THE UNIVERSITY OF UTAH GRADUATE SCHOOL

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of a thesis submitted by

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TRAVELTIME INVERSION AND MIGRATION
OF OFFSET VERTICAL SEISMIC
PROFILING DATA

by
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in
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ABSTRACT

This thesis examines the reconstruction characteristics for velocity inversion of Vertical Seismic Profiling (VSP) traveltime data. Two important contributions are: 1) an empirical relationship is established (using synthetic and real VSP data) for the accuracy of one-dimensional velocity inversion as a function of the VSP source-receiver geometry, the number of primary reflection traveltimes, and the number of layers, and 2) a new iterative traveltime inversion and migration method is developed to invert for two-dimensional velocity structure.

Using synthetic VSP traveltime data, velocity reconstructions showed that reconstruction accuracy improved for increasing number of sources, for larger source offsets (up to a 1:1 offset-to-depth ratio), for the addition of primary reflection traveltimes (even when the errors for reflection traveltimes were larger than those for direct arrival traveltimes), and for fewer layers in the inversion model (for a fixed number of geophones). Eigenvalue distributions and condition numbers suggested that resolution decreases for an increase in the number of primary reflection traveltimes. A variable bias estimator was observed to
damp parameters more equally.

To extend VSP traveltime inversion to two dimensions, an iterative traveltime inversion and migration method was developed and tested. Results with noise-free synthetics were robust and accurate, while special care was required for noisy data.

One-dimensional traveltime inversion and the iterative inversion and migration method were applied to VSP traveltime data from ARCO's Bridenstein #1 well located in the Anadarko Basin of Oklahoma. Velocity reconstructions using primary reflection traveltimes matched the sonic log more closely than the reconstructions using only direct arrival traveltimes. Application of the iterative inversion and migration method yielded somewhat poor results because of the small offset-to-depth ratio of reflectors; a definite conclusion regarding the value of the iterative method cannot be drawn from this data set.
1. INTRODUCTION

In the search for the structure of the earth, geophysicists attempt to determine the earth's velocity structure. In this endeavor, the most important tool is the seismic method. Traveltimes from teleseismic earthquakes have been used to estimate velocity structures on the scale of 100s of kilometers while traveltimes from local earthquakes have been used to estimate velocities on the scale of 10s of kilometers (Hawley et al., 1981). Traveltimes from artificial sources, such as from seismic reflection experiments, have been used to estimate velocities on the scale of 100s of meters to 100s of kilometers (Bishop et al., 1985). To provide geologic and velocity information on the scale of 10s of meters, vertical seismic profiling (VSP) experiments have been performed. A VSP experiment consists of one or more surface sources with geophones emplaced at regular depth intervals in a borehole. Seismic energy propagates from the sources and recordings are made at each depth level. Most VSP experiments consist of a single source offset between 30 and 1,500 meters from the well, with a geophone spacing of 15 to 30 meters. Multiple source VSP experiments are sometimes performed to image structures laterally and to deduce lateral changes in lithology. Details of the VSP method are found
in the books by Hardage (1983) and by Balch and Lee (1984).

Figure 1.1 depicts an idealized single source VSP experiment along with the corresponding synthetic seismogram. Dowgoing direct waves are characterized by trajectories slanted to the right and upgoing reflected waves are characterized by trajectories slanted to the left. Phase and amplitude analysis of these events can yield high resolution information about the velocity, lithology, and the dispersive, attenuative, anisotropic, and structural properties of the media near the borehole. A major advantage of VSP over common depth point (CDP) experiments is that they can resolve lithology to the order of meters, rather than hundreds of meters. This increased resolution occurs because the VSP experiment records both the transmitted (dowgoing) and reflected (upgoing) wavefields as opposed to a CDP experiment recording, where only the reflected waves are recorded. The distance traveled by VSP arrivals is shorter than that of CDP arrivals, so that the frequency bandwidth is broader and the signal-to-noise ratio is larger. In addition, depths of VSP derived parameters are exact, while depths of parameters derived from CDP seismograms are inferred. Oristaglio (1985) recently summarized the current uses of VSP.

VSP travelttime data may be used to estimate interval velocities by a linearized inversion method. The most recent
Fig. 1.1 Single source VSP experiment (a) with direct arrival raypaths (solid) and primary reflection raypaths (dashed) along with its corresponding seismogram (b).
results were presented by Stewart (1984) and by Pujol et al. (1985, 1986). Assuming a one-dimensional earth model, they applied a damped least-squares method to direct P-wave traveltimes for interval velocities. Their velocity estimates were in good agreement with the sonic logs and tests on synthetic data showed their method to be both accurate and robust. However, their inversion algorithms were limited to one-dimensional models which could not take into account the two-dimensional structures typically encountered at VSP sites. In addition, they did not examine velocity resolution as a function of source-receiver geometry, nor were primary reflection traveltimes incorporated into the inversion. These concerns are important in the design of VSP experiments. The source-receiver geometry should be designed to maximize the extraction of information for the least amount of money.

These shortcomings suggest that the inversion of VSP traveltime data can be improved in two ways, which is the motivation for this thesis. The objectives of this thesis are 1) to examine the accuracy of one-dimensional traveltime inversion as a function of source-receiver geometry and model complexity and 2) to extend the one-dimensional method to two-dimensional media.

Regarding the first objective, four questions concerning the resolution of VSP traveltime inversion are examined. Specifically, how is the accuracy of velocity reconstruction
affected by the number of layers in the inversion model (for a fixed number of geophones), the addition of primary reflection traveltimes, the source offset, and the number of sources? These questions are examined empirically and theoretically in Chapter 3 by comparing velocity reconstructions for a synthetic model to their corresponding eigenvalue distributions and condition numbers. It was found, both numerically and analytically, that reconstruction accuracy improves for fewer layers in the inversion model (for a fixed number of geophones), for larger source offsets (up to a 1:1 offset-to-depth ratio), and for multiple sources. Velocity reconstructions also improved with the use of primary reflection traveltimes.

Extension of the one-dimensional inversion to two-dimensional structures is accomplished with a new iterative travelt ime inversion-migration method. Traveltime inversion reconstructs interval velocities while the iterative application of migration maps two-dimensional reflector locations. Chapter 4 describes and tests this iterative traveltime inversion and migration method using synthetic offset VSP travelt ime data. Excellent reconstructions were obtained for noise-free synthetics while noisy synthetics required special care.
In Chapter 5 one-dimensional traveltime inversion and the iterative traveltime inversion-migration method were applied to VSP traveltime data from ARCO's Bridenstein #1 well in the Anadarko Basin of Oklahoma. One-dimensional velocity reconstructions using primary reflection traveltimes matched the sonic log more closely than reconstructions using only direct arrival traveltimes. Application of the iterative traveltime inversion-migration method yielded somewhat poor results, which may have been due to the unsuitable nature of this particular data set.
2. LINEARIZED INVERSION OF VSP TRAVELTIME DATA

For inversion problems, a mathematical model is required to relate observations to a physical property of the earth. Using this model, synthetic data are created to compare against real data. The model parameters are then changed until the synthetic data match the real data in some best-fit sense. Calculation of the synthetic data is the forward problem; inversion is the reverse problem of fitting observations to the model. Formulations of the forward and inverse problems for VSP traveltime data are now presented, followed by a discussion of how data errors affect inversion estimates.

2.1 The Forward Problem: Raytracing

In this thesis, raytracing was used to calculate synthetic traveltime data for the forward part of the inversion method.

Raytracing is based on the geometrical optics solution to the acoustic wave equation, a high frequency approximation (Langan et al., 1985). It is assumed that seismic energy propagates along rays and that transmission and reflection occurs at velocity discontinuities following Snell's law. Ray tracing is a good approximation when the wavelengths
of the propagating wave are small with respect to the spatial changes in density, the Lame constants, and the radii of curvature of reflecting interfaces. Diffracted energy can be defined as energy that does not follow Snell's law and is therefore ignored by raytracing.

In constant velocity media raypaths are straight lines. Given an initial take-off angle \( \phi \), rays are traced through the medium following Snell's law of refraction and reflection at velocity discontinuities,

\[
\sin \frac{\phi_i}{v_i} = \text{constant},
\]

(2.1)

where \( \phi_i \) is the angle of incidence with respect to the vertical and \( v_i \) is the seismic velocity of the \( i^{th} \) layer.

There are two problems to be solved in raytracing: the initial value problem and the two-point boundary-value problem. The initial value problem refers to determining the raypath through a specified velocity field given the initial orientation of a ray. The two-point problem consists of finding the raypath between a source and receiver. The two-point problem is often solved by a "shooting method," as it is in this thesis, where a series of initial value problems are solved first. A series of rays with different initial orientations are traced through the medium to the depth of the receiver until the receiver is bracketed by
rays in the horizontal direction. Once the receiver is bracketed, a linear interpolation (Langan et al., 1985) is used to find the correct take-off angle, i.e.,

$$\theta_{\text{next}} = \theta_{\text{low}} + (X_{\text{receiver}} - X_{\text{low}})(\Delta \theta)/(\Delta X),$$

(2.2)

where $\theta_{\text{low}}$ is the take-off angle for the ray that travels a distance $X_{\text{low}}$ (less than the actual offset $X_{\text{receiver}}$), $\Delta X$ is the difference between the distances traveled by the rays that bracket the receiver, and $\Delta \theta$ is the corresponding difference between take-off angles. The interpolation is stopped when a ray falls within a specified distance of the receiver. A value of one meter was sufficiently accurate and usually less than three iterations were necessary for convergence.

