ACOUSTIC WAVEFORM INVERSION OF
TWO-DIMENSIONAL GULF
OF MEXICO DATA

by

Chaiwoot Boonyasiriwat

A thesis submitted to the faculty of
The University of Utah
in partial fulfillment of the requirements for the degree of

Master of Science

in

Geophysics

Department of Geology and Geophysics
The University of Utah
May 2009
This thesis has been read by each member of the following supervisory committee and by majority vote has been found to be satisfactory.

Chair: Gerard T. Schuster

Richard D. Jarrard

Ronald L. Bruhn
To the Graduate Council of the University of Utah:

I have read the thesis of Chaiwoot Boonyasiriwat in its final form and have found that (1) its format, citations, and bibliographic style are consistent and acceptable; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the Supervisory Committee and is ready for submission to The Graduate School.

Date

Gerard T. Schuster
Chair, Supervisory Committee

Approved for the Major Department

Marjorie A. Chan
Chair/Dean

Approved for the Graduate Council

David S. Chapman
Dean of The Graduate School
Acoustic waveform tomography can provide a velocity model with higher accuracy than traveltime tomography because the forward modeling is based on the complete wave equation rather than the high-frequency approximation of ray tracing. Moreover, each trace contains both shallow and deep reflection arrivals and so is richer in information content than the first arrival traveltime. As a result, waveform tomography provides a more detailed estimate of the velocity medium than does traveltime tomography. However, acoustic waveform tomography is a highly nonlinear inverse solution that requires a good starting velocity model compared to the smooth starting model that is sufficient for traveltime tomography.

To overcome the nonlinear problems in waveform tomography, I develop a novel multiscale waveform tomography method and apply it to both synthetic and marine seismic data from the Gulf of Mexico. The inversion process is carried out using a multiscale acoustic method with a dynamic early-arrival muting window to mitigate the local minima problem of waveform tomography and elastic effects in the data. This multiscale approach is denoted as multiscale waveform tomography (MWT), and is first validated with synthetic acoustic data from the 2D SEG/EAGE salt model. Using the traveltime velocity tomogram as an initial model, MWT fails to converge to the global minimum. I found that the flooding technique normally used in subsalt migration can be used to significantly improve the convergence of MWT. The velocity model recovered by MWT using the flooding technique is more accurate and highly resolved than that obtained using conventional MWT. For the marine data, MWT can provide a more accurate velocity model than the initial model from traveltime tomography. The accuracy of the waveform velocity model is verified by comparing migration images and common image gathers.

I also present ray-based spatial resolution formulas for migration and inversion
with numerical tests on both homogeneous and heterogeneous media. Spatial resolution formulas are validated for both homogeneous and heterogeneous velocity models. For both models, the resolution of reverse-time migration images are consistent with the estimated resolution limits with respect to the Rayleigh’s resolution criterion. A long wavelength corresponds to a high velocity value and therefore both vertical and horizontal resolution limits tend to degrade with depth in the heterogeneous model — deeper regions have higher velocity values.
To my parents and my wife
CONTENTS

ABSTRACT ................................................................. iv
LIST OF FIGURES ......................................................... viii
ACKNOWLEDGMENTS ...................................................... x

CHAPTERS

1. ACOUSTIC WAVEFORM TOMOGRAPHY ......................... 1
   1.1 Introduction ................................................... 1
   1.2 Theory .......................................................... 3
      1.2.1 Acoustic Waveform Tomography ....................... 3
      1.2.2 Multiscale Early-Arrival Waveform Tomography ...... 5
   1.3 Numerical Results .............................................. 6
      1.3.1 2D SEG/EAGE Salt Model ................................. 6
      1.3.2 Gulf of Mexico ............................................ 10

2. SPATIAL RESOLUTION ANALYSIS .............................. 20
   2.1 Introduction ................................................... 20
   2.2 Spatial Resolution Formulas ................................. 20
   2.3 Numerical Results .............................................. 22
      2.3.1 Homogeneous Model ....................................... 22
      2.3.2 Smoothed SEG/EAGE Salt Model ......................... 25

3. CONCLUSIONS ....................................................... 30

REFERENCES ........................................................... 32
LIST OF FIGURES

1.1 The 2D SEG/EAGE salt model and initial models. a) The salt model. b) Traveltime velocity model. c) $v(z)$ velocity model. ................. 7

1.2 Synthetic data from the 2D SEG/EAGE salt model. a) Original 20-Hz data. b) Filtered 2.5-Hz data. c) Filtered 5-Hz data. ....................... 8

1.3 Waveform inversion results using 2.5-Hz data. a) True model. b) Waveform tomogram using the traveltime tomogram. c) Waveform tomogram using the $v(z)$ model and the flooding technique. .......... 9

