FAST LEAST SQUARES MIGRATION WITH A DEBLURRING FILTER

by

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A thesis submitted to the faculty of
The University of Utah
in partial fulfillment of the requirements for the degree of

Master of Science
in
Geophysics

Department of Geology and Geophysics
The University of Utah
May 2009
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ABSTRACT

Least squares migration (LSM) is a linearized waveform inversion for estimating a subsurface reflectivity model that, relative to conventional migration, offers improved spatial resolution of migration images. The cost, however, is that LSM typically requires 10 or more iterations, which is about 20 times or more the computational cost of conventional migration. To alleviate this expense, I present a deblurring filter that can be employed in either a regularization scheme or a preconditioning scheme to give acceptable LSM images with less than $\frac{1}{3}$ the cost of the standard LSM method. My results in applying deblurred LSM (DLSM) to synthetic data and field data support this claim. In particular, a Marmousi2 model test showed that the data residual for preconditioned DLSM decreases rapidly in the first iteration, which is equivalent to 10 or more iterations of LSM. Empirical results suggest that regularized DLSM after three iterations is equivalent to about 10 iterations of LSM. Applying DLSM to two-dimensional marine data gives a higher resolution image compared to those from migration or LSM with three iterations. These results suggest that LSM combined with a deblurring filter allows LSM to be a fast and practical tool for improved imaging of complicated structures.
To my family
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ACKNOWLEDGEMENTS

I would like to thank my advisor Gerard T. Schuster for his academic guidance, support and encouragement throughout my stay at the University of Utah. I am in debt to other members of my advisory committee, Richard D. Jarrard and Michael S. Zhdanov, for reading and correcting this thesis. I also want to thank my group colleagues for their academic discussions and personal help. I also thank JOGMEC and JAPEX for supporting my study at the University of Utah.
CHAPTER 1

INTRODUCTION

Seismic migration is the keystone imaging method in exploration seismology. It is typically applied to three-dimensional prestack data, and can be implemented as either a Kirchhoff or wave equation method. A key problem with standard migration is that the image can be distorted because of a poor acquisition geometry, a narrow source-receiver aperture, or a defocusing lens such as a salt body. To partly correct for these distortions, the data can be regularized and then migrated (Fomel and Guitton, 2006), the image can be illumination compensated (Rickett, 2003), the approximate Hessian inverse can be applied to the migration image (Hu et al., 2001; Etgen, 2002; Guitton, 2004; Yu et. al., 2006; Lecomte, 2008; Toxopeus et. al.,2008), or a least squares migration (LSM) scheme can be implemented (Nemeth et al., 1999; Chavent and Plessix, 1999; Duijndam et.al., 2000; Kuhl and Sacchi, 2004; Plessix and Mulder, 2004; Symes, 2008).

In the LSM scheme, the data are migrated and the resulting reflectivity image is used to generate simulated data. These simulated data are subtracted from the field data and the weighted residual is migrated to correct for the estimated reflectivity model. This constitutes one iteration, and a typical LSM scheme typically requires more than 15 such iterations. The benefit can be more than a doubling of lateral spatial resolution and a noticeable reduction in migration artifacts (Nemeth et al., 1999). However, the penalty is that LSM can be more than an order of magnitude more costly than standard migration.

To reduce this cost I propose the use of LSM with a deblurring filter. This deblurring filter is an inexpensive approximation to an Hessian inverse, and is used either in a regularization scheme or as a preconditioner in LSM.

This thesis is organized as follows. First, I present the theory that combines a deblurring filter with LSM, where the deblurring filter is used for regularization or
preconditioning of LSM. This is followed by a section on numerical results for a synthetic
data set from the Marmousi2 velocity model and a two-dimensional marine data set.
My LSM method is used with poststack data, but it can be applied to prestack data as
well. Finally, I present conclusions.
CHAPTER 2
DEBLURRED LEAST SQUARES MIGRATION

Forward modeling of acoustic data is mathematically represented by the modeling operator $L$ such that:

$$d = Lm,$$

(2.1)

where $d$ is the scattered seismic data, $m$ is the reflectivity model, and $L$ is the forward modeling operator associated with a specific survey geometry, source wavelet, and velocity-density model.

Inversion of the data for the reflectivity model $m$ can be represented by the following equation:

$$m = \left( L^T L \right)^{-1} L^T d,$$

(2.2)

where $L^T$ is the adjoint of the forward modeling and $(L^T L)^{-1}$ is the inverse Hessian. In comparison, the migration operator is the adjoint of the forward modeling and can be represented by the following equation (Claerbout, 1992):

$$m_{mig} = L^T d,$$

(2.3)

where $m_{mig}$ is the migration image. Equation (2.3) is computationally cheaper to use than matrix inversion of $L^T L$ in equation (2.2). However, the above equation yields the relationship between the migration and the actual reflectivity images by substituting equation (2.1) into (2.3) to get:

$$m_{mig} = L^T Lm.$$

(2.4)

This equation shows that the standard migration image is a $L^T L$ blurred version of the actual reflectivity model $m$ (Hu et. al., 2001). In addition, equation (2.2) can also be rewritten as:

$$m = (L^T L)^{-1} m_{mig},$$

(2.5)
which suggests that the migration image can be deblurred by a good estimate of 
\((L^T L)^{-1}\).

