COMPARISON OF WAVE EQUATION MIGRATION METHODS WITH PHASE ENCODING

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ABSTRACT

Prestack migration algorithms are usually divided into two classes: ray-tracing based migration methods and wave-equation methods. The latter methods provide more accurate migration images, especially in complex geological areas, but they are the relatively expensive compared with ray-based migration methods. In this work, I test the phase encoding strategy on three types of one-way wave equation methods for prestack depth migration: Split-Step Fourier method (SSF), Phase-Shift-Plus-Interpolation method (PSPI), and the Fourier Finite-Difference method (FFD). 2D algorithms were tested on the SEG/EAGE salt model data for the purpose of drastically reducing the computational time. Results show that computational effort is obviously reduced but at the expense of reduced image quality. For the 2D SEG/EAGE data, acceptable images are achieved at \( \frac{1}{5} \) the cost of standard migration if phase encoding is used. 3D algorithms are also implemented and being tested on the SEG/EAGE salt model.

INTRODUCTION

Prestack migration techniques for subsurface imaging are widely used in oil exploration and play an important role in imaging the complex subsurface structure image. The ray-based Kirchhoff migration (KM) methods are presently considered the most popular and flexible technique for 3D migration with generally good image quality. The main reason is that the KM method has the capability of target-oriented processing and efficient computation. The ray-based migration methods usually use single path ray tracing to get traveltimes but multi-path arrivals are needed for proper imaging in complex areas. Therefore KM typically generates a poor migration in area with complex geological structure. In order to remedy such drawbacks and increase the image quality, new ray tracing methods were developed to consider the multi-path arrivals and caustics.

Another standard imaging technique for seismic applications is based on solving the wave equation (Claerbout, 1974). As an example, reverse-time migration solves the two-way wave equation for imaging, which is accurate but at the cost of an increase in computation time. For shot migration, one needs to both forward propagate the source and backward propagate the receiver wavefield. In order to increase computational efficiency, the forward propagation can be implemented by a ray tracing technique. More widely-used wave equation methods are based on more efficiently solving the one-way acoustic wave equation. Unfortunately the cost of these methods for prestack 3D migration is prohibitive for industrial application when the data set is large.

Recently, some authors have adopted an encoding strategy in the frequency-space domain using the Finite Difference (FD) migration method (Morton et al, 1998; Bonomi, 1999) to reduce the cost of migration. The strategy is to simultaneously migrate several shot gathers with random sequences. For many shot gathers, the cost of wave-equation migration can be drastically reduced.
Here, I implemented three types of phase-shift based wave-equation prestack migration methods combined with three encoding techniques in the common shot domain. All tests were performed with data for the 2D SEG/EAGE SALT model. The image accuracy and computational efficiency are compared and 3D tests are currently being conducted.

METHODOLOGY

Phase shift migration was first proposed by Gazdag (1979) which forms the basis of the following three migration algorithms used in this report. By introducing the velocity \( v(z) \), the 3D scalar wave equation is:

\[
\frac{\partial P^2(x,y,z,\omega)}{\partial x^2} + \frac{\partial P^2(x,y,z,\omega)}{\partial y^2} + \frac{\partial P^2(x,y,z,\omega)}{\partial z^2} + \frac{\omega^2 P(x,y,z,\omega)}{v(z)^2} = 0, \tag{1}
\]

where \( P(x,y,z,\omega) \) is the pressure wavefield, \( z \) is the depth, \( \omega \) is the frequency and \( v(z) \) is the velocity distribution. Equation 1 can be expressed in the wavenumber-frequency domain \((k_x, k_y, \omega)\) which is a second-order ordinary differential equation,

\[
\frac{\partial^2 P(k_x,k_y,z+dz,\omega)}{\partial z^2} = k_z^2 P(k_x,k_y,z,\omega), \tag{2}
\]

in which

\[
k_z = \pm \sqrt{\frac{\omega^2}{v^2} - (k_x^2 + k_y^2)^2}, \tag{3}
\]

where \( \pm \) stands for the wavefield extrapolation direction. For a constant \( k_z \), equation 2 has two characteristic solutions relating the field at level \( z \) with that at level \( z + dz \) by a phase shift, but for depth migration, the wavefield extrapolation equation is:

\[
P(k_x,k_y,z+dz,\omega) = P(k_x,k_y,z,\omega)e^{ik_zdz}. \tag{4}
\]

Thus the wavefield extrapolation involves just the phase shift in the frequency-wavenumber domain. This formula, combined with the image condition, is the extrapolation operator for phase shift migration algorithm. It allows for the exact inverse extrapolation of seismic data inside a homogeneous layer \([z, z + dz]\) with constant velocity.