1.4 Flooding process. a) Waveform tomogram using only the $v(z)$ model. b) Waveform tomogram after a salt flood. c) Waveform tomogram after salt and sediment floods. ........................................... 11

1.5 Waveform residual plot using 2.5-Hz data. .................................. 12

1.6 Waveform residual plot using 5-Hz data. .................................... 13

1.7 Marine data. (a) An original CSG from a source at $x = 0$ km. The white line is the picked first-arrival traveltimes. (b) A filtered shot gather with a passband of 0-25 Hz. (c) A predicted shot gather obtained by using the traveltime tomogram. (d) A predicted shot gather obtained by using the waveform tomogram. ....................... 14

1.8 Inversion results from the marine data. (a) The initial velocity model obtained from traveltime tomography. (b) The velocity tomogram obtained from waveform tomography. (c) The vertical derivative of the waveform tomogram. ................................................. 16

1.9 Migration images from the marine data. (a) The Kirchhoff migration image obtained using the original data and the traveltime tomogram. (b) The Kirchhoff migration image obtained using the waveform tomogram. ................................................... 17

1.10 Zoomed views of migration images from the marine data. Using the traveltime tomogram, the Kirchhoff migration images in a) the solid box and b) the dashed box are obtained. Using the waveform tomogram, the Kirchhoff migration image in c) the solid box and d) the dashed box are obtained. ........................................ 18

1.11 Common image gathers (CIGs) obtained from the marine data migrated with the (a) traveltime tomogram and (b) waveform tomogram as the velocity model. ................................................. 19
2.1 The illumination of a diffractor at \( x \) by source/receiver pair \( r_s/r_g \) in a heterogeneous medium. The wavenumber vector \( k \) is composed of the source-side wavenumber vector \( k_s \) and the receiver-side wavenumber vector \( k_g \). ................................................................. 22

2.2 Wavenumber illumination patterns at the nine scatterer locations in the homogeneous model. ................................................................. 23

2.3 The homogeneous velocity model with resolution bars that estimate the limits of horizontal and vertical resolution at nine different locations in the model. ................................................................. 24

2.4 The reverse-time migration image for the homogeneous velocity model. 25

2.5 Smoothed SEG/EAGE salt model with scatterers. ...................... 26

2.6 Wavenumber illumination patterns at the nine scatterer locations in the smoothed SEG/EAGE salt model. The traveltimes associated with the computed wavenumbers were computed by tracing rays through the salt model. ................................................................. 27

2.7 Reverse-time migration image for the smoothed salt model with scatterers. ................................................................. 28

2.8 The reverse-time migration image line at \( x = 8 \) km in the 2D SEG/EAGE salt model. The error bars indicate the estimated vertical resolution limits at the scatterers. ................................................................. 29

2.9 The reverse-time migration image line at \( z = 2 \) km in the 2D SEG/EAGE salt model. The error bars indicate the estimated horizontal resolution limits at the scatterers. ................................................................. 29
ACKNOWLEDGMENTS

I would like to thank Dr. Gerard T. Schuster and my committee members Dr. Ronald Bruhn and Dr. Richard Jarrard for their advice and constructive criticism. I also thank Paul Valasek and Partha Routh for their advice, and the Seismic Technology Group for their help during my internship at ConocoPhillips. I am grateful for the support from the members of the University of Utah Tomography and Modeling/Migration Consortium and thank Amerada Hess for providing me with the Gulf of Mexico data set. I would like to thank the Development and Promotion of Sciences and Technology (DPST) of Thailand for financial support. Finally, I would like to thank UTAM colleagues especially Weiping Cao for useful discussion and help.
CHAPTER 1

ACOUSTIC WAVEFORM TOMOGRAPHY

1.1 Introduction

Accurate velocity models of subsurface structures are required to obtain reliable migration images of the subsurface. In areas with rugged surface topography and complex near-surface structures, conventional time-domain seismic imaging methods are likely to fail due to the severe statics problem. Using an accurate velocity model, a tomostatics method can improve the quality of the stacked section. The role of velocity models is more crucial in depth-domain seismic imaging where conventional prestack depth migration methods yield unreliable migration images unless the velocity model is accurate.

The standard tools for more accurate velocity estimation include traveltime tomography (Zhu and McMechan, 1989; Luo and Schuster, 1991; Pratt and Goulty, 1991; Schuster and Quintus-Bosz, 1993; Nemeth et al., 1997; Min and Shin, 2006) and migration velocity analysis (MVA) (Stork, 1992; Tieman, 1995; Jiao et al., 2002; Sava et al., 2005; Sava and Vlad, 2008). Inverting only first-arrival traveltimes, traveltime tomography is computationally efficient but forward models the data by ray tracing, which is a high-frequency approximation that conflicts with the band-limited nature of seismic sources. The widely used MVA methods usually require intensive quality control and typically provide just a smooth velocity model. In contrast, waveform tomography directly inverts the seismic waveform data and can provide accurate and highly resolved velocity models (Bunks et al., 1995; Zhou et al., 1995, 1997; Sheng et al., 2006).