### 2.1 LSM theory

Least squares migration (LSM) is a linearized inversion of seismic data for the 
reflectivity model, which globally deblurs the estimated model image and employs 
a gradient type optimization method such as the steepest descent or the conjugate 
gradient methods (Nemeth et al., 1999; Chavent and Plessix, 1999; Duijndam et.
al., 2000; Kuhl and Sacchi, 2004; Plessix and Mulder, 2004; Symes, 2008). The steepest 
descent method updates the estimated reflectivity model in the following way:

\[
m^{n+1} = m^n - \alpha^n g^n, \tag{2.6}
\]

where \(m^{n+1}\) and \(m^n\) are the \(n+1\) and \(n\)-th reflectivity models; 
\(-\alpha^n g^n\) is the model update for the \(n\)-th iterative step; \(\alpha^n\) is the step length that can be found by minimizing 
the misfit functional with respect to the scalar \(\alpha^n\), and \(g^n\) is the misfit gradient vector 
that is parallel to the steepest descent direction of the misfit function.

To estimate the gradient vector in equation (2.6), I first define the regularized misfit 
function \(\epsilon\) in the following form (Zhdanov, 2002):

\[
\epsilon = \frac{1}{2} \| W_d L m - W_d d \|^2 + \frac{\gamma}{2} \| W_m m - W_m m_{apr} \|^2, \tag{2.7}
\]

where \(m\) is the estimated reflectivity model; \(\| \|\) represents the \(l_2\) norm; \(\gamma\) is a regularization parameter; and \(W_d\) and \(W_m\) are weighting matrices for the data and model. 
By perturbing the misfit function with respect to the perturbation of the reflectivity 
model parameters \(m\), we get the misfit gradient:

\[
g_i = \left( \frac{\partial \epsilon}{\partial m_i} \right) = [L^T W_d^2 (L m - d)]_i + \gamma [W_m^2 (m - m_{apr})]_i, \tag{2.8}
\]

where \(g_i\) represents the \(i\)-th component of the gradient vector.

### 2.2 Deblurring filter theory

To partly deblur the migration image \(L^T d\) by an approximation to \((L^T L)^{-1}\), we 
can employ a local layered media assumption (Hu et. al, 2001; Yu et., al., 2006) or a
localized multichannel matching filter (Etgen, 2002; Guitton, 2004). For the matching filter, we first find a local filter $F \approx (L^T L)^{-1}$ that honors the following equations:

$$F \ast [L^T d'] = m',$$  \hspace{1cm} (2.9)

where $m'$ is a reference reflectivity model in a window; $L^T d'$ is the reference migration image in the filter window; and $d'$ represents synthetic data generated from the reference model by forward modeling $L m'$; and $\ast$ denotes convolution. Since the operator $F$ is designed to locally match the simulated migration response to its reflectivity model, the $F$ approximates $(L^T L)^{-1}$ in equation (2.2) (see Appendix A). This nonstationary $F$ can be computed inexpensively because its size is only several wavelengths wide and tall.

Since $L^T L$ only depends on the background velocity model and not on the actual reflectivity distribution, then $F \approx (L^T L)^{-1}$ can be computed from the reference reflectivity images and applied to the actual migration image, i.e.,

$$F \ast [L^T d] \approx m,$$  \hspace{1cm} (2.10)

where $m$ is an approximation of the actual reflectivity model in a window; $L^T d$ is the actual migration image in the filter window; and $d$ represents the actual data. The small filter window moves over the migration image with a 10% overlap in neighboring windows and the entire deblurred image is reconstructed by summing the results from all of the windows. The deblurred image as an estimate for $m$ is then used as the constraint of a regularized LSM method. Another use of $F \approx (L^T L)^{-1}$ is to use it as a preconditioning filter at each iteration.

### 2.3 Deblurred LSM theory

I employ the deblurring filter in two different ways in the deblurred LSM (or DLSM) algorithm. First, it is used to obtain a more accurate a priori model as a constraint for LSM. For my scheme, I use a deblurred a priori model in equation (2.8) such that:

$$m_{apr} = F L^T d,$$  \hspace{1cm} (2.11)
where $F \approx (L^T L)^{-1}$ is the deblurring operator computed from the reference model and the reference migration image, and $L^T d$ is the migration image. I call this LSM version regularized DLSM (or RDLSM).

The second usage of a deblurring filter is as a preconditioner. As seen in equation (2.8), the gradient vector includes the migrated data residual vector $L^T W_2 (L m - d)$, where it is blurred with $L^T L$ as well (see Appendix B). By applying the approximate Hessian inverse to the gradient vector as a preconditioner, the convergence rate of LSM should be accelerated. An algorithm for preconditioned LSM (or PDLMS) is the following:

$$m^{n+1} = m^n - \alpha^n F * g^n,$$

(2.12)

which is the same as equation 2.6 except for the preconditioned term in updating the model.
The deblurring method is tested using synthetic acoustic traces generated from the Marmousi2 model, and two-dimensional marine data from the Gulf of Mexico.

### 3.1 Marmousi2 model test

Deblurred LSM (or DLSM) is tested on poststack data obtained from the Marmousi2 model (Martin et al., 2006). To evaluate the performance of DLSM, three reference models are investigated with two types of DLSM algorithms: regularized DLSM (or RDLSM) and preconditioned DLSM (or PDLSDM). The method is tested in the following way:

1. The P-wave velocity and density models (Figures 3.1a and 3.1b) are down-sampled to a grid size of 6.25 x 6.25 m. The coarse model has 2721 x 560 grids in the x and z dimensions, which has the physical dimensions of about 17 km width x 3.5 km depth.

2. The source wavelet that mimics an air gun source signature (Figure 3.2) is generated in the following way: first, a Ricker wavelet having a dominant frequency of 20 Hz is created. Second, its phase is rotated by 60 degrees and then it is differentiated twice.