The advantage of phase shift migration is that the power spectrum of the seismic source is band-limited with the cutoff frequency far below the temporal Nyquist frequency. Mapping the seismic data into the space-frequency domain allows for a significant compression of data and decreases the computational effort. In addition, it requires less machine memory compared with the FD algorithm.

When the velocity is only a function of depth, phase shift migration can be used to efficiently extrapolate the wavefield in the frequency-domain without any approximation. Unfortunately, this migration method is valid only for weak lateral velocity variations. In order to handle strong lateral velocity variation, several methods were proposed which includes the Split-Step Fourier method (SSF), Phase Shift Plus Interpolation (PSPI) and Fourier Finite Difference method (FFD).

**Split-Step Fourier method**

SSF method is base on splitting the laterally variant velocity into two parts: a background velocity and perturbation term (Stoffa, 1990). Generally the background velocity is the average value in the layer \([z, z + dz]\). The basic equations for SSF wavefield extrapolation are:
\[ P(k_x, k_y, z + dz, \omega) = P(k_x, k_y, z, \omega)e^{ik_z dz}, \]  
\[ P(x, y, z, \omega) = P(x, y, z, \omega)e^{i\omega \left( \frac{1}{v_{ref}} - \frac{1}{v(x,y,z)} \right) dz}, \]

where \( v_{ref} \) denotes the reference velocity within the layer \([z, z + dz]\).

Equations 5 and 6 demonstrate that the implementation of SSF migration includes two parts: Perform wavefield extrapolation of the data in the frequency-wavenumber domain, then transfer the wavefield into the frequency-space domain and apply the phase shift or phase correction which accounts for the lateral velocity variations. For strong velocity lateral variations, this single perturbation is not enough for imaging and more than one reference velocity is required as the strategy used in PSPI to get a more accurate result. However, the penalty is an increase in the computational cost (Kessings, 1992; Huang et al, 1999).

**Phase Shift Plus Interpolation Method**

PSPI method is a phase-shift-like method for dealing with strong lateral velocity variations. (Gadgaz, 1984). The basic idea of PSPI is to introduce several reference velocities to account for the lateral velocity variation in each extrapolation step and obtain the multi-reference wavefields in the frequency-wavenumber domain. Based on the relationship of the local velocity and reference velocity, the final migration result is obtained by interpolating the reference wavefields in the frequency-space domain. The basic formulas are:

\[ P_0(x, y, z, \omega) = P(x, y, z, \omega)e^{i\omega \frac{v}{v_{ref}}} dz, \]

and

\[ P'(k_x, k_y, z + dz, \omega) = P_0(k_x, k_y, z, \omega)e^{i(k_z' - \frac{v}{v_{ref}}} dz, \]

where \( k_z' \) is obtained using the reference velocity. After the reference wavefield is Fourier-transformed back to the frequency-space domain, the final migration result is obtained by linear interpolation.

Obviously, the choice of the reference velocities is a crucial task for PSPI migration mainly because the cost of PSPI is proportional to the number of reference velocity values used in each extrapolation step. In order to decrease the cost, the adaptive strategy of (Bagaini, 1995) was adopted in this report. This adaptive strategy of selecting reference velocities not only reduces the cost of PSPI, but it also computes the reference velocities according to the distribution of velocities. More reference velocities will be used when the lateral velocity variation is strong and fewer velocity values will be used when the velocity contrast is small.

**Fourier Finite Difference Method**

Even though SSF and PSPI can handle lateral velocity variations, they will give less accurate results when the lateral velocity variation is strong, the correction term is only a zero-order approximation to the one-way wave equation and propagates accurately only at small angles.

However many velocity models used in exploration may contain very strong velocity contrasts. For example, salt bodies may have velocities twice or even three times larger than the surrounding media. Under such large velocity contrasts, lateral velocity variation within the extrapolation interval can not be simply expressed by several discrete velocity values. For this problem, the extended Split-step method uses a more accurate approximation for the dispersion equation by adding
additional terms, such as the extended local Born-Fourier migration and pseudo-screen propagator methods (Huang et al., 1999). In 1994, Ruth proposed a Fourier finite-difference method which is the combination of the phase shift method in the frequency-wavenumber domain and the FD method in the frequency-space domain.