Waveform tomography is a highly nonlinear inverse problem, and tends to converge to a local minimum if the starting model is not in the vicinity of the global minimum (Gauthier et al., 1986). Therefore, a good initial velocity model is
required by waveform tomography to partially overcome the local minima problem. To further mitigate the nonlinearity of waveform tomography, Bunks et al. (1995) introduced a multiscale method that sequentially inverts data band-passed from lower to higher frequencies. Once the smooth or low-wavenumber velocity structures are reconstructed, the higher-wavenumber structures are reconstructed using higher-frequency data.

The multiscale approach mitigates the local minima problem and is computationally efficient. The nonlinearity of waveform tomography depends on the frequency content of seismic data so that the misfit function at low frequencies is more linear than at high frequencies. This means that the multiscale inversion tends to converge to the global minimum (Sirgue and Pratt, 2004). At low frequencies, coarser grids can be used for computing numerical solutions of the wave equation than at high frequencies resulting in a greater computational efficiency.

In this thesis, I apply multiscale waveform tomography (MWT) to both synthetic data from the 2D SEG/EAGE salt model and marine data from the Gulf of Mexico. There are two novel features in my procedure: first, I employ a multiscale approach in the space-time domain which departs from the conventional approach in the space-frequency domain. The possible advantage is that the 3D space-time approach can be more efficient than the 3D space-frequency method. Second, I use a flooding method to overcome the convergence problem when there is a large velocity contrast in the medium. The 2D synthetic data were generated using an acoustic finite-difference code, and results show that MWT with a flooding technique can successfully invert the salt-body data for an accurate and highly resolved velocity tomogram. In the case of marine data from the Gulf of Mexico, I reduce the elastic influences in the data by using MWT with a dynamic early-arrival muting window. Most seismic events in the data are muted except the early arrivals. My results suggest that the multiscale method can provide a velocity tomogram that is significantly more accurate than the initial model.
1.2 Theory

1.2.1 Acoustic Waveform Tomography

In this section, I present the theory of time-domain acoustic waveform tomography. The acoustic wave equation is used to describe wave propagation, given by

\[ \frac{1}{\kappa(r)} \frac{\partial^2 p(r, t|\mathbf{r}_s)}{\partial t^2} - \nabla \cdot \left[ \frac{1}{\rho(r)} \nabla p(r, t|\mathbf{r}_s) \right] = s(r, t|\mathbf{r}_s), \]  

(1.1)

where \( p(r, t|\mathbf{r}_s) \) is a pressure field at position \( r \) at time \( t \) excited by a source at \( \mathbf{r}_s \); \( \kappa(r) \) and \( \rho(r) \) are the bulk modulus and density distributions, respectively; and \( s(r, t|\mathbf{r}_s) \) is the source function. The solution to equation 1.1 can be written as

\[ p(r, t|\mathbf{r}_s) = \int_{V_0} G(r, t|\mathbf{r}', 0) \ast s(\mathbf{r}', t|\mathbf{r}_s) \, d\mathbf{r}', \]  

(1.2)

where \( G(r, t|\mathbf{r}', 0) \) is the Green’s function, \( V_0 \) denotes the model volume, and the symbol \( \ast \) represents temporal convolution. In practice, the solution \( p(r, t|\mathbf{r}_s) \) is computed using a staggered-grid, explicit finite-difference method with 4th-order accuracy in space and 2nd-order accuracy in time (Levander, 1988).

The inversion scheme used in this work is based on the adjoint method proposed by Tarantola (1984). The data residual is defined as

\[ \delta p(r_g, t|\mathbf{r}_s) = [p_{obs}(r_g, t|\mathbf{r}_s) - p_{calc}(r_g, t|\mathbf{r}_s)] m(r_g, t|\mathbf{r}_s), \]  

(1.3)

where \( r_g \) is a receiver position vector, \( p_{obs}(r_g, t|\mathbf{r}_s) \) and \( p_{calc}(r_g, t|\mathbf{r}_s) \) are, respectively, the observed and calculated data, and \( m(r_g, t|\mathbf{r}_s) \) is an early-arrival window function. The velocity model is updated by minimizing the misfit function which is the \( L_2 \) norm of the data residuals and is given by

\[ E = \frac{1}{2} \sum_s \sum_g \int (\delta p(r_g, t|\mathbf{r}_s))^2 dt. \]  

(1.4)