3. The P-wave reflectivity model shown in Figure 3.3a is obtained from the P-wave velocity and density models.

4. A migrated section shown in Figure 3.3b is computed in the following way: first, a zero offset (ZO) stacked section is computed from the reflectivity model by a diffraction stack forward modeling method with the source wavelet over the x
range from the 801-th grid point to the 2000-th grid point, which is 7.5 km wide. The number of traces in this range is 300 with a trace interval of 25 m and a recording length of 5 seconds. Ray tracing in a background velocity model is used to compute traveltimes. I assume that these computed data are the actual data $d$. I also assume that the migration velocity and source wavelet are estimated properly before LSM. The data are migrated by a diffraction stack migration method with

**Figure 3.1.** Marmousi2 model. (a) P-wave velocity and (b) density model.
Figure 3.2. Source wavelet for this experiment.

Figure 3.3. Actual (a) reflectivity model and (b) migration image. (a) and (b) are also used as a reference model and migration image, respectively, in the exact model case.
the accurate velocity, which I used in forward modeling, to get \( m_{mig} = L^T d \).

5. I define a target model space below the survey line and create 2 types of simple reference reflectivity models \( m' \) for the deblurring filter test. The first one is a geological model (Figure 3.4a), which is an interpretation of the subsurface reflectivity distribution. This model promotes a smoothly changing \( F \) along the offset coordinate. Since any geological interpretation includes some errors, I provide an erroneous reflectivity distribution in the following way: Strong or moderate reflectivities are extracted from the actual model and approximated by reflectivity values of -0.2, -0.1, 0.1, 0.2, and 0.3. Some weak events in the shallow part with reflectivities between -0.1 to -0.04 or 0.04 to 0.1 are used as events with erroneous reflectivity values of -0.07 or 0.07. Events related to hydrocarbon anomalies such as oil/gas, oil/water and gas/water contacts (i.e., OGC, OWC, and GWC) and events below the unconformity around 3 km in depth are eliminated. This model has fewer reference events in the deeper part. The second model is a grid point model (Figure 3.4b), where the model space is divided into sections; each section has an area of 250 m x 250 m (40 sample x 40 sample) in this test. A scatterer having a reflectivity of 1 is placed at the center of each section. Compared with the geological model, reference scatterers are evenly distributed in the model space.

6. Synthetic data \( d' = Lm' \) are generated from each reference reflectivity model \( m' \) and then migrated to give \( m'_{mig} = L^T d' = L^T Lm' \) (Figures 3.5a and 3.5b).

7. From the migration images and its reference model, the deblurring functions \( F \) are computed. To evaluate the dependency of \( F \) on the specific model, the actual model is used as one of the reference models in this test. The parameters used are shown in Table 3.1. Since some filter windows in the geological model do not include reference events, the deblurring filters \( F \) from the above reference events are used for such windows. As for the grid model, the filter \( F \) is computed only once in each section and is used for the entire area of the section.
Figure 3.4. Reference reflectivity models for the (a) geological and (b) grid cases. Note that (b) is a zoom view around X = 10 km and Z = 1.5 km because the whole view cannot depict tiny scatterers obviously.
Figure 3.5. Reference migration images from the (a) geological and (b) grid models.
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<tr>
<td>Window size</td>
<td>19 points (118.75 m)</td>
</tr>
<tr>
<td>Sliding step</td>
<td>1 points (6.25 m)</td>
</tr>
<tr>
<td>Filter length</td>
<td>19 points (118.75 m)</td>
</tr>
<tr>
<td>Number of channel for one side</td>
<td>9 traces (56.25 m) for Exact and Geological model or 19 traces (118.75 m) for Grid model</td>
</tr>
<tr>
<td>Damping factor</td>
<td>10 %</td>
</tr>
</tbody>
</table>
8. The computed filters $F$ are applied to the actual migration section (Figure 3.3b) and the results are shown in Figures 3.6a-c. We can confirm that the local filter correctly reconstructs the reflectivity image with the exact model case (Figure 3.6a). We also see that the images from the geological and grid models (Figures 3.6b and c) unveil the anticline structures hidden under the thrust sheets. The images are much better than that seen in the conventional migration image. Less noise can be seen, especially at the left edge and around the upper anticline. It appears that the geological model provides a more accurate deblurred image. However, we can notice that the lower anticline image from the geological model is not reliable as that from the grid model. This is because no interpretation is provided for this part of the model for the estimate of $F$ in the geological model.

9. A steepest descent LSM image is computed with up to 30 iterations. The images for the 3-rd, 10-th, and 30-th iterations are shown in Figures 3.7a, 3.7b, and 3.7c, respectively. The image from LSM after three iterations (Figures 3.7a) shows significant noise around $X = 10$ km and $Z = 3$ km but the anticlines are becoming more visible. However, this image is not as accurate as the deblurred migration image that estimated $F$ from the geological model. The LSM image after 10 iterations (Figures 3.7b) is more accurate than that after three iterations, but the upper anticline image still contains artifacts. Most of these artifacts are eliminated with LSM after 30 iterations except the noise at $X = 10$ km can still be seen.

10. The same figures from the RDLMS with the exact model are displayed in Figures 3.8a, 3.8b, and 3.8c and the same figures except with the geological model are shown in Figures 3.9a, 3.9b, and 3.9c, with the grid models in Figures 3.10a, 3.10b, and 3.10c. These images resemble the same as those for standard LSM but the continuity of seismic events in the upper anticline seem slightly better in all three models. Figures 3.11a and 3.11b compare the convergence curves of the RDLMS and LSM algorithms. In this test, $\gamma = 200000 \times 0.5^{n-1}, n = 1, 2, \ldots, 30$ provides a decreasing damping parameter for the regularization term in equation (2.8). The convergence curves show that the RDLMS method expedites the convergence rate of LSM, where the residual is about $2/3$ of that for LSM at the same number of iterations.
Figure 3.6. Deblurred migration images with the (a) exact, (b) geological, and (c) grid model.
Figure 3.7. LSM images after (a) 3, (b) 10, and (c) 30 iterations.
Figure 3.8. RDLSM images with a deblurring filter estimated from the exact reflectivity model after (a) 3, (b) 10, and (c) 30 iterations.
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Figure 3.10. RDLSM images with a deblurring filter estimated from the grid reflectivity model after (a) 3, (b) 10, and (c) 30 iterations.
Figure 3.11. Comparison of RDLSM residual curves in (a) data and (b) model space.
iterations. The model residuals from RDLSM after 30 iterations are several percent lower than that of LSM.