2.2 Linear Inverse Theory

The traveltime equation for a ray through a two-dimensional slowness (reciprocal of velocity) field is given by the line integral equation

$$t_i = \int_{R_i} dl_i s(x, z),$$

(2.3)

where $t_i$ is the traveltime of the $i^{th}$ ray, $dl_i$ is the differential raypath length of the $i^{th}$ ray, $s(x, z)$ is the two-dimensional slowness function, and $R_i$ is the raypath of the $i^{th}$ ray. The traveltime equation (2.3) is nonlinear in slowness because
the raypath depends on the slowness field. If the slowness field is discretized into \( N \) layers, then equation (2.3) becomes

\[
t_i = \sum_{j=1}^{N} \int dl_{ij} s(x, z).
\]  \hspace{1cm} (2.4)

For layers of constant slowness, the integral equation (2.4) becomes

\[
t_i = \sum_{j=1}^{N} s_j t_{ij},
\]  \hspace{1cm} (2.5)

where \( t_{ij} \) is the raypath length of the \( i^{th} \) ray through the \( j^{th} \) layer and \( s_j \) is the slowness of the \( j^{th} \) layer. In this thesis, the medium is partitioned into \( N \) homogeneous layers for \( M \) rays. The traveltime equations are written in matrix form as

\[
t = Ls,
\]  \hspace{1cm} (2.6)

where \( t \) is an \( M \times 1 \) column vector of traveltimes, \( s \) is an \( N \times 1 \) column vector of layer slownesses and \( L \) is an \( M \times N \) matrix of ray path lengths, where \( l_{ij} \) is the ray path length of the \( i^{th} \) ray through the \( j^{th} \) layer.

The objective of least-squares inversion is to find a slowness distribution that minimizes \( ||t_{obs} - t_{calc}|| \), the summed squared difference between observed and calculated traveltimes. The residual \( N \times 1 \) traveltime vector is given by \( \Delta t = t_{obs} - t_{calc} \). In order to apply linear inverse theory,
equation (2.6) is linearized by making the approximation that raypaths remain stationary for small changes in slowness. If the raypath changes slowly for small changes in slowness, then the derivative of traveltime with respect to slowness is approximated as

\[
\frac{dt_i}{ds_j} = L_{ij}.
\]  \hspace{1cm} (2.7)

where \( dt_i/ds_j \) is the derivative of the \( i^{th} \) traveltime through the \( j^{th} \) layer, and \( L_{ij} \) is the raypath length of the \( i^{th} \) ray through the \( j^{th} \) layer (Langan et al., 1985). These derivatives (raypath lengths) are calculated during raytracing along with the traveltimes for an initial slowness estimate.

The inversion procedure begins with an initial slowness estimate \( s^0 \), for which traveltimes and the residual traveltime vector are calculated. The inversion procedure is iterative because the traveltime equation (2.6) is nonlinear (raypaths change when the slownesses change). The difference between observed and calculated traveltimes is assumed to be linearly related to the corresponding change in the slowness model. Changes in the slowness vector estimate are related to traveltime residuals at the \( i^{th} \) iteration by

\[
L \Delta s^i = \Delta t^i,
\]  \hspace{1cm} (2.8)

where \( \Delta s^i \) is the slowness change vector at the \( i^{th} \) iteration, the new slowness estimate is \( s^{i+1} = s^i + \Delta s^i \), and \( L_{ij} = dt_i/ds_j \).
Due to traveltime errors, an exact solution to equation (2.8) generally does not exist. Therefore, a least-squares solution is sought (Menke, 1984a) which minimizes an objective function

$$S = (\Delta t^i - L \Delta s^i)^T (\Delta t^i - L \Delta s^i)$$  \hspace{1cm} (2.9)

to give the ordinary least-squares solution

$$\Delta s^i = (L^T L)^{-1} L^T \Delta t^i.$$  \hspace{1cm} (2.10)

The updated slowness estimate $s^{i+1} = s^i + \Delta s^i$ is then used to compute a new $L$ matrix and this procedure is repeated until the RMS traveltime residual is less than the estimated noise level of the observed data or until the slowness model does not change significantly.

If the observed traveltimes are believed to have different variances, then the data may be weighted inversely to their estimated variance with a diagonal weighting matrix $W$. This leads to the weighted least-squares solution (Beck and Arnold, 1977)

$$\Delta s = (L^T W L)^{-1} L^T W \Delta t,$$  \hspace{1cm} (2.11)

where $W$ is a diagonal $M \times M$ matrix and $W_{ii}$ is the reciprocal of the estimated data variance of the $i^{th}$ traveltime.

If the matrix $L^T W L$ is singular or nearly singular (when there are zero or near zero eigenvalues), then the method of damped least-squares (Marquardt, 1963; Hoerl and
Kennard, 1970) is applied. A small positive quantity \( \theta \) is added to the diagonal of \( L^T W L \), which removes the singularities and stabilizes the problem. The parameter estimate (2.11) then becomes:

\[
\Delta s = (L^T W L + \theta I)^{-1} L^T W \Delta t,
\]

where \( \theta \) generally takes on values in the range \([0,1]\) and \( I \) is the identity matrix. The estimate (2.12) is now biased. For certain values of \( \theta \) the mean squared error, the sum of the squared bias and the total variance, will be smaller than that for the undamped least-squares estimate (Hoerl and Kennard, 1970).

A trade-off between variance and resolution exists. Variance refers to the difference between the true and estimated parameter values while resolution refers to the bias in estimated parameter values. An unbiased estimate has perfect resolution (Menke, 1984a). A proper damping parameter must be chosen because values of \( \theta \) too large will damp parameter changes \( \Delta s \) too much and degrade resolution, while for values of \( \theta \) too small the solution will be underdamped and may diverge. For linear problems it is possible to empirically select the optimal damping parameter which minimizes the mean squared error; this is known as ridge regression (Hoerl and Kennard, 1970). Ridge regression is not easily performed for nonlinear problems because the
number of experiments becomes large. In general there is no objective way to choose the damping parameter. In this thesis, a starting value for $\theta$ is chosen and its value is reduced at each step if the RMS traveltime residual decreases until some minimum value of $\theta$ is reached. This insures that the solution is damped less as the problem becomes more linear and closer to the true solution (Marquardt, 1963). The final damping parameter is chosen as some value which stabilizes the solution but not so large as to degrade resolution.

Parameter variances due to data errors can be estimated by calculating the covariance matrix. The covariance matrix, $V$, assuming linearity, is defined as (Marquardt, 1970)

$$V = \sigma_d^2 (L^T W L + \Theta I)^{-1} L^T W L (L^T W L + \Theta I)^{-1},$$

(2.13)

where $\sigma_d^2$ is the data variance, defined as

$$\sigma_d^2 = \frac{\sum_{i=1}^{M} (t_{calc,i} - t_{obs,i})^2}{(M - N)},$$

(2.14)

where there are $M$ traveltimes, $N$ parameters and that the data are uncorrelated with equal variance and zero mean. A unit covariance matrix ($\sigma_d^2 = 1$) describes how data variance maps into parameter variance (Marquardt, 1963). The diagonal elements of $V$ are the variances of the parameter estimates and the total variance of the estimate is
\[ \sum_{i=1}^{N} V_{ii}^2. \]  

(2.15)

Normalizing \( V \) yields the correlation matrix \( C \), defined as (Marquardt, 1963)

\[ C_{ij} = \frac{V_{ij}}{(V_{ii}V_{jj})^{1/2}}, \]  

(2.16)

where \( C \) is a symmetric matrix with elements that range in value between 1 and -1. The elements of \( C \), the correlation coefficients, give a quantitative measure of the correlation between parameters. If the absolute value of any off-diagonal elements is greater than about 0.8, then it is difficult to estimate the parameters separately, while correlation coefficients of absolute value less than 0.5 indicate that the estimation of parameters are practically independent of one another.

2.3 The Effect of Errors on Inversion

Real data always contain errors. The effect of traveltime errors on the solution may be analyzed by examining the eigenvalue distribution of \( L^T L \). A symmetric matrix, which \( L^T L \) is, may be decomposed into matrices composed of orthogonal eigenvectors and their associated eigenvalues. If the eigenvectors of the \( N \times N \) \( L^T L \) matrix
are denoted by \( s_i \), and the associated eigenvalues are \( \lambda_i \), then the slowness error vector \( \delta s \) may be written as

\[
\delta s = \sum_{i=1}^{N} \frac{\langle s_i, L^T \delta t \rangle s_i}{\lambda_i},
\]

(2.17)

where \( \delta t \) is the traveltime error vector. Small eigenvalues magnify data errors into large parameter errors, resulting in poor velocity reconstruction. It is therefore preferable to design an experiment where the eigenvalues are large or where the condition number is small. This last statement is apparent from (Noble and Daniel, 1977)

\[
\frac{\| \delta s \|}{\| s \|} \leq \kappa \left( \frac{\| L^T \delta t \|}{\| L^T t \|} \right),
\]

(2.18)

where \( \kappa \) is the condition number,

\[
\kappa = \| L^T L \| \cdot \| (L^T L)^{-1} \|.
\]

(2.19)

From equation (2.18) it is apparent that smaller condition numbers indicate better velocity resolution of \( s \) (small \( \| \delta s \| / \| s \| \)), while larger condition numbers indicate the possibility of worse velocity resolution. Therefore, the condition number of \( L^T L \) is a means of assessing the quality of a VSP experiment with regard to velocity resolution. Schuster (1986) derives an exact formula for the \( L \) condition number of \( L^T L \), where the \( L \) condition number can be shown to bound the \( L_2 \) condition number.
The solution error in equation (2.17) is not only affected by the condition number but also by the magnitude of $L^T \delta \bar{t}$. If the nonzero elements of $L^T$ (always positive) are of about the same magnitude, $\bar{L}$, and if the data errors $\delta \bar{t}$ have approximately zero-mean, then, as the number, $M$, of traveltime equations becomes large, $L^T \delta \bar{t}_i$ will tend towards zero. Mathematically, this limit may be expressed as

$$(L^T \delta \bar{t})_i = \lim_{M \to \infty} \sum_{j=1}^{M} L_{ij} \delta \bar{t}_j = \bar{L} \lim_{M \to \infty} \sum_{j=1}^{M} \delta \bar{t}_j = \bar{L} \langle \delta \bar{t} \rangle \to 0, \quad (2.20)$$

where $\langle \delta \bar{t} \rangle$ is the expected value of $\delta \bar{t}$. As more equations are added to constrain the solution, the solution is expected to improve because the value of $(L^T \delta \bar{t})_i$ will tend towards zero.
3. VELOCITY RESOLUTION

Computer tests using synthetic travelt ime data were conducted to examine the accuracy of velocity reconstructions as a function of source-receiver geometry and model complexity. These tests were designed to answer four questions. Specifically, how does the accuracy of velocity reconstructions depend on 1) the number of layers in the inversion model (for a fixed number of geophones), 2) the number of travelt ime equations from primary reflections, 3) the source offset, and 4) the number of sources? These questions have not been addressed before and are important regarding the optimal design of VSP experiments.

