\mathbf{x}_0, \tag{9} \]

which says that the integrals are now spatial convolutions. For 3-D DMO the integration is 2-D; for 2-D DMO the integration is 1-D.

Converting the above equation into the wavenumber domain by a Fourier transform with respect to the spatial coordinates, we get:
\[ D\tilde{MO}(\mathbf{k}, t_r) = \tilde{E}(\mathbf{k}, t_r \mid t_1) \tilde{\delta_0}(\mathbf{k}, t_1) + \tilde{E}(\mathbf{k}, t_r \mid t_2) \tilde{\delta_0}(\mathbf{k}, t_2) 
+ \ldots + \tilde{E}(\mathbf{k}, t_r \mid t_n) \tilde{\delta_0}(\mathbf{k}, t_n), \tag{10} \]

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where \( k = (k_x, k_y) \) is the wave number.

For \( n \) different imaging levels, we get a linear equation as follows:

\[
\begin{bmatrix}
\tilde{\text{DMO}}(\mathbf{k}, t_1) \\
\tilde{\text{DMO}}(\mathbf{k}, t_2) \\
\vdots \\
\tilde{\text{DMO}}(\mathbf{k}, t_n)
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{\mathbb{E}}(\mathbf{k}, t_1 | t_1) & \tilde{\mathbb{E}}(\mathbf{k}, t_1 | t_2) & \cdots & \tilde{\mathbb{E}}(\mathbf{k}, t_1 | t_n) \\
\tilde{\mathbb{E}}(\mathbf{k}, t_2 | t_1) & \tilde{\mathbb{E}}(\mathbf{k}, t_2 | t_2) & \cdots & \tilde{\mathbb{E}}(\mathbf{k}, t_2 | t_n) \\
\vdots & \vdots & \cdots & \vdots \\
\tilde{\mathbb{E}}(\mathbf{k}, t_n | t_1) & \tilde{\mathbb{E}}(\mathbf{k}, t_n | t_2) & \cdots & \tilde{\mathbb{E}}(\mathbf{k}, t_n | t_n)
\end{bmatrix}
\begin{bmatrix}
\tilde{d}_0(\mathbf{k}, t_1) \\
\tilde{d}_0(\mathbf{k}, t_2) \\
\vdots \\
\tilde{d}_0(\mathbf{k}, t_n)
\end{bmatrix}
\] (11)

The true DMO image \( \tilde{d}_0(\mathbf{k}, t_i) \) can be obtained by solving the above equation for all \( \mathbf{k} \) and \( t \). Converting \( \tilde{d}_0(\mathbf{k}, t_i) \) back to the space domain will yield the deconvolved image \( d_0(x, t) \). In practice, the system of equations is no larger than 9 by 9.

**NUMERICAL EXPERIMENTS**

The algorithm was tested on the 2-D common offset data set for a point scatterer model and the SEG/EAGE overthrust model.

**Point Scatterer Model**

The point scatterer model is shown in Figure 3a. It has the velocity of 5000 m/s, and consists of 9 point scatterers located at the depths of 1250 m, 2500 m and 3750 m; and their horizontal positions are 625 m, 1250 m, and 1875 m, respectively. The synthetic zero-offset data and stacked DMO data from 20 common offset gathers with offsets from 100 m to 2000 m are shown in Figures 2a and 2b. In the stacked DMO section, hyperbolas associated with different point scatterers match well with those in the synthetic zero-offset section; however, there are strong artifacts associated with the trails of the DMO ellipses which are not canceled by stacking. In addition, the hyperbolas do not extend to the boundary of the model which will create the artifacts in the poststack image, and therefore decrease the resolution of the image. Figure 2c shows the stacked and deconvolved DMO section, it contains fewer artifacts than the stacked DMO section in Figure 2b; and the hyperbolas extend widely to the boundary of the model which indicates a better poststack image resolution. The migrated images of these two sections are shown in the Figures 3b and 3c. As expected, the image associated with the DMO deconvolution section has fewer artifacts and higher resolution compared to the DMO section without deconvolution. From the comparison of these figures, it is obvious that the DMO deconvolution process can noticeably decrease the DMO artifacts, improve the quality of the stacked DMO image, and improve the resolution of the migrated image.

**SEG/EAGE overthrust model**

The DMO deconvolution process works well with the simple model presented above. Now we test the algorithm on a more complicated model, i.e., the SEG/EAGE overthrust model. The velocity model and the zero-offset data set are shown in Figures 4 and 5. Applying DMO and stacking to 33 common offset gathers (offsets from 300 m to 2800 m with an interval of 100 m;
from 3000 m to 4200 m with an interval of 200 m) yields the result shown in Figure 6a. Figure 6b shows the DMO image after DMO deconvolution. It is obvious that the deconvolved DMO image has fewer artifacts and better reflector continuity.

Due to a possible bug in the DMO code, the shallow part of the stacked DMO image has strong artifacts which corrupt the image. Figure 6a shows the part without the shallow corrupted noise. Compared with the synthetic zero-offset data, Figures 6a and 6b have a horizontal shift of about 250 m. This shift is due to the horizontal shift of DMO images for large offset data (offset > 2000 m).

The DMO deconvolution is implemented in the time-space domain. Considering the difficulty in obtaining the time-velocity model from the depth-velocity model, the deconvolution operator for times larger than those corresponding to the depth of the velocity model was replaced by the operator for the largest times within the depth of the velocity model. In the above two tests, a 5-layer operator was used in the calculation. Similar to the situation in migration deconvolution, an increase in the number of layers will introduce more artifacts in the result. A smoothing filter composed of a 7-point median filter and an averaging filter was used to smooth the spectrum of the Green’s function in the same way as in the migration deconvolution (Hu and Schuster, 1999). Furthermore, a regularization factor was introduced in the Green’s function coefficient matrix to enhance stability in the calculation.

CONCLUSIONS

DMO smears an impulse from data-space along an ellipse associated with a specified source and receiver. For common offset data sets, the ellipse trails away from the reflection point are not completely canceled so that artifacts and distorted waveforms are seen in the final DMO image. Applying the principle of migration deconvolution to the common offset DMO data set and using the DMO impulse response function as the DMO’s Green’s function, DMO deconvolution can be implemented. For DMO deconvolution, the DMO operator in the common offset data is laterally invariant for infinitely wide apertures. The synthetic data tests suggest that DMO deconvolution can be effective in eliminating DMO artifacts. This suggests that DMO deconvolution can be used in post-DMO processing to eliminate noise and recover waveform distortion associated with DMO processing. This is a work in progress, so we will soon test this method on 3-D data.

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REFERENCES

Figure 1: Diagram of the DMO impulse response function at (x,t) for a given half offset h.
Figure 2: (a) Point scatterer model with constant velocity of 5000 m/s; (b) poststack KM image for stacked DMO data; (c) poststack KM image for stacked and deconvolved DMO data. Compared to (b), the poststack KM image computed from the deconvolved DMO data (c) has higher resolution at all depths.
Figure 3: (a) Synthetic zero-offset section for the point scatterer model in Figure 2a; (b) stacked DMO section; (c) stacked and deconvolved DMO section. Comparing (b) with (c), it is obvious that DMO deconvolution decreases the DMO artifacts, and improves the quality of the DMO image.
Figure 4: Velocity model for the phase A data set in the overthrust model (SEG/EAGE 3-D Modeling Series No.1).
Figure 5: The synthetic zero-offset data of the SEG/EAGE overthrust model.
Figure 6: (a). Stacked DMO image; (b). same as (a), but with DMO deconvolution before stacking. The deconvolved image has fewer artifacts and better reflector continuity.
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