11. The PDLGSM images are computed. Images with the exact model are shown in Figures 3.12a, 3.12b, and 3.12c. The same figures except with the geological model are in Figures 3.13a, 3.13b, and 3.13c, with the grid models in Figures 3.14a, 3.14b, and 3.14c. Several remarkable differences can be observed in all PDLGSM images after three iterations compared with other methods. The PDLGSM image after three iterations with the exact model (Figure 3.12a) accurately shows the anticline structures below the complex faults. However, high wavenumber noise is also seen compared with the deblurred migration with the exact model case (Figure 3.6a). The same PDLGSM image except with the geological model (Figure 3.13a) also shows an acceptable upper anticline image but the lower one is not obvious. The same image except from the grid model (Figure 3.14a) shows higher wavenumber features but the seismic events are not free of noise. Figures 3.15a and 3.15b compare the convergence curves of the PDLGSM and LSM algorithms. The model residual after three iterations of PDLGSM is equivalent to about 20 iterations of LSM in the exact model case, and is similar to about 10 iterations of LSM in the geological and grid model cases. We see that PDLGSM provides the fastest convergence rates compared to LSM and RDLSM in the early iterations. However, since the deblurring filter adds some noise, the residual curves level off after 3-5 iterations. The preconditioner is not used after the 6-th iteration and the result is that the residual curves in the data and model domains start decreasing again. The model residuals after 30 iterations of the geological and grid models are slightly better than that from LSM and are not as good as those from RDLSM. However, the one from the exact model shows a lower model residual than those from the other models.

I now compare LSM, RDLSM, and PDLGSM traces for events from shallower and deeper targets. I, first, show the standard LSM traces after the 3-rd, 10-th and 30-th iterations in Figures 3.16a and 3.16b. The migration traces and the actual model traces are also plotted in these figures, where the vertical resolution of the migration image
Figure 3.12. PDLSM images with a deblurring filter estimated from the exact reflectivity model after (a) 3, (b) 10, and (c) 30 iterations.
**Figure 3.13.** PDLSM images with a deblurring filter estimated from the geological reflectivity model after (a) 3, (b) 10, and (c) 30 iterations.
Figure 3.14. PDLSM images with a deblurring filter estimated from the grid reflectivity model after (a) 3, (b) 10, and (c) 30 iterations.
Figure 3.15. Comparison of PDLSM residual curves in (a) data and (b) model space. The preconditioner is not used after the 6-th iteration.
Figure 3.16. Comparison of traces from standard LSM after 3, 10, and 30 iterations at the locations of (a) X = 9.275 km and (b) X = 10.3875 km. Traces of the migration image and actual reflectivity model are also plotted and the amplitudes are normalized by the maximum amplitude.
is much lower than that of the actual model and the migration traces do not represent the hydrocarbon anomalies. It is noticeable that the LSM image after three iterations reveals the oil-water contact (OWC) at the shallower target (Figure 3.16a). However, for the deeper events, more than 10 iterations seem to be required for acceptable accuracy.

Next, I show the RDLSM and PDSLMS images after three iterations, which is an affordable computational expense. Figures 3.17a and 3.17b show the shallow target traces from RDLSM and PDSLMS after three iterations. The PDSLMS images (Figure 3.17b) have a shorter wavelength than those from RLSM (Figure 3.17a) or LSM after 30 iterations (Figures 3.16a). Figures 3.18a and 3.18b are the same as Figures 3.17a and 3.17b except they are zoomed to the deeper targets. Images from RDLSM (Figure 3.18a) do not reveal the OWC even with the exact model. The PDSLMS with the exact reflectivity model (Figure 3.18b) clearly reveals both the GOC and OWC. The result that used the deblurring filter obtained from the geological model slightly reveals the OWC. However, the result with the grid model is noisier than the migration image.

From the above results, we learn that the preconditioning approach has fast convergence in the first couple of iterations but it also introduces noise at later iterations and I switched to non-preconditioned LSM at the 6-th iteration. In comparison, the regularization scheme provides a more stable result with a good choice of regularization parameters but at a slower convergence rate. I think that DLSM with both the regularization and preconditioning schemes could provide both stability and efficiency.

As for model type, a geological model with properly interpreted horizons provides an acceptable deblurred image, and helps to improve the LSM convergence rate. We also observe that the PDSLMS with the exact model shows significantly lower data and model residuals in the early iterations. On the other hand, the computational cost with the grid model is much less than that of the geological model because the price for the deblurring filter linearly depends on the number of local windows for the computation of $F$; the former required several CPU minutes, which is much less than the cost of one migration, and the latter required about 1 CPU hour, which is about one migration, for the seismic image computed on a dual processor 2.2 GHz computer. Moreover, a grid model does not need the additional expense of human interpretation and can provide rapid turnaround for estimating an optimal deblurring function.
Figure 3.17. Comparison of traces at the location of $X = 9.275$ km computed with (a) RDLSM after three iterations and (b) PDLSM after three iterations. The 3 test cases are compared with the migration image and actual reflectivity traces.
Figure 3.18. Comparison of traces at the location of $X = 10.3875$ km computed with (a) RDLSM after three iterations and (b) PDLSM after three iterations. The 3 test cases are compared with the migration image and actual reflectivity traces.
In practice, a balance of computational price and performance must be considered. I suggest that a useful number of LSM iterations is around three iterations because of the rapid residual decrease in the early iterations. Moreover, the cost of LSM with three iterations is about 10 times as that in standard migration, which may be an affordable range even with a three-dimensional data set. In the next section, I compare standard migration images with LSM and DLSM images after three iterations.