To answer these questions, interval velocities were reconstructed from synthetics generated from the model in Figure 3.1 and the results were compared to the eigenvalue distributions of $L^T L$. As discussed previously, condition numbers and eigenvalues are good predictors of the resolution of $L^T L$. Figure 3.1 shows the distribution of direct and primary reflected rays for an 87 layer model. The first layer is 210 meters thick and subsequent layers are 15 meters thick. Geophones are evenly spaced in depth at 15 meter intervals from 195 to 1500 meters for a total of 88
Fig. 3.1 Raypath diagrams for a one-dimensional model consisting of 87 constant velocity layers and 88 geophones, for 53 direct arrivals and 130 primary reflections. The direct (transmitted) raypaths are shown on the left and the primary reflection raypaths are shown on the right.
geophones. In practice, geophones are seldom placed in the shallowest layers because of casing difficulties and other field problems. The compressional velocities for this model and the initial velocity estimate for the inversion are shown in Figure 3.2. The velocity profile is realistic and is similar to the sonic log for the Bridenstein #1 well (presented in Chapter 5).

3.1 Number of Layers

It is desirable to know how the number of layers in the inversion model affects the velocity estimate. Too many layers may lead to a system of equations that is not very overdetermined and the estimate would be overly sensitive to data errors. Too few layers may lead to an overly coarse model that obscures fine scaled geologic features. As a working guide, Pujol et al. (1986) selected about half as many layers as geophones (about two geophones per layer) while Stewart (1984) chose two to four geophones per layer. The velocity of a layer without a geophone cannot be constrained (Pujol et al., 1986), thus one layer per geophone represents the practical upper limit to the number of layers in an inversion model (for an overdetermined system).
Fig. 3.2. The velocity profile for the model shown in Fig. 3.1 is shown as the dark line and the initial velocity estimate is shown as the light line.
To examine the sensitivity of velocity reconstructions with respect to the number of layers, direct arrival traveltimes from the model in Figure 3.1 were inverted using two different inversion models, an 87 layer model and a 44 layer model. Velocity reconstructions using 88 direct arrival traveltimes are presented in Figure 3.3a assuming an 87 layer model and in Figure 3.3b assuming 44 layers. Layers are 30 meters thick for the 44 layer model and traveltimes were rounded to the nearest millisecond. Most VSPs are recorded with a one millisecond sampling rate, thus rounding traveltimes to the nearest millisecond represents the minimum noise level for real data. The iterative solutions to equation (2.12) were stopped when the RMS traveltime residual fell below 0.5 milliseconds, otherwise the inversion method attempts to fit the model to the noise. The minimum damping factor was 0.0001 and the solutions are underdamped. The use of a larger minimum damping parameter would stabilize the estimate. The shallower layer velocities are well determined for the 87 layer model, while several of the deeper layer velocities are significantly in error. Evidently, velocity estimates for deeper layers are more prone to error. The velocity reconstruction for the 44 layer model is also good and appears to be more stable than the 87 layer estimate (the oscillations from
Fig. 3.3. Velocity reconstructions for 88 direct arrival traveltimes when the traveltimes are rounded to the nearest millisecond. The light line represents the actual model velocities and the dark line represents the reconstructed velocities for a) the 87 layer inversion model and for b) a 44 layer inversion model.
the true model are less pronounced than those for the 87 layer model). Evidently the overdetermined system of equations for the 44 layer model (twice as many equations as unknowns) stabilizes the solution more than the system of equations for 88 layers (about the same number of equations as unknowns). Stability is gained at the expense of a coarser velocity average but in this case the accuracy is quite acceptable. This result is important when the geology is more finely scaled than the geophone spacing.

More generally, as the number of layers increases, the resolution of \((L^T L)^{-1}\) or the condition number gets worse. This statement is predicted by equation (2.4) in Schuster (1986) which states that the \(L_2\) condition number grows as \(O(N^2)\). This is verified by the previous example where the \(L_2\) condition number (\(\kappa = \lambda_{\text{max}} / \lambda_{\text{min}}\)) for the 87 layer model was computed to be 8200, more than twice as large (bad) as the condition number for the 44 layer model, \(\kappa = 3800\). A smaller condition number indicates that the solution is less sensitive to data errors. This result is consistent with the velocity reconstructions in Figure 3.3.

3.2 Primary Reflection Traveltimes

Prior to this study, it was unclear whether the addition of primary reflection traveltime equations would improve the solution relative to the solution using only direct
arrival traveltimes. The addition of primary reflection traveltimes might increase resolution because they increase the overdetermination of the system of equations. On the other hand, the use of primary reflection traveltimes might decrease resolution because they provide a less direct constraint than do direct arrival traveltimes; i.e., they are sensitive to the specification of the starting velocity model as well as the location of the reflecting interface. Also, traveltime errors errors for reflection traveltimes are estimated to be larger than for direct arrival traveltimes because the signal-to-noise ratio is smaller than that for direct arrivals.

Velocity reconstructions incorporating primary reflection traveltimes are compared to reconstructions using only direct arrival traveltime equations for the 87 and the 44 layer inversion models in Figures 3.3-3.5. Traveltimes were rounded to the nearest millisecond. Velocity reconstructions using 226 traveltimes (88 direct and 138 reflected) are shown in Figure 3.4. The reconstructions incorporating primary reflection traveltimes match the model better than the reconstruction using only direct arrival traveltimes (Figure 3.3). Velocity reconstructions using 324 traveltimes (88 direct and 236 reflected) are shown in Figure 3.5 and they match the model slightly better than
Fig. 3.4. Velocity reconstructions for data with 226 traveltimes (88 direct and 138 reflected) rounded to the nearest millisecond. The light line represents the actual model velocities and the dark line represents the reconstructed velocities for a) the 87 layer inversion model and for b) a 44 layer inversion model.
Fig. 3.5. Velocity reconstructions for 324 traveltimes (88 direct and 236 reflected) rounded to the nearest millisecond. The light line represents the actual model velocities and the dark line represents the reconstructed velocities for a) the 87 layer inversion model and for b) a 44 layer inversion model.
the reconstructions using 226 traveltimes (Figure 3.4). Apparently 226 traveltimes are sufficient to constrain the solution to acceptable tolerances.

In Figures 3.6 to 3.11, velocity reconstructions are presented when the travelt ime data contain errors. Pseudo-random, zero-mean, uncorrelated noise with a standard deviation of 1.0 millisecond was added to the travelt ime data. The minimum damping parameter for all of the solutions was 0.0001, which is not optimal and is judged to be underdamped. Figures 3.6, 3.7, and 3.8 exhibit inversion results using, respectively, 88 direct arrival traveltimes, 127 traveltimes (88 direct and 39 reflected), and 226 traveltimes (88 direct and 138 reflected). In addition, parts a, b, and c in each figure represent, respectively, the reconstructions when the source is offset 500 meters, 1000 meters, and the source is offset at both 500 and 1000 meters. From the figures it is observed that the RMS velocity errors decrease as more primary reflection traveltimes are added to the inversion, i.e., for the 500 meter offset experiment the RMS velocity error was 0.239 for 88 traveltimes, 0.139 for 127 traveltimes, and 0.135 for 226 traveltimes. The corresponding results for a 1000 meter source offset show the RMS velocity error as 0.162 for 88
Fig. 3.6. Velocity reconstructions using an 87 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 88 direct arrival traveltimes from a source offset of 500 meters (RMS velocity error = 0.239), for b) 88 direct arrival traveltimes from a source offset of 1000 meters (RMS velocity error = 0.162), and for c) 88 direct arrival traveltimes for the traveltimes from the 500 and 1000 meter offsets combined (176 total traveltimes; RMS velocity error = 0.098). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise of 1.0 milliseconds.
Fig. 3.7. Velocity reconstructions using an 87 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 127 traveltimes (88 direct and 39 reflected) from a source offset of 500 meters (RMS velocity error = 0.139), for b) 127 traveltimes (88 direct and 39 reflected) from a source offset of 1000 meters (RMS velocity error = 0.098), and for c) 127 traveltimes (88 direct and 39 reflected) for the traveltimes from the 500 and 1000 meter offsets combined (254 total traveltimes; RMS velocity error = 0.069). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise was 1.0 milliseconds.
Fig. 3.8. Velocity reconstructions using an 87 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 226 traveltimes (88 direct and 138 reflected) from a source offset of 500 meters (RMS velocity error = 0.135), for b) 226 traveltimes (88 direct and 138 reflected) from a source offset of 1000 meters (RMS velocity error = 0.096), and for c) 226 traveltimes (88 direct and 138 reflected) for the traveltimes from the 500 and 1000 meter offsets combined (452 total traveltimes; RMS velocity error = 0.069). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise was 1.0 milliseconds.
Fig. 3.9. Velocity reconstructions using a 44 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 88 direct arrival traveltimes from a source of offset 500 meters, for b) 88 direct arrival traveltimes from a source offset of 1000 meters, and for c) 88 direct arrival traveltimes for the traveltimes from the 500 and 1000 meter offsets combined (176 total traveltimes). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise was 1.0 milliseconds.
Fig. 3.10. Velocity reconstructions using a 44 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 127 traveltimes (88 direct and 39 reflected) from a source offset of 500 meters, for b) 127 traveltimes (88 direct and 39 reflected) from a source offset of 1000 meters, and for c) 127 traveltimes (88 direct and 39 reflected) for the traveltimes from the 500 and 1000 meter offsets combined (254 total traveltimes). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise was 1.0 milliseconds.
Fig. 3.11. Velocity reconstructions using a 44 layer inversion model with pseudo-random, zero-mean, uncorrelated noise added to the traveltime data for a) 226 traveltimes (88 direct and 138 reflected) from a source offset of 500 meters, for b) 226 traveltimes (88 direct and 138 reflected) from a source offset of 1000 meters, and for c) 226 traveltimes (88 direct and 138 reflected) for the traveltimes from the 500 and 1000 meter source offsets combined (452 total traveltimes). Solutions are underdamped with a minimum damping parameter of 0.0001 and the standard deviation of the noise was 1.0 milliseconds.
traveltimes, 0.098 for 127 traveltimes (88 direct and 39 reflections), and 0.096 for 226 traveltimes (88 direct and 138 reflected).