The expression for wavefield extrapolation with the FFD is:

$$\frac{\partial P}{\partial z} = i\left[\omega \frac{k_z^2 v_{\text{min}}^2}{\omega^2} + d1 + d2\right]P,$$

where

$$d1 = \omega \left(\frac{1}{v} - \frac{1}{v_{\text{min}}}\right),$$

$$d2 = \frac{\omega}{v} (a - 1) \frac{u^2}{2 - bu^2},$$

where $a = \frac{v_{\text{min}}}{v}$, $b = 0.5(1 + a + a^2)$, and $u = \frac{k_z^2 v_{\text{min}}^2}{\omega^2}$; $v_{\text{min}}$ denotes the minimum velocity at the layer $[z, z + dz]$; $v$ is the true velocity. In equation 10, the first term can be implemented using the phase shift method; the second term is the phase correction. The combination of these two terms are implemented using the SSF method. The last term is the finite difference operator. Here, I give the FFD another name: SSF+FD because it is implemented by using the combination of SSF and FD algorithms.

**Encoding Strategy for Wave Equation Migration**

To drastically reduce the cost of prestack migration, a possible solution is to randomly compress the source and recording data in the frequency-space domain. The one pseudo-source term is generated by a linear combination of all single source terms. In a similar way the super common shot gather is also obtained by superimposing all shot gathers. The resulting seismic data are considered as the new input shot gather for migration.

$$S_s(x, \omega) = \sum_n \alpha_n S_n(x, \omega),$$

$$D_s(x, \omega) = \sum_n \alpha_n D_n(x, \omega),$$

where $S_n$ and $D_n$ denotes the $nth$ source term and shot gather respectively; $S_s$ and $D_s$ are the encoded source term and shot gather; the coefficients $\alpha_n$ denotes a complex number which can be obtained in several ways (Romero, 2000; Bonomi, 1999). The following three encoding strategies were used to generate $\alpha_n$. First randomly set one of the $\pm 1$ values to every $a_n$ with probability of 50 percent. The second way is linear encoding, which generates $a_n = e^{i\theta}$, where $\theta$ is uniformly distributed over the range $[0, 2\pi]$. In the third algorithm, $a_n$ is obtained by randomly sampling from a Gaussian distribution with mean zero and variance of 1.
NUMERICAL RESULTS

All the migration methods were tested on data for the SEG/EAGE salt model. The data set consists of 325 shots, each shot contains 176 records with a recording length of 5 s and a sampling interval of 8 ms. Shot and receiver intervals are 160 ft and 80 ft, respectively. The velocity model contains $645 \times 150$ grids with a gridpoint spacing of 80 ft. The velocity model is shown in Figure 1.

First we tested the SSF, PSPI and SSF+FD and KM method with two types of data: one free from multiples and another that includes multiples as seen in the Figure 2 shot gathers.

The migration results are shown in Figure 3 which correspond to the migrations images obtained from the salt model data without multiples. All results were achieved using one PII 450 MHz processor with 512 Mbytes of memory. The comparison of computational times for these methods is shown in Figure 5. As we noted previously, the cost of KM is the lowest. The wave equation migration cost is several times that of the KM. However, the SSF+FD migration provides superior image quality, especially in the subsalt part where geologists are most interested. SSF and PSPI give comparable results to that of the SSF+FD method. In the PSPI method, its cost is dependent on the number of references velocities used in migration. The computational time of the PSPI method is about twice that of SSF, whereas SSF+FD is slightly more costly than SSF.

In order to drastically reduce the computational time of the wave equation method, an encoding strategy was applied (Morton et al., 1998). Figure 6 shows the results of SSF+FD migration with three encoding algorithms. All three encoding algorithms yield almost the same results. In the following tests, the second phase encoding algorithm was adopted. Note, the input data for these migration comparisons includes data with multiple reflections.

In the following tests of wave equation migration, two strategies for implementing phase encoding migration were used: in the first, we encode several adjacent shot gathers into one pseudo shot gather. After that, the total number of shot gathers is reduced and the migration method was applied to these new pseudo shot gathers. Another strategy is to encode a total of 320 shot gathers into one supergather, then apply the migration method to this unique supergather. After one migration, repeat the encoding and the migration steps with different random phase encoding until a satisfactory migration image is obtained.