### 3.2 Two-dimensional marine data test

I now migrate a two-dimensional marine data set recorded in the Gulf of Mexico (GOM). There are 515 shot gathers with a source interval of 37.5 m and for each shot gather, there are 480 traces with a trace interval of 12.5 m. The stacked section is obtained in PROMAX with a simple pre-processing flow:

1. Shot gathers are loaded and the survey geometry is defined.
2. Traces are sorted into common midpoint gathers (CMP) with the trace interval of 6.25 m.
3. The stacking velocity is estimated from a velocity analysis applied to the CMPs.
4. NMO is applied with a stretch mute percentage of 30%.
5. A top mute function is defined and applied to the data.
6. CMP stacking is applied with a mean method for trace summing.
7. A stacked section with a 7.5 km width and 4.0 sec in two way time is extracted.

Notice that no amplitude compensation is applied because it is accounted for in the deblurring filter. A conventional wavelet deconvolution is not necessarily because the deblurred filter and LSM deconvolve the source wavelet. If a standard deconvolution is applied to the input data previously, an estimated wavelet after the deconvolution must be provided for in the deblurring filter and LSM.

These pre-processing steps are followed by some other processing steps:
1. The source wavelet is estimated from the sea floor reflection in the following way: Several traces from the line, after flattening of the direct arrival, are summed together, and then the wavelet is extracted from the result.

2. A velocity model is obtained with traveltime tomography and resampled to a 6.25 m x 6.25 m of spacing grid.

3. Traveltimes are computed for the velocity model with a ray tracing method.

From the experience of the Marmousi2 model test, the following DLSM processing flow is employed:

1. A deblurred migration image is obtained with a grid model.

2. A geological model is created by skeletonizing the deblurred migration image.

3. Both the regularization and preconditioning schemes are employed with the DLSM.

In this test, the preconditioner is used for all three iterations to increase the LSM convergence rate. To avoid noise from the deblurring filter, a regularization scheme is also employed. In Figures 3.19 and 3.20, results from migration (Figures 3.19a and 3.20a) and LSM after three iterations (Figures 3.19b and 3.20b) are compared with the DLSM images after three iterations (Figures 3.19c and 3.20c).

Zoom views of the migration, LSM, and DLSM images from the box A and B in Figure 3.19 are shown Figure 3.21a-c and 3.22a-c, respectively. The same views from the box C and D in Figure 3.20 are also shown Figure 3.23a-c and 3.24a-c. The DLSM image near the seafloor in Figure 3.21c shows an improved image of the recognizable strata compared to LSM. The normal faults in the shallow sediments (Figure 3.22c) are more clearly imaged with DLSM. Figures 3.23c and 3.24c at the deeper parts also show the highest resolution in these three images. The DLSM method successfully provides a quality image after only three iterations.
Figure 3.19. Comparison of images from the shallow part after (a) standard migration, (b) LSM after three iterations, and (c) DLSM after three iterations.
Figure 3.20. Comparison of images from the deeper part after (a) standard migration, (b) LSM after three iterations, and (c) DLSM after three iterations.
Figure 3.21. Zoom views of the box A.
Figure 3.22. Zoom views of the box B.
Figure 3.23. Zoom views of the box C.
Figure 3.24. Zoom views of the box D.
3.3 Conclusions

A migration deblurring function can be efficiently approximated by a local non-stationary filter for estimating the Hessian inverse. My results suggest that this filter can significantly expedite the computation of an LSM image.

After estimating $F$ from synthetic data, the migration image is deblurred to give $m \approx F \ast m_{mig}$, which is the starting model for an iterative LSM method. Tests on the Marmousi2 data sets show that the deblurred migration image after three iterations is roughly equivalent to that of LSM after 10-20 iterations. However, some additional noise is introduced into the migration image by the deblurred filter. This noise can be eliminated by controlling the model dependency with a regularization parameter or not using the preconditioner after several iterations.

The concept of the deblurring filter with a grid model is similar to migration deconvolution in Hu et al, (2001). Both methods deconvolve the estimated impulse response of the migration operator at each imaging point. The deblurred result is not good as that with a geological model but these errors can be corrected by a few iterations of LSM.

For a deblurring filter with a geological model, I employed a skeletonized reference model interpreted from the migration model, instead of the migration image used by Etgen (2002) and Guitton (2004). This approach provides acceptable results but at the expense of a human interpretation of the reflectivity model. It is expected that this human intervention can be replaced by a computer-aided interpretation. Seismic attributes such as amplitude and phase should help to extract lines along constant phases of major seismic events. Tests on a synthetic data set and a field data set show acceptable LSM images in a few iterations and suggest deblurred LSM as a practical method for mitigating errors due to the acquisition footprint and improving spatial resolution in the migration image.