The velocity reconstructions assuming a 44 layer inversion model are presented in Figures 3.9 to 3.11. Velocity reconstructions for a 500 meter source offset are displayed in Figure 3.9a for 88 traveltimes (direct only), in Figure 3.10a for 127 traveltimes (88 direct and 39 reflected), and in Figure 3.11a for 226 traveltimes (88 direct and 138 reflected). Using the same minimum damping parameter (0.0001), the solutions become more accurate with an increase in the number of primary reflection traveltimes; the errors from the true solution are less pronounced than for the 88 layer inversion model. The corresponding velocity reconstructions for the 44 layer inversion model and a 1000 meter offset are displayed in Figure 3.9b for 88 traveltimes (direct only), in Figure 3.10b for 127 traveltimes (88 direct and 39 reflected), and in Figure 3.11b for 226 traveltimes (88 direct and 138 reflected). As in the 88 layer case, the velocity errors decrease as more reflection traveltimes from primary reflection traveltimes are added to the inversion.

The RMS velocity errors for the 87 layer models are summarized in Figure 3.12 for up to 240 primary reflection
Fig. 3.12. RMS velocity errors for the velocity reconstructions in Figures 3.6 - 3.11. The reconstructions are for a single source offset of 500 meters (circle), a single source offset of 1000 meters (X), and for the two sources combined (square). The multiple source experiments have an overall smaller error. The solution improves as more primary reflection traveltimes are used. The solutions do not necessarily represent the optimal solutions.
traveltimes per source, for 500 meter offsets, 1000 meter offsets, and for the data from the two sources combined. For the same minimum damping parameter (0.001), the error decreases as more primary reflection traveltimes are used. These results suggest that the solutions are less sensitive to noise as more primary reflection traveltimes are added for the same minimum damping parameter. However, it is not clear whether the increased velocity resolution is due to an increase in the resolving capabilities of $L^T L$ (i.e., smaller or better condition numbers) or by the increase in error averaging from the addition of primary reflection traveltime equations (see equation 2.21).

To clarify this issue, eigenvalue distributions of $L^T L$ and $L_2$ condition numbers as a function of the number of primary reflection traveltimes were calculated. Eigenvalue distributions for 87 and 44 layer inversion models with source offsets of 500 and 1000 meters are presented in Figures 3.13 and 3.14. From these figures it is observed that the addition of primary reflection traveltimes results in a decrease in the magnitude of the average eigenvalue, which suggests the possibility of poorer velocity resolution (see the error analysis remarks in section 2.3). This observation is contrary to the observed velocity reconstructions. In Figure 3.15 it is observed that $L_2$ condition
Fig. 3.13. Eigenvalue distributions of an 87 layer inversion model for a) a source offset of 50 meters and for b) a source offset of 1000 meters using 88 rays (circle), 127 rays (X), 226 rays (square), and 324 rays (triangle). Note the logarithmic scale.
Fig. 3.14. Eigenvalue distributions for a 44 layer inversion model with a) a source offset of 50 meters and for b) a source offset of 1000 meters using 88 rays (circle), 127 rays (X), 226 rays (square), and 324 rays (triangle). Note the logarithmic scale.
Fig. 3.15. Spectral $L_2$ condition numbers plotted against the number of primary reflection traveltimes for a) 87 layer inversion model and for b) a 44 layer inversion model for four source offsets: 50 meters (circle), 500 meters (X), 1000 meters (square), and 1500 meters (triangle). Zero reflections correspond to the direct arrivals only (88 traveltimes), 39 reflections correspond to 127 total rays, 74 reflections correspond to 162 traveltimes, 138 reflections correspond to 226 traveltimes, 192 reflections correspond to 280 traveltimes, and 244 correspond reflections to 324 traveltimes. Note that the smallest (best) condition numbers are for the 1000 and 1500 meters source offset experiments and for the 44 layer model (b).
numbers increase as more primary reflection traveltimes are added to the inversion, which predicts decreased resolution. Both smaller average eigenvalue magnitudes and large $L_2$ condition numbers predict possibly poorer velocity resolution, contrary to the observations. A possible explanation for this apparent contradiction is that the addition of primary reflection traveltime equations averages data errors in $L^T_6t$ (see equation 2.21).

It is important to know if the velocity reconstructions are improved when the noise level of reflection traveltimes is larger than that of direct arrival traveltimes. Using synthetic tests it was found that the reconstruction is still improved over that using direct arrival traveltimes only. Pseudo-random, zero-mean, uncorrelated noise with a standard deviation of 1.5 milliseconds was added to the direct arrival traveltimes and 4.0 milliseconds noise was added to the primary reflection arrival traveltimes. Traveltime errors for primary reflections may be as large as eight milliseconds. Stewart (1984) estimated an average error of 1.5 milliseconds for direct arrival traveltimes, but a noise level for reflected traveltimes has not been reported. This noise level may be pessimistic with respect to noise contained in real data. Figures 3.16 and 3.17 show velocity reconstructions for 88 traveltimes (direct
Fig. 3.16. Velocity reconstructions using an 87 layer inversion model with pseudo-random, uncorrelated noise added to the traveltime data for a source offset of 1000 meter for a) 88 direct arrival travel times and for b) 226 travel times (88 direct and 138 reflected). The standard deviation of the noise was 1.5 milliseconds for direct arrival travel times and 4.0 milliseconds for primary reflection travel times. The solution using reflected travel times with more noise matches the model more closely than the solution using only direct arrival travel times.
Fig. 3.17. Velocity reconstructions using a 44 layer inversion model with pseudo-random, uncorrelated noise added to the traveltime data for a source offset of 1000 meter for a) 88 direct arrival traveltimes and for b) 226 traveltimes (88 direct and 138 reflected). The standard deviation of the noise was 1.5 milliseconds for direct arrival traveltimes and 4.0 milliseconds for primary reflection traveltimes. The solution using reflected traveltimes matches the model more closely than the solution using only direct arrival traveltimes.
only) and for 226 traveltimes (88 direct and 138 reflected) for 87 and 44 layer models, respectively. The reconstructions using 226 traveltimes are the most accurate. The RMS velocity errors for the 87 layer estimates were 0.0608 using 88 direct arrival traveltimes and 0.0472 using 226 direct arrival traveltimes. This is an important result that suggests that primary reflection traveltimes should be used in VSP velocity inversion (when the layers are reasonably flat).

3.3 A Variable Bias Estimator

Most inversion methods use a constant damping parameter so that all parameters are damped equally, regardless of how well individual parameters are resolved. Ideally one would not add a constant damping parameter but would add the covariance matrix multiplied by the damping parameter. In this way poorly determined parameters are damped more than well determined parameters (Crosson, 1976). As an example, the velocity reconstructions in Figures 3.3 to 3.11 become worse with depth. This observation suggests that parameters should be damped more with increasing depth. However, variable damping using the covariance matrix is generally not used because it is computationally expensive (it requires the matrix inversion and multiplication in equation 2.13).
A relatively inexpensive variable damping was proposed by Clawson (1981), who approximated the covariance matrix as the inverse of the diagonal of $L^T L$. In this case the estimator (2.13) becomes

$$\Delta s = (L^T L + \theta K)^{-1} L^T \Delta t.$$  \hfill (3.1)

where $K$ is a diagonal matrix and $K_{ii} = 1.0 / (L^T L)_{ii}$. Clawson (1981) demonstrated that the minimum of the mean squared error was smaller and broader using the estimator in equation (3.1). This variable bias matrix was tested for VSP traveltime inversion, but the bias varied too much; some parameters were over- or underdamped. Clawson's approach is a good one when $L^T L$ is diagonally dominant. A variable bias approximation was made for the VSP experiment by damping parameters proportional to the square root of the diagonal elements of $L^T L$. The estimate of the slowness errors become

$$\Delta s = (L^T L + \theta K)^{-1} L^T \Delta t,$$  \hfill (3.2)

where $K$ is a diagonal matrix and $K_{ii} = 1 / \sqrt{(L^T L)_{ii}}$. This variable bias damped the parameter estimates more equally. Figure 3.18 presents velocity reconstructions using the three estimators, i.e., constant damping, $1 / (L^T L)_{ii}$, and $1 / \sqrt{(L^T L)_{ii}}$. Figure 3.18a is the reconstruction using a constant bias estimator (equation 2.12); the estimated
Fig. 3.18. Velocity reconstructions using a) a constant damping parameter, b) a variable bias estimator where the damping is proportional to the diagonal elements of $L^TL$, and c) a variable bias estimator where the damping is proportional to the square root of the diagonal elements of $L^TL$. The bars represent one standard deviation due to data errors. The size of the error bars increase with depth in (a), are large in the middle for (b), and are more equal in size for (c).
variances increase with depth, as indicated by the size of the error bars. The reconstruction in Figure 3.18b uses the variable bias estimator (equation 3.1) suggested by Clawson (1981). The size of the errors varies considerably; the middle parameters are damped too little. Figure 3.18c shows the reconstruction using the variable bias estimator in equation 3.2; in this case the parameter variances were judged to be more equal in size and the reconstruction showed the least error. Therefore, this variable bias estimator was used for the remaining tests.

3.4 Source Offset and Multiple Sources

The source offset has been shown to be important regarding the subsurface illuminated by the VSP seismogram (Balch and Lee, 1984). The resolution of VSP traveltime inversion may also be sensitive to source offset, a question whose answer is not clearly understood.

To understand the velocity resolution as a function of source offset, velocity reconstructions for several source offsets are examined. The velocity reconstructions in Figures 3.6-3.11 are examined again. Recall that pseudo-random noise with a standard deviation of 1.0 millisecond was added to the traveltimes and that the solutions are underdamped (the minimum damping parameter was 0.0001) and that the RMS velocity errors for the 87 layer estimates
are presented in Figure 3.12. In Figure 3.6a the velocity reconstruction for 88 traveltimes (direct only) for a source offset of 500 meters is presented (RMS velocity error = 0.239) while that for a 1000 meter source offset is presented in Figure 3.6b (RMS velocity error = 0.162). The solution for the larger offset is better. The same result is observed for the corresponding reconstructions using 127 traveltimes (88 direct and 39 reflected) in Figures 3.7a (500 meter source offset; RMS velocity error = 0.139) and 3.7b (1000 meter source offset; RMS velocity error = 0.098). For 226 traveltimes (88 direct and 138 reflected) the same improvement for larger offsets is observed in Figures 3.8a (500 meter source offset; RMS velocity error = 0.135) and 3.8b (1000 meter source offset; RMS velocity error = 0.096).