A nonlinear preconditioned conjugate-gradient method (Sheng et al., 2006) is used to minimize the misfit function and obtain a velocity model \( v(r) = \sqrt{\kappa(r)/\rho(r)} \). The gradient of the misfit function in equation 1.4 with respect to slowness perturbation is computed by the zero-lag correlation between the forward-propagated wavefields
and the back-projected wavefield residuals (Tarantola, 1984; Luo and Schuster, 1991; Zhou et al., 1995, 1997; Sheng et al., 2006)

\[ g(r) = \frac{2}{v(r)} \sum_s \int \dot{p}(r, t|r_s) \dot{p}'(r, t|r_s) dt, \]  

where \( \dot{p} \) denotes the time derivative of \( p \), \( p(r, t|r_s) \) represents the forward-propagated wavefields, and \( p'(r, t|r_s) \) represents the back-projected wavefield residuals given by

\[ p'(r, t|r_s) = \int G(r, -t|r', 0) * \delta s(r', t|r_s) dr', \]  

and the perturbed slowness field is

\[ \delta s(r', t|r_s) = \sum_g \delta (r' - r_g) \delta p(r_g, t|r_s). \]  

The velocity model is iteratively updated along the conjugate directions defined by

\[ d_k = -P_k g_k + \beta_k d_{k-1}, \]  

where iterations \( k = 1, 2, ..., k_{max} \), \( g_k \) is the gradient direction, and \( P \) is the conventional geometrical-spreading preconditioner (Causse et al., 1999). At the first iteration \( d_0 = -g_0 \). The parameter \( \beta_k \) is obtained using the Polak-Ribiére formula (Nocedal and Wright, 1999)

\[ \beta_k = \frac{g_k^T (P_k g_k - P_{k-1} g_{k-1})}{g_{k-1}^T P_{k-1} g_{k-1}}. \]  

The velocity model is updated by

\[ v_{k+1}(r) = v_k(r) + \lambda_k d_k(r), \]  

where \( \lambda_k \) is the step length, which is determined by a quadratic line-search method (Nocedal and Wright, 1999), and \( d_k(r) \) is the component of the conjugate-direction vector \( d_k \) at position \( r \). At each iteration, one forward-propagation and one back-projection are needed for computing the gradient direction. Additional forward propagations are required for the line search. In this work, the starting model \( v_0(r) \) is obtained from traveltime tomography with dynamic smoothing filters (Nemeth et al., 1997).
1.2.2 Multiscale Early-Arrival Waveform Tomography

In this section, I describe a time-domain implementation of multiscale waveform tomography (MWT). In the frequency domain, the data are decomposed into separate frequency components and it is straightforward to apply the multiscale method. In contrast, time-domain inversion simultaneously uses multiple frequency components of the data in a frequency band. Thus, the data must be band-pass filtered into multiple frequency bands with various peak frequencies, and the inversion process can proceed using low-frequency data and then high-frequency data.

A Wiener filter is used for low-pass filtering the data, where Boonyasiriwat et al. (2009) show that Wiener filtering is more accurate than the filtering method proposed by Bunks et al. (1995). A low-pass Wiener filter (Boonyasiriwat et al., 2009) can be computed by

\[
f_{\text{Wiener}}(\omega) = \frac{W_{\text{target}}(\omega)W_{\text{original}}^\dagger(\omega)}{|W_{\text{original}}(\omega)|^2 + \epsilon^2},
\]

where \(f_{\text{Wiener}}\) is the Wiener filter, \(W_{\text{original}}\) is the original wavelet, \(W_{\text{target}}\) is the low-frequency target wavelet, \(\omega\) is an angular frequency, \(\epsilon\) is a damping factor to prevent numerical instability, and \(^\dagger\) denotes the complex conjugate. The Wiener filter is applied to the source wavelet and data in the frequency domain.

Once the source and data are filtered to a low frequency band, the spacing of the finite-difference grid points can be determined by the maximum frequency of the band. The numerical dispersion condition for the finite-difference scheme used in this work requires at least 5 grid points per minimum wavelength (Levander, 1988). A square grid \((dx = dz)\) is utilized in the finite-difference scheme so that the grid spacing \(dx\) is determined by

\[
dx \leq \frac{\lambda_{\text{min}}}{5} \leq \frac{c_{\text{min}}}{5f_{\text{max}}},
\]

where \(\lambda_{\text{min}}\) is the minimum wavelength, \(c_{\text{min}}\) is the minimum velocity, and \(f_{\text{max}}\) is the maximum frequency of the band. At low frequencies, coarser grids can be used than at high frequencies. Therefore, low-frequency inversions will be fast and efficient compared to high-frequency inversions, and can afford to take a large
number of iterations in order to obtain an accurate estimate of low-wavenumber components in the velocity model.

The multiscale approach has the ability to mitigate the local minima problem commonly encountered in waveform tomography (Bunks et al., 1995; Boonyasiriwat et al., 2009). The velocity model with accurate low-wavenumber components is a good initial model for higher-frequency inversions while higher-frequency data progressively recover the higher-wavenumber parts of the model.