Future research should explore the use of RDLSM and PDLSM on three-dimensional prestack data. I should also explore the possibility of using the deblurring method to accelerate convergence of an iterative migration deconvolution scheme that updates the reflectivity model by, e.g., a steepest descent method:
\[ m^{k+1} = m^k - \alpha (L^T L)(L^T Lm^k - m_{\text{mig}}), \]  

(3.1)

where \( \alpha \) is the deblurring filter.
APPENDIX A

HESSIAN INVERSE BY NONSTATIONARY MATCHING FILTER

A nonstationary matching filter can be used to approximate the inverse Hessian in the following way:

1. An input reflectivity model $m$ is defined, the synthetic data are generated by $d = Lm$, and are then migrated to get $m_{mig} = L^T d$. For computing the synthetic data and migration image, the velocity model and source wavelet should be estimated properly so that the synthetic migration image closely resembles the actual migration image obtained from field data. With a grid model, a constant $F$ is applied within a predefined section. It assumes the Hessian does not change within a section. On the other hand, a geological model provides a space-variant $F$ that changes rapidly across the section. It can be obtained by taking the original data, migrating it to get the migration image, and then skeletonizing it to a line model of reflector interfaces denoted by $m$.

2. We seek a non-stationary filter $F$ such that it approximates $(L^T L)^{-1}$, i.e., $F m_{mig} \approx m$. This filter can be found by defining the $p$th small window, e.g., the window has dimensions of the 2 x 3, i.e., dimensions of depth x offset, matrix $F_p = [a \, c \, e; b \, d \, f]$. The filter coefficients must satisfy

$$F_p \ast m_{mig}^p = m^p,$$  \quad (A.1)

where the $\ast$ denotes convolution only in depth (not in offset), $m^p$ is the vector that represents the discrete reflectivity trace centered in the $p$th window, and $m_{mig}^p$ represents the three migration traces in that window.
3. We know that a local convolution operation is commutative so that \( m \ast F = F \ast m \). This means that the matrix \( F \) in a window can be reformed into a vector and the vector \( m_{mig} \) can be formulated as a convolution matrix. Therefore, reformulating \( F \) as a 6 x 1 vector and \( m_{mig} \) as a convolution matrix yields the 6 x 1 vector \( f = (a \ b \ c \ d \ e \ f)^T \) and the normal equations given by

\[
[m^p_{mig}]^T [m^p_{mig}] F^p = [m^p_{mig}]^T m^p,
\]

where \( m^p_{mig} \) is the convolution matrix that contains the coefficients for three migration traces, with each trace being two samples long, and \( m^p \) is the vector that contains one reflectivity trace.

4. The filter \( F^p \) is found for all the predefined windows by solving equation A.2 for all \( p \) values.

5. The synthetic images were generated from synthetic data having the same source-receiver configuration as the original field experiment. Therefore the application of these non-stationary filters to the field migration image should be an approximation to the actual inverse Hessian to give a good approximation to the actual reflectivity distribution.
APPENDIX B

PRECONDITIONING BY
AN APPROXIMATE
HESSIAN INVERSE

The solution we seek is defined by:

\[ m_1 = m_0 + \Delta m, \quad (B.1) \]

where \( m_1 \) and \( m_0 \) are the 1st and initial model; and \( \Delta m \) is the optimal model update. To determine the specific direction of \( \Delta m \), we define the following misfit function:

\[
\phi(m_1) = \frac{1}{2} \| Lm_1 - d \|^2, \quad (B.2)
\]

\[
= \frac{1}{2} \| L(m_0 + \Delta m) - d \|^2, \quad (B.3)
\]

where \( L \) is the forward modeling operator, \( d \) represents the data, and \( \| \| \) denotes the \( l_2 \) norm.

The first variation of the misfit function with respect to \( \Delta m \) is:

\[
\delta \phi(m_1) = \delta \Delta m^T L^T [L(m_0 + \Delta m) - d], \quad (B.4)
\]

where \( L^T \) is the adjoint of the forward modeling. Since the first variation of the misfit function at the minimum must be equal to zero, we obtain the following condition:

\[
L^T L \Delta m = -L^T (Lm_0 - d). \quad (B.5)
\]

Here, we notice that the right hand side of the equation has the same form as the 1st step of the gradient vector of steepest descent method. This equation says that the gradient, which is the migrated data residual, is a \((L^T L)\) filtered version of \( \Delta m \).
From the above, a preconditioned steepest descent takes the following form:

\[ \mathbf{m}_{k+1} = \mathbf{m}_k - \alpha_k \mathbf{F} \mathbf{L}^T (\mathbf{L} \mathbf{m}_k - \mathbf{d}), \]  

(B.6)

where \( \mathbf{m}_{k+1} \) and \( \mathbf{m}_k \) are the \( k+1 \)-th and \( k \)-th models, respectively, \( \alpha_k \) is the \( k \)-th step length that can be found by using a line search, and \( \mathbf{F} \) is the approximated \( \mathbf{L}^T \mathbf{L}^{-1} \) that acts as a preconditioner. This equation represents an iterative Newton method except the inverse Hessian is locally approximated.
APPENDIX C

DLSM RESOLUTION

Deblurred LSM (or DLSM) provides a higher spatial resolution than the classical Rayleigh resolution limit. In this appendix, the above statement is validated on the 9-scatterer model in the following way: First, the test model, acquisition geometry, and test cases are explained. Second, the workflow of a deblurring filter is described and the results are presented. Third, the prestack DLSM results are compared with those from a prestack migration and a prestack LSM.

C.1 9-scatterer model test

The model (Figure C.1) has 9 scatterers located on a 3 x 3 grid in the center of the model, where the depth of scatterers is about 1250 m depth, each scatterer has a reflectivity of 0.1, and there is a 100 m horizontal and vertical spacing between scatterers. The model has a grid interval of 25 m, which is equal to each scatterer’s width and height. The details of the model are described in Table C.1.

The synthetic tests are conducted with Ricker source wavelets having dominant frequencies ($F_{dom}$) of 20, 10, and 5 Hz. The resolution of the migration images are compared to the lateral Rayleigh resolution limits (Table C.2).