The 44 layer velocity reconstructions in Figures 3.9-3.11 are also better for larger offsets. Velocity reconstructions using 88 traveltimes (direct only) are presented in Figure 3.9a for a 500 meter source offset and in Figure 3.9b for a 1000 meter offset. The velocity estimate for the larger offset is more accurate mainly for the upper (shallow) layers and is actually worse for the deepest layers. Reconstructions using 127 traveltimes (88 direct and 39 reflected) for the 44 layer model are presented in Figure 3.10a for a source offset 500 meters and in Figure
3.10b for a source offset 1000 meters. Again, the estimate for the larger offset is better, especially for the upper layers. The reconstructions for 226 traveltimes (88 direct and 138 reflected) are shown in Figure 3.11a for a source offset 500 meters and in Figure 3.11b for a source offset 1000 meters. The reconstruction for the larger offset matches the true model more closely, especially in the upper layers.

The effect of offset is also examined in Figures 3.19 and 3.20 when the optimal damping parameter (about 0.10) is used. For 88 direct arrival traveltimes with pseudo-random, zero-mean, uncorrelated noise (with a standard deviation of 1.5 milliseconds) added to the traveltimes, the velocity reconstruction for a source offset 500 meters is shown in Figure 3.19a and the reconstruction for a source offset 1000 meters is shown in Figure 3.19b. Both of the reconstructions assumed an 87 layer model with the starting velocity estimate in Figure 3.2. The RMS velocity error for the 1000 meter offset reconstruction (0.059) is smaller than for the 500 meter offset reconstruction (0.070). Figures 3.20a and 3.20b present the 44 layer velocity reconstructions for source offsets of, respectively, 500 meters and 1000 meters. The reconstruction for the larger offset matches the true model more closely.
Fig. 3.19 Velocity reconstructions using an 87 layer inversion model with zero-mean, uncorrelated noise added to the traveltime data for 88 direct arrival travel times for a source offset of a) 500 meters, b) 1000 meters, and c) 500 and 1000 meters combined. The estimate for the larger offset is better and the final damping parameter was 0.1.
Fig. 3.20 Velocity reconstructions using an 44 layer inversion model with zero-mean, uncorrelated noise added to the traveltime data for 88 direct arrival traveltimes for a source offset of a) 500 meters, b) 1000 meters, and c) 500 and 1000 meters combined. The estimate for the larger offset is better and the final damping parameter was 0.1.
Better velocity reconstructions for larger source offsets suggest that the condition number is smaller and that the average eigenvalue of $L^T L$ is larger for larger source offsets. To verify this statement, eigenvalue distributions as a function of source offset are shown in Figure 3.21 for 87 layers and in Figure 3.22 for 44 layers. The average eigenvalue is larger for larger offsets, which predicts better resolution and is consistent with the velocity reconstructions. Computed spectral condition numbers for several source offsets are plotted in Figure 3.15. Smaller condition numbers, which predict better resolution, are observed for larger offsets, except that slightly smaller condition numbers are observed for the 1000 meter offset compared to the 1500 meter offset. This observation indicates that the accuracy of velocity reconstruction should take into account not only the condition number, but also the average eigenvalue of $L^T L$.

Multiple source VSP data has been shown to improve VSP migration (Wiggins and Levander, 1984), but it is not clear how multiple sources will affect VSP traveltime inversion. Velocity reconstructions may improve with multisources since they help overdetermine the system of equations. Velocity reconstructions for multiple source data are presented in Figures 3.6-3.11 and in Figures 3.19 and 3.20. In Figures
Fig. 3.21. Eigenvalue distributions for an 87 layer inversion model for a) 88 direct arrival travel times and for b) 226 travel times (88 direct and 138 reflected) for source offsets of 50 meters (circle), 500 meters (X), 1000 meters (square), and 1500 meters (triangle). The average eigenvalue is larger for larger offsets. Note the logarithmic scale.
Fig. 3.22. Eigenvalue distributions for a 44 layer inversion model for a) 88 direct arrival travel times and for b) 226 travel times (88 direct and 138 reflected) for source offsets of 50 meters (circle), 500 meters (X), 1000 meters (square), and 1500 meters (triangle). The average eigenvalue becomes larger for larger offsets. Note the logarithmic scale.
3.6c, 3.7c, 3.8c, 3.9c, 3.10c, and 3.11c, velocity reconstructions combining the 500 and 1000 meter offset data are presented for the 87 and 44 layer models. It is clear from an examination of the multiple source velocity estimates that the solution is more stable and closer to the true model than for the single source estimates. The RMS velocity errors for the 87 layer estimates are summarized in Figure 3.12, which supports the previous statement. Reconstructions combining the data from the 500 and 1000 meter source offsets are displayed in Figure 3.19c for 87 layers and in Figure 3.20c for 44 layers. The multiple source (two source) reconstructions match the model better, which is not unexpected because more equations were used to constrain the solution.

To explain these results, eigenvalue distributions for the multiple source data are displayed in Figure 3.23. The average eigenvalue becomes larger (better) and the spectral condition number becomes smaller (better) as the number of sources increase. In general, this behavior is predicted by equation (2.4) in Schuster (1986), which shows a decreasing condition number and an increasing average eigenvalue for an increase in the number of sources.

In this chapter the accuracy of offset VSP traveltime inversion was examined as a function of source-receiver
Fig. 3.23. Eigenvalue distribution using an 87 layer inversion model for a) 88 direct arrival traveltimes from each source and for b) 226 traveltimes (88 direct and 138 reflected) from each source at offsets of 50 meters only (circle), 50 and 500 meters (X), 50, 500 and 1000 meters (square), and 50, 500, 1000 and 1500 meters (triangle). The average eigenvalue is larger for larger offsets. Note the logarithmic scale.
geometry. Velocity reconstructions improved for fewer layers in the inversion model (for a fixed number of geophones), for an increase in the number of primary reflection traveltimes (even when the error of primary reflection traveltimes was greater than that of direct arrival traveltimes), for larger source offsets (up to a 1:1 offset-to-depth ratio), and for increasing the number of sources. A variable bias matrix was observed to damp parameters more equally and give better velocity reconstructions. It should be noted that the analytic formulae in Schuster (1986) confirm that reconstruction accuracy should improve for increasing number of sources and decreasing number of layers. However, these formulae do not generalize the statement that resolution improves for increasing number of reflection traveltimes and increasing source offset. Chapter 5 uses the results in this chapter in inverting traveltime data from the Bridenstein #1 VSP data set.
4. ITERATIVE TRAVELTIME INVERSION AND MIGRATION

The previous chapters were restricted to one-dimensional earth models. To extend the inversion to two- or three-dimensional models, a new iterative traveltime inversion-migration method is developed and tested.

4.1 Ray Map Migration

The object of migration is to determine the spatial position \((x, z)\) of reflectors from seismic data recorded in time and in one spatial dimension (depth for VSP data). To correctly migrate reflections requires knowledge of the velocity field, which may be found from sonic logs, stacking velocities, or velocity inversions.

Offset VSP data sets have been migrated with "prestack" acoustic wave equation methods by several workers (Oristaglio, 1985; Wiggins and Levander, 1984; and others). With these migration methods, wave fields are extrapolated from both the source and receiver arrays and the image forms where the two wavefields coincide in space and time. The resolution is limited by the poor raypath coverage inherent in the VSP experiment and by the presence of strong vertical velocity variations. Migration results improve when
multiple source data are used. Wiggins and Levander (1984) presented a synthetic study which demonstrated that the subsurface illuminated by the migration is a function of the experiment geometry and of the subsurface. Miller (1983) presented a method for the reconstruction of reflectors using the method of ovals, but only results using perfect data for two sources and three layers were presented.

Traveltimes may be migrated using a ray map migration. The imaging condition for VSP migration is that the sum of the traveltimes from the source to the reflector and from the reflector to the receiver is equal to the observed traveltime. This condition is expressed as

\[ T_{\text{source-reflector}} + T_{\text{reflector-receiver}} = T_{\text{observed}}. \] (4.1)

The locus of all points satisfying equation (4.1) is a pseudo-ellipse (Miller, 1983). If the arrival angle at the receiver can be determined, then a single reflection point can be imaged rather than the entire locus of reflection points. This approach has a great computational advantage in that only a few rays need to be traced. The arrival angle at the receiver is found from

\[ \frac{dt}{dz} = -\frac{\cos \theta}{v}, \] (4.2)

where \( \theta \) is the arrival angle at the receiver, \( dt/dz \) is the
vertical ray parameter, and \( u \) is the seismic velocity at the receiver. This migration method is similar to the migration presented by Reshef and Kosloff (1986) for synthetic common shot gather traveltime data, where the arrival angle was determined from measurements of the horizontal ray parameter.

To determine \( \theta \) correctly, accurate measurements of \( dt/dz \) and accurate estimates of \( u \) are necessary. To measure the vertical ray parameter, traveltimes are smoothed and the slope \( (dt/dz) \) at the \( i^{th} \) geophone is calculated with the central difference formula

\[
\left( \frac{dt}{dz} \right)_i = \frac{t_{i+1} - t_{i-1}}{z_{i+1} - z_{i-1}}.
\]  

(4.3)

When noise is present, traveltimes for rays from the same reflector are fit to a straight line in a least-squares sense, where the slope of the line is equal to \( dt/dz \), a plane wave approximation. The velocity estimate can be obtained from a fixed layer traveltime inversion, as presented in Chapter 3.

To implement this migration, a single ray is traced at an angle \( \theta \) from the receiver into the medium, and then several rays are traced from the source until they intersect the ray from the receiver. The reflection point is defined
as the intersection point where the total traveltime of the two rays is equal to the observed traveltime. Figure 4.1 illustrates this concept, and the locus of reflection points defines a reflector.

4.2 Iterative Inversion and Migration

To combine the inversion and migration, iterations through three steps are required:

1) Use $u^{(0)}(x,z)$ (initially derived from sonic log data) to migrate reflections in the VSP data to locate reflecting interfaces.