In practice, my acoustic modeling does not account for elastic effects in the data, attenuation, unknown density, unknown source wavelet, and source radiation patterns, which can lead to a poor convergence. I try to partly overcome these problems by using a multiscale method with a dynamic early-arrival muting window, which partly corrects for attenuation and wavelet distortion effects. The inversion initially inverts data low-pass filtered and windowed about a short time-window. After some iterations, higher-frequency data are used in the inversion with the same muting window. Then, the inversion proceeds with longer time-windows.

1.3 Numerical Results

To demonstrate its effectiveness, I apply MWT to synthetic data from the 2D SEG/EAGE salt model and to marine data from the Gulf of Mexico.

1.3.1 2D SEG/EAGE Salt Model

The 2D SEG/EAGE salt model (Figure 1.1a) has dimensions of 16 km × 3.7 km with a source spacing of 40 m and a receiver spacing of 20 m. Sources and receivers are located along the free surface and I employ two initial models. The first one is from traveltime tomography (Figure 1.1b) and the second one is the $v(z)$ model (Figure 1.1c) obtained from a smooth 1D sediment velocity profile of the true model. The original data are generated using a 20-Hz Ricker wavelet. A common shot gather from a source at the center of the free surface is shown in Figure 1.2a. To use the multiscale method, the original data are low-pass filtered to 2 frequency bands with peak frequencies of 2.5 Hz and 5 Hz; the filtered data are shown in Figures 1.2b and 1.2c, and the MWT results are shown in Figure 1.3.
Figure 1.1. The 2D SEG/EAGE salt model and initial models. a) The salt model. b) Traveltime velocity model. c) $v(z)$ velocity model.
Figure 1.2. Synthetic data from the 2D SEG/EAGE salt model. a) Original 20-Hz data. b) Filtered 2.5-Hz data. c) Filtered 5-Hz data.
Figure 1.3. Waveform inversion results using 2.5-Hz data. a) True model. b) Waveform tomogram using the traveltime tomogram. c) Waveform tomogram using the $v(z)$ model and the flooding technique.
Using the travelt ime tomogram (Figure 1.1b), MWT converges to a local minima and provides an inaccurate velocity tomogram (Figure 1.3b). Using the $v(z)$ model and the flooding techniques, MWT can provide an accurate and highly resolved velocity tomogram (Figure 1.3c).

Now I describe how the flooding technique can yield an accurate tomogram. Using the $v(z)$ model, MWT yields the tomogram in Figure 1.4a. After the top of the salt boundary is picked, the salt velocity of 4500 m/s is used to flood the region below the salt top and the resulting velocity (Figure 1.4b) is used to migrate the data. The bottom of the salt body is picked from the migration image (not shown here) and the velocity model in Figure 1.4b is flooded with a sediment velocity of 3000 m/s below the salt bottom to obtain the velocity model shown in Figure 1.4c. Then the velocity model obtained after the flooding process is used in the inversion process to give the residual plot using 2.5-Hz data shown in Figure 1.5. The $v(z)$ model is used in the first 100 iterations and the flooding model (Figure 1.4c) is used in the last 30 iterations. The waveform tomogram is then used to invert 5-Hz data and the final velocity model is obtained as shown in Figure 1.3c. The residual plot using 5-Hz data is shown in Figure 1.6.

1.3.2 Gulf of Mexico

A streamer data set from the Gulf of Mexico was acquired using 515 shots with a shot interval of 37.5 m, a time-sampling interval of 2 ms, a trace length of 10 s, and 480 active hydrophones per shot. The hydrophone interval is 12.5 m with a near offset of 198 m and a far offset of about 6 km.

The data are transformed from 3D to 2D format by applying the filter $\sqrt{i/\omega}$ in the frequency domain to correct for 3D geometrical spreading (Zhou et al., 1995). The attenuation factor $Q$ is estimated by the spectral ratio method (Maresh et al., 2006), and the attenuation effect is compensated by applying an inverse-$Q$ filter (Wang, 2006) to the data. The source wavelet is estimated by stacking along the water-bottom reflection. Then, the data are low-pass filtered to 2 frequency bands with passbands of 0-15 Hz and 0-25 Hz. Figure 1.7a and 1.7b show the original and filtered shot gathers, respectively.
Figure 1.4. Flooding process. a) Waveform tomogram using only the $v(z)$ model. b) Waveform tomogram after a salt flood. c) Waveform tomogram after salt and sediment floods.
Figure 1.5. Waveform residual plot using 2.5-Hz data.
Figure 1.6. Waveform residual plot using 5-Hz data.
Figure 1.7. Marine data. (a) An original CSG from a source at $x = 0$ km. The white line is the picked first-arrival traveltimes. (b) A filtered shot gather with a passband of 0-25 Hz. (c) A predicted shot gather obtained by using the traveltime tomogram. (d) A predicted shot gather obtained by using the waveform tomogram.
Traveltime tomography is utilized to provide an initial velocity model for waveform tomography (Figure 1.8a). The inversion process is composed of 3 parts and in each part a muting window with a different length is used for inversion of both the low-pass and high-pass data. In the first part, a muting window of length 1 s is applied to the filtered data, and the inversion sequentially proceeds using data with passbands of 0-15 Hz and 0-25 Hz. The reconstructed velocity from the first part is used as an initial model in the second part where a 2-second window is used, and, in the last part, a 3-second window is applied. Figure 1.8b shows the reconstructed velocity tomogram from waveform tomography which has a higher resolution than the initial model (Figure 1.8a). To reveal the high-wavenumber details of the waveform tomogram, the derivative with respect to depth is applied to the waveform tomogram (Figure 1.8c). The predicted shot gathers obtained by using traveltime tomogram and waveform tomogram are shown in Figures 1.7c and 1.7d, respectively.