The Rayleigh resolution limit is defined to be the smallest lateral separation $\Delta x$ of two point scatterers such that they can be distinguished from one another in the migration image (Schuster, 2009, chapter 11). The limit is described by the following equation:

$$\Delta x = \frac{z_0 \lambda}{2L}, \quad (C.1)$$

where $z_0$ is the distance between the center of the geophone array, $\lambda$ is a dominant wavelength, and $L$ is a half geophone array length.
Figure C.1. 9-scatterer model (a) in a two-dimensional plot, where the source (●) and receivers (○) are denoted at the surface, and (b) the same model as (a) except plotted in three-dimension.
Table C.1. Parameters for the actual model.

<table>
<thead>
<tr>
<th>(a) Actual Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Model name</td>
<td>9-scatterer model</td>
</tr>
<tr>
<td>Model grid size</td>
<td>$100 \times 100$ at 25 m interval</td>
</tr>
<tr>
<td>Velocity</td>
<td>1000 m/s</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>0.1 at gridpoint coordinates $(47, 47), (47, 51), (47, 55), (51, 47), (51, 51), (51, 55), (55, 47), (55, 51), (55, 55)$, 0 otherwise</td>
</tr>
</tbody>
</table>

Table C.2. Data acquisition parameters.

<table>
<thead>
<tr>
<th>Data acquisition parameters.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>6 locations at a 200 m interval (1000 m aperture length)</td>
</tr>
<tr>
<td>Receiver</td>
<td>14 locations at a 100 m interval (1300 m aperture length)</td>
</tr>
<tr>
<td>Source Wavelet</td>
<td>Ricker wavelet having a dominant frequency of 20, 10, and 5 Hz.</td>
</tr>
</tbody>
</table>
The data acquisition parameters are chosen so that $\Delta x$ can be about 50 m, 100 m, and 200 m, which are 0.5x, 1x, and 2x of the scatterer separation interval.

C.2 Deblurring Filter Results

The deblurring filter is validated in the following way:

1. Synthetic data $\mathbf{d}$ are used as the actual data for this test and are computed for the 9-scatterer model $\mathbf{m}$ with a coarse data acquisition geometry having 6 shots with a shot interval of 200 m and 14 receivers spaced at 100 m intervals. The aperture size of the source and receiver lines are about the same as the scatterers’ depth. The velocity is a constant value of 1000 m/s. These acquisition parameters are designed so that the wavelengths associated with the source wavelets are 50, 100 and 200 m at the scatterers’ depth, which are the same as the Rayleigh resolution limits of the migration operator. Examples of those synthetic common shot gathers (CSG) are shown in Figures C.2a- C.2c.

2. Each CSG is migrated to get $\mathbf{m}_{\text{mig}} = \mathbf{L}^T \mathbf{d}$ with the correct velocity of 1000 m/s. The migration images are shown in Figures C.3a- C.3c, where the amplitude is normalized so that the maximum amplitude is 0.1. The scatterers in these images seem to be connected horizontally except for the $F_{\text{dom}} = 20$ Hz ($\Delta x = 50$ m) case. The $l_2$ norm of the model residual are 0.14, 0.35, and 0.59.

3. A reference reflectivity model $\mathbf{m}'$ for the deblurring filter has a scatterer in the center (Figure C.4 and Table C.3). The aim in using the single scatterer model is to evaluate the effect of an erroneous reference model on deblurred image.

4. Synthetic data $\mathbf{d}' = \mathbf{Lm}'$ are generated from the reference reflectivity model $\mathbf{m}'$ and then migrated to give reference migration images $\mathbf{m}'_{\text{mig}} = \mathbf{L}^T \mathbf{d}' = \mathbf{L}^T \mathbf{Lm}'$. Figures C.5a-C.5c show the reference migration images of the single scatterer having $F_{\text{dom}} = 20$, 10, and 5 Hz (i.e., $\Delta x$ of 50 m, 100 m, and 200 m).

5. A deblurring filter $\mathbf{F}$ is found such that $\mathbf{F} * \mathbf{m}'_{\text{mig}} = \mathbf{m}'$, where $\mathbf{F}$ is a local nonstationary multichannel filter that minimizes the error between $\mathbf{F} * \mathbf{m}'_{\text{mig}}$ and
Figure C.2. Example of CSGs having a dominant frequency of (a) 20 Hz, (b) 10 Hz, and (c) 5 Hz, which provide the Rayleigh resolution limits ($\Delta x$) of 50 m, 100 m, and 200 m, respectively.
Figure C.3. Migration images having the Rayleigh resolution limits ($\Delta x$) of (a) 50 m, (b) 100 m, and (c) 200 m.
Figure C.4. The single scatterer model as the reference model for a deblurring filter.

Table C.3. Parameters for the reference model.

<table>
<thead>
<tr>
<th></th>
<th>Reference Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model name</td>
<td>Single scatterer model</td>
</tr>
<tr>
<td>Model grid size</td>
<td>The same as that of actual model</td>
</tr>
<tr>
<td>Velocity</td>
<td>The same as that of actual model</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>0.1 at a gridpoint coordinate (51,51), 0 otherwise</td>
</tr>
</tbody>
</table>
Figure C.5. Reference migration images of the single scatterer model for $\Delta x$ equal to (a) 50 m, (b) 100 m, and (c) 200 m.
\( \mathbf{m}' \). The filter \( \mathbf{F} \) is then applied to \( \mathbf{m}_{\text{mig}} \) to give the deblurred migration image. Figures C.6a- C.6c show the filtering results. Their \( l_2 \) norms are 0.11, 0.23, and 0.47, respectively. Migration noise is reduced in all cases and no additional noise is observed on the images. The parameters used for the deblurring filter are shown in Table C.4.