2) With these reflection interfaces fixed, invert the traveltimes from direct waves and primary reflections to construct the new velocity estimate $u^{(1)}(x,z)$.

3) If $u(x,z)^{(1)}$ is "close enough" to $u^{(0)}(x,z)$, then stop. If not, set $u^o(x,z)$ equal to $u^1(x,z)$ and repeat steps 1-3 until convergence in $u(x,z)^{(1)}$ is achieved.

Inversion may be the first step rather than migration, and it may be necessary if the initial velocity estimate is poor. This method is presented as a flow chart in Figure 4.2. A conceptually similar method was applied to synthetic CDP reflection data by Stork and Clayton (1985), except that only reflected waves were available.
Fig. 4.1 Ray map migration for VSP traveltime data. The arrival angle at the receiver, $\theta$, is found from $\theta = \cos^{-1}(-dt/dz \cdot v)$. The intersection point of this ray from the receiver and the ray from the source defines the reflection point, when the sum of the traveltimes of both rays is equal to the observed traveltime.
Set $i = 0$
Initialize $\nu(x,z)^{(0)}(x,z)$ from sonic log
Calculate $t^{(0)} = Ls^{(0)}$

Compute $\Delta s = (L^TL)^{-1} L^T \Delta t$
Update $s^{(i+1)} = s^{(i)} + \Delta s$
$v^{(i+1)} = 1/s^{(i+1)}$

Migrate primary reflections to reposition interfaces

Constrain $\nu(x,z)^{(i+1)}$ with updated reflector positions

Calculate new traveltimes $t^{(i+1)}$
Calculate RMS traveltimes residual $\Delta t_{RMS}$

If $\Delta t_{RMS} < \epsilon$
or $v^{(i+1)} = v^{(i)}$

NO

$i = i + 1$
calculate $t^{(i)} = Ls^{(i)}$ and $\Delta t = t^{(i)} - t_{ obs}$

YES

END

Fig. 4.2 Flow chart illustrating the iterative traveltime inversion and migration method.
4.3 Computer Tests

The iterative inversion-migration method is first tested for synthetic traveltimes data without noise. A six-layer unconformity model with primary reflections from five interfaces was used to create 183 traveltimes (53 direct and 130 reflected). The raypaths for this model are shown in Figure 4.3. The initial model velocity estimate is shown in Figure 4.4. Traveltimes were smoothed with a five point, low pass filter, and the vertical ray parameter was calculated with a central difference formula (equation 4.3). The migrated reflection points from each interface were fit to a line in a least-squares sense. The result after one iteration (one inversion and one migration) and the final result after eight iterations are shown in Figure 4.5. The velocities, dips, and well intercepts are well imaged.

If the traveltimes data are not migrated and the layers are assumed to be horizontal, then the inversion estimate may be seriously incorrect. An inversion was performed using the starting model in Figure 4.4 with a 300 meter source offset, assuming the correct well intercepts but fixing the layers to be horizontal. The inverted result without migration is shown in Figure 4.6. The RMS traveltimes residual was 2.66 milliseconds and the velocities are
Fig. 4.3 Raypath diagrams for the synthetic model. Direct arrival raypaths are shown on the left and primary reflection raypaths are shown on the right. The source offset is 300 meters.
Fig. 4.4. The diagram on the left shows the model with the P-wave interval velocities. The diagram on the right displays the starting estimate for the travel-time inversion and migration results in Fig. 4.5. The layers are horizontal and the well intercepts are incorrect, as are the velocities.
Fig. 4.5. Results of the iterative inversion and migration method for 183 traveltimes (53 direct and 130 primary reflections) without noise using the starting estimate shown in Fig. 4.4. Diamonds represent migrated reflection points, which are fitted in a least-squares sense to yield the dashed lines. Solid lines represent the true reflectors. The result after one iteration is shown in the diagram on the left. The final image after eight iterations is shown in the diagram on the right. The RMS traveltime residual was 0.60 milliseconds. The dips and depths are well imaged. The estimated velocities of the bottom two layers are incorrect. Velocity units are meters per second.
Fig. 4.6. The initial estimate for the results shown in Figs. 4.6 to 4.13 is shown in the diagram on the right. The layers are horizontal and they intersect the well (at 300 meters) at the correct depths. The velocities are incorrect (compare to the true model in Fig. 4.4). The results of a traveltime inversion (no migration) assuming flat layers is shown in the diagram on the right. The RMS traveltime residual after five iterations was 2.66 milliseconds, which is poor because an incorrect starting model was chosen for the inversion. The velocity estimates are also poor (compare to Fig. 4.4).
significantly in error. Using migration to determine the correct reflector positions is necessary to obtain good results, as shown by the poor horizontally layered result in Figure 4.6.

The iterative method is next tested with travel times rounded to the nearest millisecond. A low pass filter was insufficient to smooth travel times when measurements of $dt/dz$ were corrupted by noise. Fitting the travel times to a straight line in a least-squares sense, where the slope of the line is equal to $dt/dz$, was found to be better.

The reflector interfaces in Figure 4.7, for a source offset 300 meters, were constrained to pass through the correct well intercepts. After applying the iterative procedure, the RMS traveltime residual was 2.93 milliseconds and several of the reflector dips were poorly determined. The actual reflection points line up at about the correct dip, but their depths are incorrect because the velocities are incorrect. Constraining the reflectors to pass through the correct well intercepts imaged dips poorly. In Figure 4.8 the well intercepts were not constrained, and the final result after four iterations is an improvement, with a RMS traveltime residual of 1.85 milliseconds. However, the well intercepts and velocities are slightly incorrect, even though the dips are well determined. This observation
Fig. 4.7. The reflection points were fitted to a line constrained to pass through the correct well intercepts. The traveltimes for a 300 meter source offset were rounded to the nearest millisecond. The result after one iteration is shown in the diagram on the left and the final result after three iterations is shown in the diagram on the left. The RMS travel-time residual was 2.93 milliseconds. The dips are poorly imaged, particularly for the second layer. Constraining the reflectors to pass through fixed well intercepts yielded poor results.
Fig. 4.8. The reflectors in these diagrams were not constrained to pass through fixed intercepts, as in Fig. 4.7. The dips and reflector intercepts were allowed to "float." The source offset was 300 meters and the traveltimes were rounded to the nearest millisecond. The result after one iteration is shown in the diagram on the left. The final result after four iterations is shown in the diagram on the right. The RMS traveltime residual was 1.85 milliseconds. The dips are well determined but the depths are wrong.
suggests that an improved result might be obtained if the well intercepts were constrained after the dips were estimated by the iterative inversion-migration method, a "float and fix" approach. In this "float and fix" approach, dips are determined by fitting the migrated reflection points to a line, but well intercepts of the reflectors are fixed (from well logs).

The results using this "float and fix" approach are shown in Figure 4.9. The RMS travelt ime residual was 0.39 milliseconds; the dips and velocities are imaged almost exactly. This result is superior to the result assuming horizontal layers (Figure 4.6).

This iterative inversion and migration method is partially similar to the inversion of VSP travelt ime data for planar layer dips by Lines et al. (1984). Layer dips were parameterized in terms of VSP travelt imes, for direct and primary reflected arrivals. Velocities and well intercepts were fixed from the sonic log. Lines et al. (1984) found that travelt imes were less sensitive to dip with increasing reflector depth (meaning that the method is more sensitive to noise with increasing reflector depth), particularly for offset to depth ratios less than 0.4, and that errors propagate downward. Fixing the well intercepts is realistic for VSP data, particularly when the sonic log
Fig. 4.9. The reflector intercepts at the well were fixed after the reflection points were fit to a planar reflector, a "float and fix" approach. The source offset was 300 meters and the traveltimes were rounded to the nearest millisecond. The result after one iteration is shown in the diagram on the left. The final image after ten iterations is shown in the diagram on the right. The RMS traveltimes residual was 0.39 milliseconds. The reflectors and velocities are well imaged. This "float and fix" approach, which was more successful than the approaches used for the results in Figs. 4.7 and 4.8, was used for the results in Figs. 4.10 to 4.13.
is available. The iterative inversion and migration method differs from the method of Lines et al. (1984) in that velocities are estimated and layer dips are determined from migration. Their method may be invalid if velocities are incorrectly estimated or if velocities vary considerably between reflectors, while the iterative traveltime inversion and migration method is robust. Moreover, the iterative traveltime inversion-migration is applicable to irregular interfaces, while the method of Lines et al. (1984) is limited to planar interfaces.

Results using the "float and fix" approach for data containing pseudo-random, zero mean noise with a standard deviation of 2.0 milliseconds are shown in Figure 4.10 for a 300 meter source offset. The results are good, with a RMS traveltime residual of 1.95 milliseconds. Note the scatter of the reflection points due to noise in the data. The layer dips are well imaged and the velocities are well determined even with two milliseconds of noise because the inversion problem is well constrained (183 traveltimes and 6 layers). Velocity reconstructions for a single source offset 150 meters with two milliseconds of noise are presented in Figure 4.11. The RMS traveltime residual after 10 iterations was 2.05 milliseconds.
Fig. 4.10. The method is tested when noise is present for a single source offset 300 meters using 183 traveltimes. Pseudo-random, zero mean, uncorrelated noise with a standard deviation of 2.0 milliseconds was added to the traveltime data. The reflectors were imaged using the "float and fix" approach. The result after one iteration is shown in the diagram on the left and the final image after six iterations is shown in the diagram on the right. The RMS traveltime residual was 1.95 milliseconds and the image is close to the true model.
Fig. 4.11. The iterative method using the "float and fix" approach is tested for a source offset of 150 meters when noise with a standard deviation of 2.0 milliseconds is added to the traveltimes. The image after one iteration is shown in the diagram on the left and the final image after nine iterations is shown on the right. The RMS traveltime residual was 2.05 milliseconds.
It cannot be stated without qualification that larger offsets yield better results. Just as the traveltime inversion is more sensitive to dip with increasing offset, migration is more sensitive to incorrect velocities with increasing offset. This iterative method diverges when the starting estimate is poor. When the starting estimate is good, data from larger offsets should yield better images.