To verify that the reconstructed velocity tomogram is more accurate than the initial model, I compare the migration images and common image gathers (CIG) obtained by using the initial model and the final model. The original data were migrated using Kirchhoff migration, and the migration images using the traveltime and waveform tomograms are shown in Figures 1.9a and 1.9b, respectively. The zoomed views of the migration images are shown in Figure 1.10 for more detailed comparisons. Using the waveform tomogram as the migration velocity, the resulting migration image appears to be better focused than that obtained by using the traveltime tomogram as the migration velocity. Comparing the CIGs in Figure 1.11, the waveform tomogram is more accurate than the traveltime tomogram since the corresponding CIGs are flatter. Horizontal reflectors in a common image gather are an indication that the migration velocity model is accurate (Yilmaz, 2001).
Figure 1.8. Inversion results from the marine data. (a) The initial velocity model obtained from traveltime tomography. (b) The velocity tomogram obtained from waveform tomography. (c) The vertical derivative of the waveform tomogram.
Figure 1.9. Migration images from the marine data. (a) The Kirchhoff migration image obtained using the original data and the traveltime tomogram. (b) The Kirchhoff migration image obtained using the waveform tomogram.
Figure 1.10. Zoomed views of migration images from the marine data. Using the traveltime tomogram, the Kirchhoff migration images in a) the solid box and b) the dashed box are obtained. Using the waveform tomogram, the Kirchhoff migration image in c) the solid box and d) the dashed box are obtained.
Figure 1.11. Common image gathers (CIGs) obtained from the marine data migrated with the (a) traveltime tomogram and (b) waveform tomogram as the velocity model.
CHAPTER 2

SPATIAL RESOLUTION ANALYSIS

2.1 Introduction

The theory of spatial resolution is well-established in seismic migration and inversion (Beylkin et al., 1985; Cohen et al., 1986; Bleistein, 1987; Vermeer, 1999; Chen and Schuster, 1999). Using the Born approximation, Beylkin et al. (1985) presented a formula that connects the source frequency and acquisition geometry to the spatial resolution of the migration image. This spatial resolution formula is quite simple and, in theory, applicable to heterogeneous media. In contrast, Chen and Schuster (1999) used the far-field approximation to derive spatial resolution limits for homogeneous media so that the resolution formula of Beylkin et al. (1985) is more general. Consequently, I choose to apply the formula of Beylkin et al. (1985) for resolution analysis tests in this work.

2.2 Spatial Resolution Formulas

In this section I present some formulas that map acquisition geometry configuration to spatial resolution in both the wavenumber and the space domains. Since 2D surface-seismic-profile (SSP) data are only dealt with in this work, I will limit my derivation for 2D SSP acquisition geometry. The source/receiver configuration can be described by coordinates $\xi$ so that for fixed $X$, $r_s = (X, 0)$ and $r_g = (\xi, 0)$ describe a 2D common-shot gather.

Beylkin et al. (1985) derived a formula to compute the acoustic impedance contrast as a function of position $r$ from seismic measurements with limited aperture. In 2D the limited aperture is defined by the range of $\xi$. Their formula is a mapping from $(\xi, \omega)$ (the coordinates of the observed data; $\omega$ is the angular frequency of
the source signature) to \((k_x, k_z)\) (the coordinates of the migration image). This mapping is given by

\[
k(r) = \omega \nabla \phi(r, \xi),
\]

where \(k = (k_x, k_z)\) is the wavenumber vector in the reconstructed (migration) domain, and \(\phi(r, \xi)\) is the traveltime surface of a diffractor in \(r\) for shot/receiver pairs described by \(\xi\). The traveltime surface \(\phi(r, \xi)\) is computed from the background velocity model and \(\nabla \phi(r, \xi)\) represents the derivative of \(\phi(r, \xi)\) with respect to the point of reconstruction \(r\). Equation 2.1 determines the region of coverage \(D\) in the spatial wavenumber domain which is the estimate of spatial resolution (Beylkin et al., 1985; Vermeer, 1999). Vermeer (1999) states, “the larger the region of coverage in \(k\), the better the spatial resolution.”