### C.3 Deblurred LSM Results

The deblurred image \( \mathbf{F} * \mathbf{m}'_{\text{mig}} \) is used as the a priori model for a DLSM. A regularized conjugate gradient method (Zhdanov, 2002, pp.149) is used for the optimization algorithm. In this test, \( \gamma = \frac{||\text{data}||^2}{||\text{model}||^2} \), where \( || \) represents the \( l_2 \) norm, is used for a regularization parameter. The results after three iterations and 10 iterations are shown in Figures C.7a- C.7c and Figures C.8a- C.8c, respectively. The DLSM images with \( \Delta x = 50 \) m and 100 m show fine scatterers’ images even after three iterations. The DLSM images with \( \Delta x = 200 \) m after three iteration is as the same as the migration image, and the one after 10 iterations shows dominant sidelobes. Their \( l_2 \) norms are 0.06, 0.17, and 0.52 after three iterations, and 0.04, 0.13, and 0.55 after 10 iterations.

Figures C.9a and C.9b show the model and data residuals for up to 10 iterations compared with that of the standard LSM residuals. The model residual curve decreases with increasing number of iterations in all cases and the smaller \( \Delta x \) provides the smaller model residual. However, we observed that the data residual does not always decrease in these tests (e.g., DLSM with \( \Delta x = 200 \) m, DLSM and SLSM with \( \Delta x = 50 \) m).

Figures C.10a and C.10b compare the reflectivity values estimated by 10 iterations of DLSM and standard migration. Both the DLSM and migration images reveal an accurate estimate of the scatterer’s locations along the horizontal line at \( Z = 47 \) and the vertical line at \( X = 47 \). As for Figure C.10b, no noise can be seen in both the migration and DLSM images.

In the \( \Delta x = 100 \) m case, the lateral resolution of the migration image shown in Figure C.11a decreases to about half of that in the \( \Delta x = 50 \) m case but that of the DLSM image is still the same as that in Figure C.10a. The migration and DLSM images shown in Figure C.11b show an accurate estimate of the scatterer’s locations even though the noise slightly increases in both images compared to the noise in Figure C.10b.
Figure C.6. Deblurred migration images for $\Delta x$ equal to (a) 50 m, (b) 100 m, and (c) 200 m.
Table C.4. Deblurring parameters.

<table>
<thead>
<tr>
<th>Deblurring parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Window size</td>
<td>50 points</td>
</tr>
<tr>
<td>Sliding step</td>
<td>25 points</td>
</tr>
<tr>
<td>Filter length</td>
<td>49 points</td>
</tr>
<tr>
<td>Number of channel</td>
<td>20 traces for one side</td>
</tr>
<tr>
<td>Damping factor</td>
<td>1 %</td>
</tr>
</tbody>
</table>
Figure C.7. DLSM images after 3 iterations for $\Delta x$ equal to (a) 50 m, (b) 100 m, and (c) 200 m.
Figure C.8. DLSM images after 10 iterations for $\Delta x$ equal to (a) 50 m, (b) 100 m, and (c) 200 m.
Figure C.9. Comparison of (a) model and (b) data residual between DLSM and LSM.
Figure C.10. The migration and DLSM images after 10 iterations with $\Delta x = 50$ m along the lines at (a) $Z = 47$ and (b) $X = 47$. 
Figure C.11. The migration and DLSM images after 10 iterations with $\Delta x = 100$ m along the lines at (a) $Z = 47$ and (b) $X = 47$. 
In the \( \Delta x = 200 \) m case, Figure C.12a shows the scatterers are not recognisable in the migration image. However, the DLSM image still estimates the three distinct scatterers, which means that the DLSM spatial resolution can exceed the Rayleigh limit. The vertical images shown in Figure C.12b show that sidelobes increase in both images compared to the former cases.

Next I compare the DLSM images with the LSM images. Figures C.13a and C.13b show the images after 10 iterations, where the DLSM images in these figures are the same as those in Figures C.12a and C.12b. Figure C.13a shows that the LSM image from \( Z = 47 \) after 10 iterations does not clearly identify three distinct scatterers. Figure C.13b shows that noise levels in the DLSM and the LSM images are about the same. These results show that DLSM gives higher resolution than that from LSM if the number of iterations is the same.

\section*{C.4 Discussion}

Comparison of the DLSM images with those from standard migration for the 9-scatterer model shows that DLSM resolution limit can be better than the classical resolution limit. Comparison of DLSM and LSM shows that DLSM provides a more reliable image if the number of iterations is the same. A trade-off of DLSM is larger side lobes that increase when the scatterers are closer than \( \Delta x \).

Comparison of the residual curves between DLSM and LSM shows a non-linear relationship between the model and data residual curves. The model residual decreases over the number of iterations in all cases and the smaller \( \Delta x \) provides a smaller model residual. DLSM images have a lower model residual than those from LSM images except for the \( \Delta x = 200 \) m case.

The data residual curves show the following.

1. In the case of \( \Delta x = 100 \) m, the data residual in the DLSM image decreases rapidly in the first three iterations. This is the case for which DLSM reduces migration noise most effectively.

2. In the case of \( \Delta x = 200 \) m, the data residual with DLSM image increases gradually as the image of the scatterers distinguishes one scatterer from the other.
Figure C.12. The migration and DLSM images after 10 iterations with $\Delta x = 200$ m along the lines at (a) $Z = 47$ and (b) $X = 47$. 
Figure C.13. Comparison between the DLSM and LSM images after 10 iterations with $\Delta x = 200$ m along the lines at (a) $Z = 47$, and (b) $X = 47$. 
The latter result suggests that too fine of a model grid spacing compared to $\Delta x$ potentially introduces additional noise. From the point of view of quality and efficiency, it is recommended that the spacing of the model gridpoints should be defined based on the Rayleigh formula.
REFERENCES