4.4 Multiple Source Inversion

Results for the combined 150 and 300 meter source offset data are shown in Figure 4.12. The RMS traveltime residual was 1.91 milliseconds. Several of the reflector points are clearly in error, separated from the other well-grouped reflection points. The results might be improved if the outlying points were not included, which could be easily done in an interactive graphics mode.

In Figure 4.13 the results for data from three sources (150, 300 and 450 meter source offsets) with a noise level of 2.0 milliseconds are shown. The RMS traveltime residual was 1.41 milliseconds. The final image is better than the result using two sources or a single source offset 150 meters.

These tests with the six layer model have demonstrated that the iterative method, using a "float and fix" approach, can accurately image layer dips and velocities. With a
Fig. 4.12. The iterative method is tested using the "float and fix" approach for two sources offset 150 and 300 meters from the well. The traveltime data contain random noise with a standard deviation of 2.0 milliseconds. The image after one iteration is shown in the diagram on the left and the final image after nine iterations is shown on the right. The RMS traveltime residual was 1.91 milliseconds.
Fig. 4.13. The iterative method is tested using the "float and fix" approach using traveltime data from a three source experiment with offsets of 150, 300 and 450 meters. Noise with a standard deviation on 2.0 milliseconds was added to the traveltime data. The result after one iteration is shown on the left. The final image after nine iterations is shown on the right. The RMS traveltime residual was 1.41 milliseconds. The third reflector is "pulled up" because the velocity of the third layer is slower than the true velocity.
more sophisticated raytracing and migration algorithm, this method could be extended to nonplanar layers. It is expected that this iterative method will work well when the vertical ray parameter can be measured accurately and when a good initial velocity estimate is available. The method will be more prone to errors when velocity contrasts are large and when layers are inhomogeneous. Intelligent interactive application of the iterative inversion-migration procedure, however, may greatly alleviate these problems.
5. APPLICATION TO BRIDENSTEIN #1 VSP DATA

A high quality VSP data set was provided by ARCO Research and Technology Corporation from their Bridenstein #1 well (API # 35-007-22227) in Beaver County, Oklahoma (8-3N-20ECM). Figure 5.1 shows the location of the well, which is situated on the northwest edge of the Anadarko Basin. The data were recorded by Seismograph Service Company in November, 1981. A vibroseis source offset 305 meters (1000 feet) north 60 west of the well was used to record traces at 135 depths ranging from 305 to 2270 meters (1000 to 7450 feet), with most geophones spaced 15 meters (50 feet) apart. A vibroseis down sweep with frequencies from 81 to 18 hertz was used with a sweep period of 18 seconds and a record length of three seconds. Three component data were recorded, one vertical and two orthogonal horizontal, using the Seismograph Service Corporation IDPC-1100 System. The ground elevation of the well site was 832 meters (2729 feet). The observer reports are included in the Appendix.

Figure 5.2 is the vertical component of the VSP data. Most of the P-wave energy is contained in the vertical component because of the small offset-to-depth ratio for
Fig. 5.1. Map showing the location of ARCO's Bridenstein #1 well in Beaver County, Oklahoma.
Fig. 5.2. Relative amplitude VSP seismogram for the vertical component of particle velocity.
most geophones. Both upgoing and downgoing energy is present, including several strong primary reflections and a strong multiple reflection lagging the direct arrival by 225 milliseconds. The downgoing energy is much larger in amplitude than the upgoing energy.

To accurately pick primary reflection traveltimes it was necessary to separate the upgoing and downgoing components; the downgoing waves contain the direct arrivals while the upgoing waves contain the primary reflections. The wavenumber of the upgoing and downgoing waves have opposite sign, so that the two components can be separated by setting half of the frequency-wavenumber plane equal to zero (Seeman and Horowicz, 1983). The actual data were filtered with a Digicon filtering program by Dr. William P. Iverson of the University of Wyoming using a narrow reject band. Figure 5.3 shows the upgoing wavefield after velocity filtering. The downgoing energy has been effectively removed, and primary reflection traveltimes may now be picked. Figure 5.4 shows the downgoing wavefield after velocity filtering.

Well logs available for this well are the sonic, density, and gamma ray logs, and are shown in Figure 5.5. No lithologic log is available. The geology consists of Paleozoic clastics and carbonates. Formation tops are
Fig. 5.3. Relative amplitude seismogram of the upgoing wavefield after velocity filtering. Upgoing waves have been enhanced and several reflections may be seen.
Fig. 5.4. Relative amplitude seismogram of the downgoing wavefield after velocity filtering. A multiple reflection lags the direct arrival by about 225 milliseconds.
Fig. 5.5. The density, velocity, and gamma ray logs plotted together (logs courtesy of ARCO Research and Technology Corporation).
marked next to the velocity log in Figure 5.6. The Council Grove group consists of Permian limestones, the Douglas group marks the bottom of the Virgil Series of the Pennsylvanian, and the Lansing-Kansas City group marks the top of the Missouri Series of the Pennsylvanian. The Marmaton group is the top of the Des Moines Series of the Pennsylvanian while the Cherokee marks the lower group of the Des Moines Series. The Morrow is a clastic, often gas-bearing, group at the base of the Pennsylvanian, unconformably overlying the Chester group, the upper series of the Mississippian. The St. Genevieve is a Mississippian limestone group beneath the Chester.

A total of 599 traveltimes were picked, including 130 direct arrival traveltimes, 70 multiple reflection traveltimes, and 399 primary reflection traveltimes from seven reflectors. The seven reflectors are marked in Figure 5.7. The traveltime data are included in the Appendix. Because peaks of the vibroseis wavelets are picked, the traveltime data may be static shifted. The static shift problem can be overcome by including static shift terms in the inversion or by VSP deconvolution (Lines et al., 1984). The static shift method was used. The traveltime equation (2.5) becomes
Fig. 5.6. Velocity log with formation tops marked.
Fig. 5.7. Relative amplitude VSP seismogram of the upgoing wavefield with seven reflections marked, from which 399 primary reflection traveltimes were picked.
\[ t_i = \sum_{j}^{N} s_j l_{ij} + T_k, \]  

(5.1)

where \( t_i \) is the \( i \)th traveltime corresponding to the primary reflection from the \( k \)th reflector, \( N \) is the number of layers, \( s_j \) is the slowness of the \( j \)th layer, \( l_{ij} \) is the raypath length of the \( i \)th ray through the \( j \)th layer, and \( T_k \) is the appropriate static shift term. There is a static shift term for the direct arrivals and one term for each set of reflections from the same reflector; a total of eight time shift terms for the real data (one for direct arrivals and seven for primary reflection traveltimes). The computed static shift terms were observed to be less than ten milliseconds and were uncorrelated with other parameters. This static shift term also takes into account reflector depth errors, correlated geophone depth errors, and source elevation errors.

An important practical problem encountered for this data set is that there are no receivers for the first 305 meters (1000 feet). This section of earth has a large effect on travel times because the seismic velocities are the slowest and it is probably the most variable. The sonic log is only available for depths below 625 meters (2050 feet), thus no independent velocity information is available for the first 625 meters (2050 feet). Pujol et al. (1986)
examined this problem, where there are no receivers in the upper layers, and using synthetic data they concluded that the upper section is best represented by a single layer.

The presence of a strong multiple reflection indicates that a strong impedance contrast exists in the upper section and that a single layer cannot accurately represent the upper 300 meters. Assuming that this multiple reflection reflects off the surface, then some velocity information for the layers above the shallowest geophone may be inferred. This assumption provides a constraint which is the delay time between the direct arrival and ghost multiple. In this case both the depth to the impedance contrast and the velocities are unknowns. By inverting the ghost multiple traveltimes for a suite of reflector depths, a set of velocity profiles can be constructed. A reasonable estimate for the depth can be made by examining the polarity of the multiple reflections and velocity profiles obtained from inversion of multiple reflection traveltimes. For the Bridenstein #1 data, the polarity of the ghost multiple is opposite to that of the direct arrival. Assuming that this multiple reflects off the earth's surface, then the large impedance contrast must be a low impedance layer above a high impedance layer. It is thus reasonable to choose a reflector depth that yields a velocity profile increasing
with depth. A reflector depth of 170 meters (550 feet) yielded a velocity profile increasing with depth when direct and ghost multiple arrival traveltimes were inverted.

5.1 One-Dimensional Traveltime Inversion

One-dimensional inversions using starting models of 64 and 126 layers were performed on 130 direct arrival traveltimes only and on 529 traveltimes (130 direct and 399 reflected) from the Bridenstein #1 VSP data. The multiple reflection traveltimes were not included because the depth of the associated reflector is unknown. The starting velocity estimate is shown with the velocity log in Figure 5.8. The velocity reconstruction assuming 64 layers (about two geophones per layer) and a first layer thickness of 305 meters (1000 feet) using 130 direct arrival traveltimes is shown in Figure 5.9a while the velocity reconstruction using direct and primary reflected traveltimes is shown in Figure 5.9b. Both reconstructions match the sonic log well. The solution using the primary reflection traveltimes matches the sonic log better, especially between 600 and 900 meters depth. The RMS traveltime residual using direct arrival traveltimes only was 1.41 milliseconds (using a minimum damping parameter of 0.01) while it was 5.13 milliseconds using both direct and primary reflection traveltimes (using
Fig. 5.8. The initial velocity estimate for the traveltime inversion is shown with a 15 meter (50 feet) block averaged velocity log.
Fig. 5.9. One-dimensional velocity estimates assuming 64 layers (about two geophones per layer) and a first layer 305 meters (1000 feet) thick. The reconstruction using direct arrivals only (130) is shown in (a) with a RMS traveltime residual of 1.41 milliseconds and a RMS velocity error of 0.144 and the reconstruction using direct and primary reflected arrival traveltimes (130 direct and 399 reflected) is shown in (b) with a RMS traveltime residual of 5.13 milliseconds and a RMS velocity error of 0.141. A larger RMS traveltime error does not necessarily indicate a worse solution because there are more traveltimes constraining the estimate. Bars indicate one standard deviation due to data errors. The velocity estimate using primary reflection traveltimes matches the sonic log more closely. Arrows mark reflector depths.
a minimum damping parameter of 0.004). The smaller traveltime residual associated with the direct arrival traveltimes does not necessarily indicate a better solution; a smaller traveltime residual may be obtained because there are fewer traveltimes to constrain the solution. The traveltime residual becomes zero when the number of traveltimes equals the number of layers. Assuming that the sonic log is accurate, the RMS velocity errors for the inverted results were 0.144 (using direct arrival traveltimes only) and 0.141 (using direct and primary reflected arrival traveltimes). This indicates that the use of both direct and reflected arrivals will give a better velocity estimate.