Equation 2.1 makes resolution analysis quite simple since resolution can be obtained by analyzing the spatial gradients of the diffraction traveltime surface \(\phi(r, \xi)\) in the given experiment configuration (Vermeer, 1999). The maximum wavenumber that corresponds to the maximum gradient of \(\phi(r, \xi)\) gives a fair indication of resolution.

The diffraction traveltime \(\phi(r, \xi)\) can be described as

\[
\phi(r, \xi) = \tau(r, r_s) + \tau(r, r_g) = \tau_s + \tau_g,
\]

where \(\tau(x, y)\) is the traveltime from surface position \(y\) to subsurface position \(x\). Similarly, \(k\) can be written as the vectorial sum

\[
k = k_s + k_g,
\]

where \(k_s\) and \(k_g\) are the contributions of shot and receiver, respectively, to the wavenumber vector \(k\) as illustrated in Figure 2.1. Each shot/receiver pair in the acquisition geometry corresponds to a point \(k\) in the wavenumber space. Taking all shot/receiver pairs of a configuration leads to a collection of points in wavenumber space.

Using the Beylkin’s formula, horizontal resolution \(\Delta x\) and vertical resolution \(\Delta z\) can be written as
Figure 2.1. The illumination of a diffractor at \( \mathbf{x} \) by source/receiver pair \( \mathbf{r}_s/\mathbf{r}_g \) in a heterogeneous medium. The wavenumber vector \( \mathbf{k} \) is composed of the source-side wavenumber vector \( \mathbf{k}_s \) and the receiver-side wavenumber vector \( \mathbf{k}_g \).

\[
\Delta x = \frac{\pi}{\max |k_x(r, \xi, \omega)|}, \quad (2.4)
\]

and

\[
\Delta z = \frac{\pi}{\max |k_z(r, \xi, \omega)|}, \quad (2.5)
\]

where \( k_x \) and \( k_z \) are the horizontal and vertical components of the wavenumber vector \( \mathbf{k} \).

2.3 Numerical Results

2.3.1 Homogeneous Model

I first validate the resolution formulas for a homogeneous model (velocity = 2000 m/s) with a source spacing of 20 m and a receiver spacing of 10 m. Sources and receivers are located along the free surface, and there are nine scatterers in the model. The wavenumber illuminations for this model are shown Figure 2.2 and the estimated spatial resolutions are given in Table 2.1 and illustrated in Figure 2.3 as...
Figure 2.2. Wavenumber illumination patterns at the nine scatterer locations in the homogeneous model.

Table 2.1. Estimated spatial resolutions at nine scatterer locations in the homogeneous model for a Ricker source wavelet with a peak frequency of 20 Hz. The values in the parentheses are the horizontal resolution limit $\Delta x$ and the vertical resolution limit $\Delta z$, respectively.

<table>
<thead>
<tr>
<th>Scatterer Position</th>
<th>$x = 2$ km</th>
<th>$x = 8$ km</th>
<th>$x = 14$ km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = 1$ km</td>
<td>(26 m, 25 m)</td>
<td>(27 m, 25 m)</td>
<td>(26 m, 25 m)</td>
</tr>
<tr>
<td>$z = 2$ km</td>
<td>(30 m, 25 m)</td>
<td>(36 m, 25 m)</td>
<td>(30 m, 25 m)</td>
</tr>
<tr>
<td>$z = 3$ km</td>
<td>(36 m, 25 m)</td>
<td>(49 m, 25 m)</td>
<td>(36 m, 25 m)</td>
</tr>
</tbody>
</table>
Figure 2.3. The homogeneous velocity model with resolution bars that estimate the limits of horizontal and vertical resolution at nine different locations in the model.
error bars. According to the wavenumber illumination patterns, it is expected that the horizontal resolution will degrade with depth. The reverse-time migration image of this model is shown in Figure 2.4. Comparing the migration image with the estimated resolutions, it is clear that the spatial resolutions of the migration image at nine scatterer locations are consistent with the estimated resolutions — the vertical resolution limits are about the same and the horizontal resolution limits are decreasing with depth.