Figures 7 and 8 show the SSF+FD migration results using the two phase encoding strategies. The CPU times of the SSF+FD migration method using two phase encoding strategies are shown in Figure 9. From this comparison, phase encoding of the SSF+FD migration still generates good migration result, especially in the part of subsalt region, with a running time reduced by a factor two or three times. The SSF migration tests also give the same conclusion as shown in Figures 10, 11 and 12 except that the subsalt image is not as clear as that obtained by SSF+FD migration. Parallel encoding of the wave equation migration codes were developed with MPI Fortran and accomplished on ICEBOX cluster. Figure 13 shows the CPU time for the SSF+FD encoding with 10 processors. From these tests, encoded wave-equation migration can reduce the computational time by a factor of around $5 - 7$ and achieve a better image than Kirchhoff migration.

The 3D migration algorithm was implemented and tested on the 3D SEG/EAGE salt model. We hope to soon develop an efficient 3D wave equation based method for providing an accurate image of the subsurface geology.

CONCLUSIONS

Several encoded wave equation migration methods were implemented and tested on the SEG/EAGE salt model data. For the prestack wave-equation migration tests, SSF is the most computationally efficient compared with other wave-equation based methods for our codes. The PSPI method yields a migration image of comparable quality but at the twice the cost of SSF migration. The SSF+FD
migration generates the most accurate image when the velocity model has strong lateral variations but requires more CPU time than SSF migration and less CPU time than PSPI migration.

We also implemented these algorithms in parallel and with phase-encoding. Results demonstrate that phase-encoding will reduce the cost of wave-equation migration by a factor of 2 or 3. In some cases, the encoded wave-equation migration is competitive with the cost of UTAM’s Kirchhoff migration code. We also note that the encoding technique has more influence on the shallow part of the migration image.

The extension of SSF and PSPI migration algorithms to 3D is easily implemented by incorporating an additional Fourier transform in the cross-line direction. The 3D phase encoding algorithm is currently being tested on the 3D salt model and future work should mainly focus on the random encoding of data for 3D prestack migration.

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REFERENCES


Figure 1: SEG/EAGE 2D velocity model.
Figure 2: Common shot gathers No. 100 and 200: (a). shot 100 without multiples; (b). shot 200 without multiples; (c). shot 100 with multiples; (d). shot 200 with multiples.
Figure 3: Comparison of four migration methods using data from the SEG/EAGE salt model without multiples. Images are from: (a). Kirchhoff migration, (b). SSF migration, (c). PSPI migration, and (d). SSF+FD migration.
Figure 4: Comparison of four migration methods using data from SEG/EAGE salt model without multiples. Images are from: (a). Kirchhoff migration, (b). SSF migration, (c). PSPI migration, and (d). SSF+FD migration.
Figure 5: Computational time of migration algorithms tested on SEG/EAGE salt model. The Kirchhoff method is faster at the cost of worst equality. The SSF+FD method gives a most accurate migration image, and the CPU time of the PSPI method is dependent on the number of reference velocities used.
Figure 6: SSF+FD migrations images with different encoding strategies: (a). uniquely random distribution; (b). linear encoding; (c). Gaussian distribution. Here the migration results were obtained by migrating a supergather stack a total of 60 times.
Figure 7: Migration comparison of SSF+FD method using different numbers of encoded shot gathers: (a). 2; (b). 4; (c). 10; (d). no encoding.
Figure 8: Migration comparison of SSF+FD method using different number for migrating an encoded super shot gather: (a). 100; (b) 80; (c). 60 and (d). no encoding.
Figure 9: (Top). CPU time comparison for encoded SSF+FD migration with different number of shot gathers; *Encoding Number* denotes the number of adjacent shot gathers to be phase encoded. (Bottom) CPU time comparison for encoding SSF+FD migration with the second strategy. Note, the CPU time of no-encoding SSF+FD migration and Kirchhoff migration are also presented for comparison.
Figure 10: Migration comparison of SSF method with different encoded shot numbers: (a). 2; (b); 4; (c). 10 and (d). standard migration result.
Figure 11: Migration comparison of encoded SSF method using different number of migration of a super encoded shot gather: (a). 80; (b). 60; (c). 40 and (d). standard migration result.
Figure 12: (Top). CPU time comparison for encoded SSF migration with different number of shot gathers; *Encoding Number* denotes the number of adjacent shot gathers that were phase encoded. (Bottom) CPU time comparison for encoding SSF migration with the second strategy. Note, the CPU time of no-encoding SSF+FD migration and Kirchhoff migration are also presented for comparison.
Figure 13: CPU time for SSF+FD migration method applied to the SEG/EAGE salt dome data. The solid line denotes the CPU time SSF+FD migration using one node. The dashed line is the encoded SSF+FD migration method executed on 10 nodes.