Reconstructions for 64 layers assuming a first layer thickness of 170 meters (550 feet) are shown in Figure 5.10. The results are nearly identical to those assuming a first layer thickness of 305 meters. The RMS traveltime residual using only direct arrival traveltimes was 0.86 milliseconds (using a minimum damping parameter of 0.01) and that using direct and primary reflection traveltimes was 5.24 milliseconds (using a minimum damping parameter of 0.004). The RMS velocity error from the block averaged sonic log was 0.144 using only direct arrival traveltimes and was 0.143 using direct and primary reflected arrival traveltimes.
Fig. 5.10. One-dimensional velocity estimates assuming 64 layers (about two geophones per layer) and a first layer 170 meters (550 feet) thick. The reconstruction using direct arrivals only (130) is shown in (a) with a RMS traveltime error of 0.86 milliseconds and a RMS velocity error of 0.144 and the reconstruction using direct and primary reflected arrival traveltimes (130 direct and 399 reflected) is shown in (b) with a RMS traveltime error of 5.24 milliseconds and a RMS velocity error of 0.141. Bars indicate one standard deviation due to data errors. The velocity estimate using primary reflection traveltimes matches the sonic log more closely. Arrows mark reflector depths.
Velocity reconstructions assuming 126 layers (about one geophone per layer) and a first layer thickness of 305 meters (1000 feet) are shown in Figure 5.11a using only direct arrival traveltimes (with a RMS traveltime residual of 3.01 milliseconds using a minimum damping parameter of 0.0075) and in Figure 5.11b using all 529 traveltimes (with a RMS traveltime residual of 4.73 milliseconds using a minimum damping parameter of 0.004). The RMS velocity error was 0.157 using only direct arrival traveltimes and 0.152 using direct and primary reflected arrivals. The results are nearly identical to the 64 layer solutions. In fact, the 64 layer solutions appear to be an averaging of the 126 layer solutions.

One-dimensional inversion of VSP traveltime data for interval velocities matched the block averaged sonic log well (less than 16 percent RMS error), even though no traveltime data were available for the first 300 meters. Reconstructions including primary reflection traveltimes matched the sonic log better (smaller RMS velocity errors). Velocity estimates for 64 layers appeared to be an average of the 126 layer estimates.

Two-dimensional velocity estimates for dipping planar reflectors are considered next.
Fig. 5.11. One-dimensional velocity estimates assuming 126 layers (about one geophone per layer) and a first layer 170 meters (550 feet) thick. The reconstruction using direct arrivals only (130) is shown in (a) with a RMS traveltime error of 3.01 milliseconds and a RMS velocity error of 0.156 and the reconstruction using direct and primary reflected arrival traveltimes (130 direct and 399 reflected) is shown in (b) with a RMS traveltime error of 4.73 milliseconds and a RMS velocity error of 0.152. Bars indicate one standard deviation due to data errors. The velocity estimate using primary reflection traveltimes matches the sonic log more closely.
5.2 Iterative Traveltime Inversion and Migration

The iterative inversion and migration method described in Chapter 4 was applied to the Bridenstein #1 data. A seismic reflection line near the well is shown in Figure 5.12. The layers are essentially flat, thus the one-dimensional velocity reconstructions in Figures 5.9-5.11 are probably valid. Raypath diagrams for direct and primary reflected arrivals using the 64 layer velocity estimate (Figure 5.9) are shown in Figure 5.13. The raypath aperture is relatively narrow, with reflection points offset less than 150 meters from the well.

This is a poor data set with which to apply the iterative inversion and migration method. The offset-to-depth ratio is small (1:7 for the deepest reflector). Lines et al. (1984) reported poor dip estimates for reflectors with an offset-to-depth ratio of less than 0.4. The offset-to-depth ratio for the shallowest reflector (1036 meters) is less than 0.3. Jones and Jovanovich (1985) showed that raypaths from ray parameter measurements are highly nonlinear, especially for near vertical incidence rays. Following their method of analysis, the dependence of θ on \( v \) (the velocity of the layer containing the geophone) and \( dt/dz \) (the vertical ray parameter), expressed in equation (4.2), is written as
Fig. 5.12. Seismic reflection section located near the well with the VSP Extracted Trace (VET) in the middle (courtesy of ARCO Research and Technology Corporation). The VET is formed by time shifting the upgoing wavefield to align reflections and summing the traces together. This transforms the data from depth to time, so that the VSP data can be directly compared to the seismic reflection seismogram. Note that the reflectors are essentially flat. Spacing between traces is 220 feet and several formation tops and the total depth (TD) of the well are marked.
Fig. 5.13. Raypath diagrams for direct arrivals (left) and primary reflection arrivals (right) for the 64 layer velocity estimate in Fig. 5.9. The reflection points are offset less than half the source offset; thus the rays sample a relatively narrow column of earth. The horizontal exaggeration is 2.4.
\[ \cos \theta = a, \] (5.2)

where

\[ a = v \cdot \frac{dt}{dz}, \] (5.3)

then

\[ \frac{d\theta}{da} = \frac{-1}{\sqrt{1 - a^2}}. \] (5.4)

From equation (5.4) it is apparent that a small error in \( a \) (or \( v \) or \( dt/dz \)) generates a much larger error in \( \theta \) when \( a \) approaches 1, which occurs when the ray arrives at near vertical incidence. It is clear that measuring the arrival angle is highly nonlinear or unstable for nearly vertical rays. This analysis predicts poor results for near vertical incidence arrivals which is the case for the Bridenstein #1 data. In this case the iterative method is not expected to give a better velocity reconstruction than the one-dimensional estimates, but they serve to illustrate the method as applied to real data.

The starting model for inversion was divided into 34 layers (about four geophones per layer) and the starting velocity estimate was an average of the one-dimensional solution (Figure 5.10). Well intercepts were fixed and the "float and fix" approach (as described in Chapter 4) was used. The image after one iteration (one inversion and one
migration) is displayed in Figure 5.14. The dips of the nonreflecting layers were constrained to vary smoothly between reflectors, and the top and bottom layers were constrained to be horizontal. The migrated reflection points are offset as far as 1300 meters from the well and most migrated reflection points decrease in depth as their offset increases. This observation indicates that the reflection points are falling along the pseudo-ellipse of points that satisfy the imaging condition [equation (4.1)] and that vertical ray parameter measurements are in error. To obtain reasonable dip estimates, only reflection points offset less than 200 meters from the well were fit to a straight line. The RMS traveltime residual was 5.12 milliseconds, no larger than the residual assuming a horizontally layered model. The final image after four iterations, where the RMS traveltime residual reached a minimum of 3.42 milliseconds, is shown in Figure 5.15. This traveltime residual, which is smaller than the residuals for the one-dimensional models, indicates that the velocity estimates are nonunique (that other models may fit the traveltime data equally well). The image after four iterations is nearly the same as the image after one iteration. The layers dip less than 10. From the seismic reflection line in Figure 5.12 it is not clear whether there
Fig. 5.14. Image of the Bridenstein #1 VSP data after one iteration of the iterative inversion and migration method using 34 layers (about four geophones per layer) with the starting velocity estimate taken from Fig. 5.8. The RMS traveltime residual was 5.12 milliseconds. Diamonds represent migrated reflection points. Only reflection points offset less than 200 meters from the well were fit to planar reflectors. Many of the reflection points are probably far from their true reflection points (compare to Fig. 5.12).
Fig. 5.15. Image after three iterations. The results are nearly identical to those in the previous figure. The RMS traveltime residual was 3.42 milliseconds.
is any dip at all. The velocity profile for the final image is shown with the block averaged velocity log in Figure 5.16.

The conclusion drawn from these results is that the iterative inversion and migration works poorly for the Bridenstein #1 data because the method is highly sensitive to errors in the measurement of $v$ and $dt/dz$, particularly for near vertical incidence arrivals. The method may work well for data with larger offset to depth ratios, for multiple source data, and when the vertical ray parameter can be accurately measured, as suggested by the synthetics in Chapter 4. The advantage of the ray map migration is that only travelt ime data is required, no more than one source is required, and it is relatively fast compared to other migration methods. The disadvantage is that it is highly nonlinear or unstable when $\theta$ is obtained from noisy measurements of $dt/dz$. A VSP data set with dipping layers and a larger source offset or multiple source data is necessary to test the method.
Fig. 5.16. Velocity profile corresponding to the image in Fig. 5.15 plotted against the 15 meter (50 feet) block averaged velocity log.
6. CONCLUSIONS

The resolution of VSP traveltime inversion (using a damped least-squares method) was examined as a function of source-receiver geometry using synthetic traveltime data from one-dimensional models. Velocity reconstructions were compared to computed eigenvalue distributions and spectral ($L_2$) condition numbers and to analytic ($L_\infty$) condition numbers. Improved velocity reconstructions were observed for fewer layers in the inversion model (for a fixed number of geophones), when primary reflection traveltimes were added to the system of equations (even when the noise level of the primary reflection traveltimes was larger than that of direct arrival traveltimes), for larger source offsets (up to a 1:1 offset-to-depth ratio), and for multiple source data. A variable bias estimator was found to damp parameters more equally.

An iterative traveltime inversion and migration method was developed and tested using synthetic traveltime data, with encouraging results. The method appears to be accurate and robust when the vertical ray parameter can be measured accurately. The traveltime inversion method is robust while the ray map migration is sensitive to $dt/dz$ measurements.
One-dimensional traveltime inversion and the iterative inversion and migration method were applied to VSP traveltime data from ARCO's Bridenstein #1 well. One-dimensional velocity reconstructions incorporating primary reflection traveltimes matched the block averaged velocity log more closely than velocity reconstructions using direct arrival traveltimes only, confirming the results from synthetic tests. Application of the iterative inversion and migration method to the Bridenstein #1 data yielded somewhat poor results because of the small offset to depth ratio of reflectors (less than 0.3).
APPENDIX

BRIDENSTEIN #1 VSP DATA SET