### 2.3.2 Smoothed SEG/EAGE Salt Model

In this experiment I validate the resolution formulas for a heterogeneous model which is a smoothed version of the 2D SEG/EAGE salt model (Figure 2.5). The

![Migration Image for Homogeneous Model](image)

**Figure 2.4.** The reverse-time migration image for the homogeneous velocity model.
acquisition geometry for this experiment consists of 201 sources and 806 receivers per shot with a source spacing of 80 m and a receiver spacing of 20 m. Sources and receivers are located along the free surface, and there are nine scatterers in the model. Using the Beylkin’s spatial resolution analysis, the wavenumber illuminations at the nine locations are computed as shown in Figure 2.6, and the resolution limits are estimated as shown in Table 2.2 and illustrated in Figure 2.5 at the scatterers’ locations as the error bars. By using the velocity model in Figure 2.5 as the migration velocity, the reverse-time migration image of this model is obtained as shown in Figure 2.7. Since it is difficult to compare the scatterers’ responses in the migration image with the estimated resolution limits, I plot the migration image along a vertical line at \( x = 8 \) km (Figure 2.8) and along a horizontal line at \( z = 2 \) km (Figure 2.9). Along the vertical line the scatterer at \( z = 2 \) km has the poorest vertical resolution and the scatterer at \( z = 1 \) km has the best vertical resolution. This result is consistent with the vertical resolution limits estimated from the Beylkin’s formula. Similarly, along the horizontal line, the scatterer at \( x = 8 \) km has the poorest resolution which is also consistent with the estimated horizontal resolution limit. The scatterer at the middle of the model (\( x = 8 \) km)
Figure 2.6. Wavenumber illumination patterns at the nine scatterer locations in the smoothed SEG/EAGE salt model. The traveltimes associated with the computed wavenumbers were computed by tracing rays through the salt model.

and $z = 2$ km) has the poorest resolution in both the horizontal and the vertical directions because it is located within the salt body which has a very high velocity value compared to the surrounding sediments. The wavelength of seismic wave is longer in regions with high velocity values than lower-velocity regions. For this heterogeneous model, both horizontal and vertical resolution limits tend to degrade with depth. This is also due to the increasing velocity with depth.
Table 2.2. Estimated spatial resolutions at nine scatterer locations in the smoothed SEG/EAGE salt model for a Ricker source wavelet with a peak frequency of 20 Hz. The values in the parentheses are the horizontal resolution limit $\Delta x$ and the vertical resolution limit $\Delta z$, respectively.

<table>
<thead>
<tr>
<th>Scatterer Position</th>
<th>$x = 2\text{ km}$</th>
<th>$x = 8\text{ km}$</th>
<th>$x = 14\text{ km}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = 1\text{ km}$</td>
<td>(32 m, 33 m)</td>
<td>(18 m, 28 m)</td>
<td>(30 m, 31 m)</td>
</tr>
<tr>
<td>$z = 2\text{ km}$</td>
<td>(39 m, 40 m)</td>
<td>(56 m, 55 m)</td>
<td>(35 m, 32 m)</td>
</tr>
<tr>
<td>$z = 3\text{ km}$</td>
<td>(43 m, 41 m)</td>
<td>(35 m, 32 m)</td>
<td>(46 m, 35 m)</td>
</tr>
</tbody>
</table>

Figure 2.7. Reverse-time migration image for the smoothed salt model with scatterers.
Figure 2.8. The reverse-time migration image line at $x = 8$ km in the 2D SEG/EAGE salt model. The error bars indicate the estimated vertical resolution limits at the scatterers.

Figure 2.9. The reverse-time migration image line at $z = 2$ km in the 2D SEG/EAGE salt model. The error bars indicate the estimated horizontal resolution limits at the scatterers.
CHAPTER 3

CONCLUSIONS

Acoustic waveform tomography is used to invert both 2D synthetic and field data for the velocity models. In the case of 2D synthetic data from the SEG/EAGE salt model, the traveltime velocity model is not a good starting model for waveform inversion as it causes artifacts at the tip of the salt body. Consequently, waveform inversion converged to a local minimum which was inaccurate compared to the true model. To overcome this problem, I discovered that the flooding technique, commonly used in subsalt imaging, can be used to improve the convergence of waveform inversion. Surprisingly, the $v(z)$ velocity model, which is only a 1D velocity profile, when combined with the flooding technique can provide an accurate velocity model by multiscale waveform tomography.

In the marine data case, multiscale waveform tomogram with a dynamic early-arrival muting window successfully inverted the marine data set to obtain a velocity tomogram that is more accurate than the initial model from traveltime tomography. Since in this case, the true velocity structure is not known, the accuracy of the waveform tomogram is assessed by comparing the migration images and common image gathers. The results showed that acoustic waveform tomography can be used to invert these elastic field data. This success is attributed to the fact that marine data are simpler than land data which are usually corrupted by surface waves, strong random noise, and strong elastic effects.

Beylkin’s spatial resolution formulas are validated for both homogeneous and heterogeneous velocity models. For both models, the resolution of reverse-time migration images are consistent with the resolution limits estimated using Beylkin’s resolution analysis with respect to the Rayleigh’s resolution criterion. A long wavelength corresponds to a high velocity value and therefore both vertical and
horizontal resolution limits tend to degrade with depth in the heterogeneous model — deeper regions have higher velocity values.
REFERENCES


